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$K_L \rightarrow \mu^+ \mu^-$, TOP MASS AND BOTTOM LIFETIME

L. Bergström and E. Massó *)
CERN - Geneva

P. Singer +)
CEN - Saclay

and

D. Wyler
CERN - Geneva

A B S T R A C T

We study a class of diagrams, that has previously been overlooked, contributing to the (dispersive) electromagnetic part of the $K_L \rightarrow \mu^+ \mu^-$ amplitude. The strength of this extra contribution can be estimated in different models and could be measured in related processes. By combining these results with those coming from the $K^0 - \bar{K}^0$ transition (evaluated in the Kobayashi-Maskawa scheme) we obtain rather strong constraints on the t quark mass and the B meson lifetime. The recently measured value for the B meson lifetime is difficult to accommodate unless the t quark is very heavy.

*) On leave of absence from Departament de Física Teòrica, Universitat Autònoma de Barcelona, Bellaterra, Spain.

+*) On leave from Technion - Israel Institute of Technology, Haifa, Israel.

The observed suppression of the $K_L \rightarrow \mu^+ \mu^-$ decay and the $K^0 - \bar{K}^0$ transition indicates not only the absence of a direct neutral $|\Delta S| = 1$ weak current, but also that a cancellation mechanism^{1),2)} is at work when higher order interactions effectively induce such currents. Therefore, both processes constrain the parameters of the underlying theory of the weak interactions. In order to calculate the above processes one must, after evaluating the free quark diagrams²⁾⁻⁴⁾, also take into account the strong confining forces which bind the quarks inside the mesons. The K_L vacuum matrix element, relevant for the purely weak part of the $K_L \rightarrow \mu^+ \mu^-$ amplitude, can be reliably estimated and expressed in terms of the kaon decay constant f_K . The hadronic matrix element of the $K^0 - \bar{K}^0$ amplitude, on the other hand, has been a subject of great concern and uncertainty in the literature^{2),4)-7)}. From this it would follow that the $K_L \rightarrow \mu^+ \mu^-$ process would be more suitable for quantitative considerations. However, also in this case, there is a potentially dangerous ambiguity: the magnitude of the dispersive electromagnetic contribution to the decay amplitude. Previously, this was assumed to be small, either ad hoc⁸⁾ or by comparison to $\eta \rightarrow \mu^+ \mu^-$ ⁹⁾, where a recent measurement¹⁰⁾ indicates a small value for the real part of the amplitude^{*)}. But, as recently pointed out¹⁵⁾, the weak Hamiltonian induces a class of diagrams which are absent in $\eta \rightarrow \mu^+ \mu^-$. In the present paper, we study the influence of these on the weak parameters.

To start with, we define a reduced $K_L \rightarrow \mu^+ \mu^-$ amplitude, A , such that^{**)}

$$\Gamma(K_L \rightarrow \mu^+ \mu^-) / \Gamma(K_L \rightarrow \gamma \gamma) = \frac{2\beta}{\pi^2} \left(\frac{m_\mu}{m_K} \right)^2 |A|^2, \quad (1)$$

where $\beta^2 = 1 - 4(m_\mu^2 / m_K^2)$. The imaginary (absorptive) part of the amplitude comes mainly from the $\gamma\gamma$ intermediate state (Fig. 1). It is model-independent and given by¹⁶⁾

$$\text{Im } A = - \frac{e^2}{8\beta} \ln \left(\frac{1+\beta}{1-\beta} \right) \simeq - 3.8 \cdot 10^{-2}, \quad (2)$$

where e is the unit charge. We write the real part as

$$\text{Re } A \equiv A_{\text{e.m.}} + A_w. \quad (3)$$

*) An old measurement¹¹⁾ of the $\eta \rightarrow \mu^+ \mu^-$ decay implied a substantial contribution of the real part of the amplitude. This is inconsistent with several conventional models¹²⁾⁻¹⁴⁾ which agree with the most recent measurements¹⁰⁾.

**) The CP violating component in K_L can be safely neglected.

Here, $A_{e.m.}$ is the real part of the two-photon amplitude (Fig. 1) and A_w stands for additional contributions to the dispersive part of A . Experiment ¹⁷⁾ fixes $|A|$, as given in Eq. (1) :

$$|A|_{exp} = (4.7 \pm 0.5) \cdot 10^{-2}. \quad (4)$$

Since we know the imaginary part of the amplitude, Eq. (2), we can also fix the dispersive part :

$$|Re A|_{exp} = |A_{e.m.} + A_w|_{exp} = (2.8 \pm 0.8) \cdot 10^{-2}. \quad (5)$$

Several authors have used the $K_L \rightarrow \mu^+ \mu^-$ process, together with other information, to bound some definite combination of the KM angles and phases and/or the top quark mass. In order to use (5) one must assume a value for $A_{e.m.}$. In Refs. 9)-10), the approximations discussed before were made. But as we will see now, these are not all justified.

In evaluating $A_{e.m.}$ one assesses first the transition $K_L \rightarrow \gamma^* \gamma^*$; the problem here seems to be the presence of important long-distance contributions ¹⁸⁾⁻²⁰⁾. We therefore adhere to the calculational scheme proposed in Ref. 15), where pole dominance is assumed and where experimental knowledge of several couplings reduces the problem to a calculation of non-leptonic weak meson-meson transitions ¹⁵⁾. The resulting effective diagrams are given in Figs. 2a and 2b. It is evident that diagram 2b has no counterpart in the decay $\eta \rightarrow \mu^+ \mu^-$ and thus destroys the analogy between the two processes.

The contribution of the diagram 2a (virtual photons) to $A_{e.m.}$ involves the transition $P \rightarrow \gamma^* \gamma^*$, where P is a non-strange pseudoscalar meson which is off-mass-shell. This has been recently evaluated in a quark triangle model ¹⁴⁾. The results coincide with the ones obtained using vector dominance (VDM) ¹³⁾. As a consequence, we can estimate reliably the contribution from the graph of Fig. 2a to the $P \rightarrow \ell^+ \ell^-$ decays. For consistency, we work in the long-distance (VDM) approach. The saturation of one virtual photon by a vector meson (corresponding to a $PV\gamma$ vertex) gives

$$A_1^{(a)} = -2.3 \cdot 10^{-2}, \quad (6a)$$

while if one saturates both virtual photons (PVV vertex) one finds

$$A_2^{(a)} = -1.3 \cdot 10^{-2}. \quad (6b)$$

All realistic models for $P \rightarrow \ell^+ \ell^-$ give numbers between (6a) and (6b) ¹²⁾⁻¹⁴⁾. The form factor corresponding to the diagram 2b is given by ^{15),21)}

$$F(s) = \alpha \sqrt{2} e G_F f_{K^*} \frac{m_s^2}{f_{K^*} f_s^2} \left(\frac{1}{1 - s/m_{K^*}^2} \right) \times$$

$$\left\{ \frac{4}{3} - \frac{1}{1 - s/m_s^2} - \frac{1}{9} \left(\frac{1}{1 - s/m_w^2} + \frac{2}{1 - s/m_\varphi^2} \right) \right\}, \quad (7)$$

where s is the squared mass of the photon, coupled to the vector meson. Note $F(0) = 0$, as required by gauge invariance ^{2),15),19),21)}. The parameter α depends on the model used to calculate the transition K^*-V . For instance, in Ref. 15), the weak vector-vector $\Delta S = 1$ transition was evaluated in the KM model, including gluonic corrections. Penguin diagrams are essentially inoperative to enhance it. The result is ¹⁵⁾ $|\alpha| \approx 1.2 \sin \theta_c \cos \theta_c \approx 0.2-0.3$, to be inserted in Eq. (7). Details may be found in Ref. 15). On the other hand, $|\alpha| = 1$, if one uses the assumptions of Sakurai ^{21),22)}. In both cases we see that the previous assertions on $A_{e.m.}$ which correspond to $\alpha = 0$ are not tenable. We will therefore take $|\alpha|$ to be a free parameter in the following, $|\alpha| \lesssim 1$, hoping for its quick experimental determination.

Introducing now the experimental $K^* \rightarrow K \gamma$ rate ²³⁾ in Eq. (7), one finds that the contribution of the graph 2b to $A_{e.m.}$ is ^{*)}

$$A_1^{(b)} = -5.3 \cdot 10^{-2} \times \alpha. \quad (8a)$$

Here, we used $f_{K^*} = f_\rho = 5.09$. We have also evaluated the diagram 2b with a vector meson coupled also to the photon emerging from the $K^* K \gamma^*$ vertex. With minor changes in (7) we find

$$A_2^{(b)} = -4.9 \cdot 10^{-2} \times \alpha. \quad (8b)$$

Notice that the results expressed in Eqs. (6a) and (8a) correspond to diagrams with only one vector meson dominated photon, while Eqs. (6b) and (8b) reflect the saturation of both photons.

*) To our knowledge, the only similar calculation is done in Ref. 21). We were not able to reproduce that result. The discrepancy may be due to the fact that in Ref. 21) the results of Ref. 13) are heavily used. However, in Ref. 13), there is an error in the $PV\gamma$ calculation (Ll. Ametller, A. Bramon, L. Bergström and E. Massó, unpublished).

With this,

$$A_{e.m.} = A^{(a)} + A^{(b)} \quad (9)$$

and thus depends on α . The allowed values for $A_{e.m.}/\text{Im } A$ are shown in Fig. 3a. The boundaries of the band correspond to Eqs. (6a) and (8a), and to Eqs. (6b) and (8b). The maximum value of $A_{e.m.}$, corresponding to constructive interference Eq. (9), is about twice the absorptive part.

As anticipated, the usual assumptions about $A_{e.m.}$ are not justified in general. As seen from Fig. 3b, obtained by combining Eqs. (4), (5) and (9), the ratio $A_W/\text{Im } A$ can be as high as ≈ 3 ^{*)}. Consequently, the results found by using the available $K_L \rightarrow \mu^+ \mu^-$ information might differ appreciably from published results.

Knowledge of α , associated with the strength of the weak transitions between vector mesons would obviously improve the situation. As has been noticed in Ref. 15), the $K_L \rightarrow \mu^+ \mu^- \gamma$ Dalitz decay is very sensitive to α . A good measurement of this decay would therefore determine α and thus improve the usefulness of $K_L \rightarrow \mu^+ \mu^-$ to bound mixing angles and quark masses, to which issue we now turn.

In the three-generation standard model A_W depends on the unknown KM angles θ_2, θ_3 , on the phase δ , and on the top quark mass m_t . Fixing α we get an equation for these parameters. On the other hand, they also enter the calculations ^{4),24)} of the K_L-K_S mass difference and the ϵ parameter of CP violation ^{**)} , and therefore provide independent further equations for the unknown parameters. As mentioned before, the K_L-K_S mass difference and ϵ unfortunately depend strongly on an unknown hadronic matrix element, B . In the recent literature ^{9),26)-28)}, a quite small value $B \sim 0.3-0.4$ has been used ; this value being favoured by a PCAC ⁶⁾ and chiral perturbation analysis ²⁸⁾. However, whether the relative consensus about this small value stems from the belief that it is correct or from the fact that it tends to give stronger (and hence more interesting) bounds on the parameters is difficult to judge. Nevertheless, we will also take this optimistic stand and mainly present results for $B \sim 0.3-0.4$ and only occasionally mention how the picture is changed if, for example, the vacuum insertion value ²⁾ $B = 1$ is used. With this proviso, the three equations then restrict the allowed

^{*)} Errors in (4) are taken into account.

^{**)} We do not consider here ϵ' , since its evaluation ²⁵⁾ is made difficult because of Penguin graphs.

values of θ_2 , θ_3 , δ and m_t quite considerably. A new ingredient in our analysis is that we use them also to predict the bottom lifetime.

We define $\lambda_i = U_{is}^* U_{id}$, where U_{ij} are Kobayashi-Maskawa matrix elements, and hence

$$\begin{aligned} \operatorname{Re} \lambda_c &= -s_1 c_2 (c_1 c_2 c_3 - s_2 s_3 c_\delta) \\ \operatorname{Im} \lambda_c &= -\operatorname{Im} \lambda_t = s_1 s_2 s_3 c_2 s_\delta \\ \operatorname{Re} \lambda_t &= -s_1 s_2 (c_1 s_2 c_3 + c_2 s_3 c_\delta), \end{aligned} \quad (10)$$

where $s_i = \sin \theta_i$, etc.

Also needed are the usual functions ⁴⁾ related to the $K^0 - \bar{K}^0$ box diagram

$$B(x) = x \left(\frac{1}{4} + \frac{9}{4(1-x)} - \frac{3}{2(1-x)^2} \right) + \frac{3}{2} \left(\frac{x}{x-1} \right)^3 \ln x \quad (11)$$

$$\begin{aligned} \bar{B}(x, \gamma) &= \frac{xy}{y-x} \left\{ \left(\frac{1}{4} + \frac{3}{2} \frac{1}{(1-\gamma)^2} - \frac{3}{4} \frac{1}{(1-\gamma)^2} \right) \ln \gamma - (x \leftrightarrow \gamma) \right. \\ &\quad \left. - \frac{3}{4} \frac{1}{(1-x)(1-\gamma)} \right\}, \end{aligned} \quad (12)$$

and to the weak $K_L \rightarrow \mu^+ \mu^-$ diagrams

$$G(x) = \frac{3}{4} \left(\frac{x}{x-1} \right)^2 \ln x + \frac{x}{4} + \frac{3}{4} \frac{x}{1-x}. \quad (13)$$

We then have, for $\Delta m = m_{K_L} - m_{K_S}$, ϵ and $K \rightarrow \mu^+ \mu^-$, respectively

$$\frac{\Delta m}{m_K} = B \frac{G_F^2}{6\pi^2} m_w^2 f_K^2 F(x_c, x_t, \theta_i, \delta) = 0.71 \cdot 10^{-14}, \quad (14)$$

$$\epsilon = B \frac{G_F^2}{6\pi^2 \sqrt{2}} m_w^2 f_K^2 \left(\frac{m_K}{\Delta m} \right) D(x_c, x_t, \theta_i, \delta) e^{i\frac{\pi}{4}} = 2.27 \cdot 10^{-3} e^{i\frac{\pi}{4}}, \quad (15)$$

$$|\operatorname{Re} \lambda_t| G(x_t) \cdot \eta = 5.1 \cdot 10^{-2} \cos \theta_3 |A_w|, \quad (16)$$

where $x_i = m_i^2/m_W^2$,

$$\begin{aligned} F &= ((\operatorname{Re} \lambda_c)^2 - (\operatorname{Im} \lambda_c)^2) B(x_c) \eta_1 + ((\operatorname{Re} \lambda_t)^2 - (\operatorname{Im} \lambda_t)^2) B(x_t) \eta_2 \\ &\quad + 2(\operatorname{Re} \lambda_c \operatorname{Re} \lambda_t - \operatorname{Im} \lambda_c \operatorname{Im} \lambda_t) \bar{B}(x_t, x_c) \eta_3, \end{aligned} \quad (17)$$

$$D = (\text{Re}\lambda_c \text{Im}\lambda_c) B(x_c) \eta_1 + (\text{Re}\lambda_t \text{Im}\lambda_t) B(x_t) \eta_2 \\ + (\text{Re}\lambda_c \text{Im}\lambda_t + \text{Re}\lambda_t \text{Im}\lambda_c) \bar{B}(x_t, x_c) \eta_3. \quad (18)$$

Finally, the bottom lifetime is ²⁷⁾⁻²⁹⁾

$$\tau_B \approx 0.41 \cdot 10^{-14} \left\{ |U_{bc}|^2 + 0.1 \sin^2 \theta_3 \right\}^{-1} \text{ sec.} \quad (19)$$

Above, the $\eta_1, \eta_2, \eta_3, \eta$ are QCD correction factors calculated by the renormalization group technique ^{3),24)}. We have used the values $\eta_1 \approx 0.7, \eta_2 \approx 0.6, \eta_3 \approx 0.4, \eta \approx 1$, corresponding to a value of $\Lambda_{\text{QCD}} \approx 100$ MeV. For our evaluations we further took $m_c = 1.4$ GeV, $m_b = 4.6$ GeV and $\sin\theta_1 = 0.23$.

The procedure is now as follows. First, θ_2, θ_3 are bound by $0.02 \leq \theta_2 \leq 0.3, 0 \leq \theta_3 \leq 0.18$ according to a recent analysis ³⁰⁾. For given values of θ_2, θ_3 in this range and all δ , (14) is solved for m_t . If a solution exists, the set $\theta_2, \theta_3, \delta$ and m_t is inserted into (16) and is accepted as a solution if (16) is satisfied for an R_W lying within the bands of Fig. 3b. Finally, consistency with (15) is checked. For those $\theta_1, \theta_2, \delta, m_t$ which survive these constraints, τ_B is calculated [Eq. (19)]. In this way we obtain, by scanning the whole allowed range of the angles, a lower and an upper bound on m_t . The result is shown in Figs. 4a and 4b as a function of $R \equiv \Gamma(K_L^+ e^+ e^- \gamma) / \Gamma(K_L^+ \mu^+ \mu^- \gamma)$ and, equivalently ¹⁵⁾, of α , for $B = 0.42$ (bag model ⁵⁾) and $B = 0.33$ (PCAC ⁶⁾). Note that the same value of m_t can be an upper or a lower bound, depending on α . For instance $m_t \approx 40$ is an upper bound for $\alpha = 0$ ⁸⁾, but for a reasonable value of $\alpha \approx 0.45$ it is a lower bound. This underscores the importance of α . Finally, we have considered the relation between τ_B and m_t , for all the allowed angles using $B = 0.42$ and $B = 0.33$. The result is shown in Fig. 5 ; as is seen, a quite small region in the m_t - τ_B plane is allowed ^{*)}. An interesting result is that the lifetime of the B meson is rather short, between $(0.2-1.5) \times 10^{-13}$ sec. This reflects the fact that, in order to satisfy (16), $\sin\theta_2$ cannot be too small ; a lower bound is $\theta_2 \geq 0.08$, for the presumably too high values $\alpha > 0.6, B = 1$. For the favoured values $B = 0.42, 0.33$ the bound is much stronger ; we find $\theta_2 > 0.2$ for any α . In order to get a small enough ϵ , δ must then be very close to zero, or π ; only if $s_3 \approx 0, \delta$ can be larger.

*) m_t and τ_B are not strongly correlated only for large values of B , e.g., $B = 1$.

Finally, we have to discuss the uncertainties in our analysis. One, which fortunately can be eliminated through experimental measurements, concerns the value of α . In a quark model analysis, which is really our framework here, a value of $|\alpha| \approx 0.25$ is obtained¹⁵⁾. As can be seen in Figs. 4a and 4b, the bound $20 \lesssim m_t \lesssim 50$ GeV then follows. (By simply and erroneously neglecting the contribution, i.e., by putting $\alpha = 0$ like in all previous analyses, one would get the more narrow bound $20 \lesssim m_t \lesssim 36$ GeV.) However, there are as yet no data on non-leptonic weak transitions that exclude, e.g., the Sakurai model²²⁾ which corresponds to $|\alpha| = 1$. Given this uncertainty of α , at present only the rather weak bound $20 \lesssim m_t \lesssim 80$ GeV can really be given. We can only stress once again the importance of an experimental measurement of $\Gamma(K_L \rightarrow \mu^+ \mu^- \gamma)$, which seems to be the cleanest process to determine α .

The theoretical uncertainties are more troublesome. The main problem is the B factor which unfortunately also influences the results very strongly. A new determination, e.g., using lattice methods, would be very welcome. On the other hand, we see from Fig. 5 that a B meson lifetime greater than, say, $2 \cdot 10^{-13}$ sec and/or a t quark mass greater than 80 GeV would not be consistent with the small value $B \sim 0.3-0.4$ used in this work. In fact, a B lifetime as long as $\sim 10^{-12}$ sec, which has been indicated by two very recent SLAC experiments³¹⁾, is on the edge of making the whole scheme inconsistent. We find that such values of τ_B are only possible for a B factor in the high (and narrow) range $1.4 \lesssim B \lesssim 2$; furthermore, a quite heavy t quark is required, $m_t \gtrsim 100$ GeV. This is in itself an interesting result, but owing to the large experimental errors³¹⁾ and bearing in mind the complicated history of the charmed meson lifetimes, one should perhaps wait for more accurate data before drawing any firm conclusions.

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FIGURE CAPTIONS

- Fig. 1 General two-photon graph contributing to $A_{e.m.}$.
- Fig. 2 Pole diagrams for the $K_L \rightarrow \gamma^* \gamma^*$ form factor. The weak $\Delta S = 1$ transition occurs either on the mesonic leg (a) or on the photon leg (b).
- Fig. 3 (a) Allowed band for the $A_{e.m.}$ contribution (normalized to the absorptive part of the amplitude) as a function of the parameter α .
(b) Allowed bands for the non-photon contribution to the $K_L \rightarrow \mu^+ \mu^-$ amplitude as a function of α . The over-all unknown sign of $\text{Re } A$ gives rise to the two bands.
- Fig. 4 Allowed bands for the top-quark mass as a function of α and of the $\Gamma(K_L \rightarrow e^+ e^- \gamma) / (K_L \rightarrow \mu^+ \mu^- \gamma)$ ratio (see text). The differently hatched areas come from the two separate bands shown in Fig. 3(b). The value used for B is 0.42 (a), and 0.33 (b), corresponding to the bag model and PCAC calculations, respectively.
- Fig. 5 Correlation between the top quark mass and the B meson lifetime as emerging from our analysis.

Fig,1

Fig,2a

Fig,2b

Fig.3a

Fig.3b

Fig.4a $\Gamma(K_L \rightarrow e^+e^-\gamma) / \Gamma(K_L \rightarrow \mu^+\mu^-\gamma)$

Fig.4b $\Gamma(K_L \rightarrow e^+e^-\gamma) / \Gamma(K_L \rightarrow \mu^+\mu^-\gamma)$

Fig.5