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THE FUNDAMENTAL SU(5) MONOPOLE :
CLASSICAL CONFIGURATION AND MASS *)

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ABSTRACT

We summarize a numerical analysis of the Yang-Mills equations corresponding to the fundamental monopole in SU(5) broken spontaneously by the usual 24-plet and 5-plet of scalars.

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Dedicated to Yuri Orlov.

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In 1979 Dokos and Tomaras¹⁾ have minimally extended the 't Hooft-Polyakov monopole²⁾ from SU(2) to the SU(5) algebra of Georgi and Glashow's unified model³⁾ of strong and electroweak interactions. Recently, this so-called fundamental SU(5) monopole has attracted much attention after Rubakov's discovery⁴⁾ that, if existent, it might catalyze nucleon decay with cross-sections typical for strong interactions. Let us recall that 't Hooft-Polyakov type monopoles are not "elementary" particles like Dirac monopoles, but rather complicated field configurations of gauge bosons and Higgs scalars characterized by finite (calculable) energy interpreted as monopole mass.

In the following we summarize in a - we hope - convenient form the major results of a numerical analysis devoted to the classical configuration and mass of the fundamental SU(5) monopole⁵⁾.

Our starting point is Georgi and Glashow's Lagrangian without fermions.

$$\mathcal{L} = -\frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \text{Tr} D_\mu \phi D^\mu \phi + (D_\mu H)^\dagger (D^\mu H) - V(\phi, H) \quad (1)$$

where $F_{\mu\nu} := \partial_\mu W_\nu - \partial_\nu W_\mu + ig [W_\mu, W_\nu]$, $W_\mu \in \underline{24}$

$$D_\mu \phi := \partial_\mu \phi + ig [W_\mu, \phi], \quad \phi \in \underline{24}$$

$$D_\mu H := \partial_\mu H + ig W_\mu \phi, \quad H \in \underline{5}$$

The scalar potential breaking SU(5) down to $SU(3)_c \times U(1)_{\text{e.m.}}$ is given by Magg and Shafi

$$\begin{aligned} V(\phi, H) := & -\frac{1}{2} \mu^2 \text{Tr} \phi^2 + \frac{a}{4} (\text{Tr} \phi^2)^2 + \frac{b}{2} \text{Tr} \phi^4 \\ & - \frac{1}{2} v^2 H^\dagger H + \frac{\lambda}{4} (H^\dagger H)^2 \\ & + \alpha (H^\dagger H) \text{Tr} \phi^2 + \beta H^\dagger \phi^2 H + C \end{aligned} \quad (2)$$

where the constant C is chosen such that the potential vanishes at its minimum.

The fundamental SU(5) monopole corresponds to the following static ansatz with maximal symmetry and topologically non-trivial asymptotic behaviour¹⁾:

$$W_0(\vec{x}) = 0 \quad (3)$$

$$\vec{W}(\vec{x}) = \frac{k(r)-1}{2gr} \vec{e}_r \times \begin{pmatrix} 0 \\ 0 \\ \vec{\tau} \\ 0 \end{pmatrix} \quad (4)$$

$$\begin{aligned} \phi(\vec{x}) = \frac{1}{g} & \begin{pmatrix} f_1(r) \\ f_1(r) \\ f_2(r) \mathbb{1}_2 \\ -2(f_1(r) + f_2(r)) \end{pmatrix} \\ & + \frac{f_3(r)}{g} \vec{e}_r \cdot \begin{pmatrix} 0 \\ 0 \\ \vec{\tau} \\ 0 \end{pmatrix} \end{aligned} \quad (5)$$

$$H(\vec{r}) = \frac{h(r)}{g} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad (6)$$

Note that the scalar 5-plet H is not twisted.

A first question comes to mind here: has the monopole mass,

$$m_M := - \int d^3x \mathcal{L}(x) \quad (7)$$

considered as a functional of K , ϕ_i and H , a finite local minimum? Or asked differently: do the corresponding Euler-Lagrange equations (which we do not write here), with boundary conditions given below, admit solutions? The existence of such solutions has been shown in the limit of vanishing potential⁷⁾ (BPS limit)

$$K(0) = 1$$

$$\lim_{r \rightarrow \infty} K(r) = 0$$

$$p_3(0) = 0$$

$$\lim_{r \rightarrow \infty} p_3(r) = \left(\frac{5}{4} - \frac{\epsilon}{2}\right) g v$$

$$\lim_{r \rightarrow \infty} p_1(r) = g v$$

$$\lim_{r \rightarrow \infty} p_2(r) = \left(-\frac{1}{4} + \frac{\epsilon}{2}\right) g v \quad (8)$$

$$\lim_{r \rightarrow \infty} h(r) = g x$$

v and x are the vacuum expectation scales of the 24-plet and 5-plet of scalars. They are, as well as ϵ , functions of the parameters in the potential⁶⁾.

A second question is that of uniqueness: is there more than one finite local minimum? If so, then they would be interpreted as excited monopole states. To our knowledge, this is still an open question. In the case of $SU(2)$, in the BPS limit, Maison⁸⁾ has shown that there is only one solution. However, Kerner⁹⁾ claims the existence of a one-parameter-family of solutions and gives their asymptotic behaviour.

We have solved numerically the differential equations described above. Our method consists of dividing the integration domain in three parts:

- small r : the Euler-Lagrange equations have a weak singularity at the origin, which we avoid by an asymptotic expansion;

- intermediate r : here we use the Runge-Kutta method of order 7 proposed by Fehlberg¹⁰⁾;

- large r : we use again an asymptotic expansion. In order to satisfy the boundary conditions both at $r = 0$ and $r \rightarrow \infty$ we use Newton's method to adjust the initial values and slopes of the functions at the origin.

Based on some 50 runs with different sets of parameters $a, b, \alpha, \beta, \lambda, x/v$ we observe:

- existence: our procedure always yields a stable numerical solution indicating an affirmative answer to the first question. The lower and upper bounds for the monopole mass given by Scott¹¹⁾ and Schellekens and Zachos¹²⁾

$$\frac{m_X}{g^2/4\pi} \leq m_M \leq 1.787 \frac{m_X}{g^2/4\pi} \quad (9)$$

are respected by our solutions. A typical one is shown in the figure. Concerning the second question, we only have weak evidence for uniqueness: in a few cases we tried to get a second solution by varying the initial values in a rather large domain - in vain.

- monotony of K , ϕ_i and h is always observed;
- weak dependence of the solutions on x : by keeping $a, b, \alpha, \beta, \lambda$ fixed and varying x/v , we found a non-vanishing but extremely small dependence of the solutions on the untwisted vacuum expectation of the 5-plet. Upon variations of x/v from 0 to 1, the monopole mass is a decreasing function and changes by less than 5%.

According to the gauge hierarchy x/v has an extremely small value $\sim 10^{-12}$ and we can safely put $x = 0$. This implies that h is identically zero

$$h \equiv 0 \quad (10)$$

and the parameters α, β and λ drop out.

In particular, at this level we do not see the electroweak barrier, which could suppress the Rubakov effect, as suggested by Grossman, Lazarides and Sanda¹³⁾.

For the two remaining parameters a and b we introduce polar co-ordinates:

$$b =: \eta \cos \theta \quad a =: \eta \sin \theta \quad (11)$$

with

$$\eta > 0 \quad \text{and} \quad -\frac{7}{15} < \tan \theta \leq \infty \quad (12)$$

such that the potential breaks $SU(5)$ to $SU(3)_C \times SU(2)_L \times U(1)$ ⁶⁾ and is positive¹⁴⁾. Along the same lines as for the $SU(2)$ case¹⁵⁾, we can now show analytically that for any θ subject to the above restriction (12), the monopole mass is a differentiable, strictly increasing function of η . Furthermore, its lower bound is saturated in the limit $\eta \rightarrow 0$ ⁷⁾ with the well-known BPS solution

$$K = \frac{5}{2} \frac{g v r}{\sinh(\frac{5}{2} g v r)} \quad , \quad P_3 = \frac{5}{4} g v \coth(\frac{5}{2} g v r) - \frac{1}{2 r} \quad (13)$$

with ϕ_1 and ϕ_2 identically equal to their vacuum expectation:

$$p_1 \equiv g v \quad , \quad p_2 \equiv -\frac{1}{4} g v \quad (14)$$

On the other hand, our numerical analysis also shows that the upper bound (12) is saturated in the limit $\eta \rightarrow \infty$, the solutions tending to

$$p_1 \equiv g v \quad , \quad p_2 \equiv -\frac{1}{4} g v \quad , \quad r > 0 \quad (15)$$

$$p_3 \equiv \frac{5}{4} g v \quad \text{for } r > 0 \quad , \quad p_3(0) = 0 \quad (16)$$

and K solution of

$$r^2 K'' = K(K^2 - 1) + \frac{25}{4} g^2 v^2 r^2 K^2 \quad (17)$$

The discontinuity of ϕ_1 at the origin is harmless. The analogous result for $SU(2)$ was already found by Kirkman and Zachos¹⁵⁾ using the numerical solutions of Bogomolnyi and Marinov¹⁶⁾.

Finally, we note that in the general case $h \neq 0$, the dependence of the monopole mass on σ decreases with η indicating that the upper bound is still saturated.

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$\begin{array}{c} b/g^2 \\ \tan \theta \end{array}$	0.1	1	10	100	1000
-0.3	1.061	1.154	1.310	1.509	
-0.1	1.071	1.172	1.337	1.532	
-0.01	1.074	1.178	1.343	1.537	1.702
0.	1.074	1.178	1.344	1.537	1.702
0.01	1.074	1.179	1.345	1.537	1.703
0.1	1.077	1.183	1.349	1.541	
1.	1.092	1.205	1.370	1.556	
10.	1.126	1.232			

Table caption: The classical monopole mass in units of m_X/α for several values of b and θ , which are the only relevant parameters in the limit of small weak scale.

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FIGURE CAPTION

A typical solution of the field equations for $a = -3g^2$, $b = 10g^2$, $\alpha = 0$,
 $\beta = -2g^2$, $\lambda = 3g^2$ and $x^2 = 0.1v^2$. $\hat{\phi}_i = \phi_i/(gv)$, $i = 1,2,3$ and $\hat{\phi}_4 = h/(gx)$.