NON-INTERLEAVED SEXTUPOLE SCHEMES FOR LEP
AND RELATED EFFECTS OF SYSTEMATIC MULTPOLE COMPONENTS IN THE LATTICE MAGNETS

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Summary

Non-interleaved sextupole arrangements (i.e. sextupole families free from geometric aberrations) have been used for the chromaticity correction of LEP. It is shown here that care had to be taken for optimizing the sextupole arrangement. Under this condition, the performance obtained was substantially better than that of the standard interleaved scheme. On the other hand, the perturbation of the non-interleaved scheme by undesired multipoles helped to specify tolerances on the field quality of the LEP cell magnets.

Introduction

Non-interleaved sextupole schemes have been proposed\(^1\) for the chromaticity correction of large electron machines such as LEP. In such schemes, the sextupoles are grouped into pairs of identical elements so that firstly only linear components sit between the sextupoles of each pair and secondly the betatron phase advances in a pair are \(\pi\). Under these conditions, the non-linear motion is exactly localized inside the pairs and the complete machine behaves like a linear machine, i.e. betatron oscillations with any amplitude are stable.

In an actual electron machine, the synchrotron oscillations produce an uninterrupted change of the particle momentum so that the phase advance between the sextupoles of each pair is practically never \(\pi\) and the above nice property of the non-interleaved scheme breaks down, i.e. the betatron amplitudes associated with synchrotron motion are limited.

A method has recently been proposed\(^2\) for determining the influence of this limitation on the luminosity of the machine. In the light of this analysis, which will be shortly recalled below, it is shown in this paper that different non-interleaved schemes may lead to considerably different performance. On the other hand, the reduction of performance associated with the introduction of perturbing multipole components can be estimated, which is useful for tolerance specifications.

1. Analysis\(^2\) of performance limitation due to the stability limits

The stability limit of the non-linear betatron motion is a geometrical property of a given machine. It can be conveniently expressed as a maximum stable emittance \(\varepsilon(\delta)\) which is a function of the amplitude of the synchrotron oscillation \(\delta\). For our purpose, this is restricted to the case where the vertical emittance is equal to half the horizontal emittance, which will be referred to as the emittance in what follows. The maximum stable emittance is that of a particle whose transverse coordinates remain bounded over a certain number of turns. In our case, the trajectories are tracked for 400 turns with the program PATRICIA\(^3\).

As the actual standard deviations of the transverse and longitudinal particle distributions are functions of the beam energy and the damping partition numbers, the latter can be adjusted as a function of the energy \(E\) so that an arbitrary number \(F_1\) of standard transverse emittance \(\varepsilon_x\) is linked to an arbitrary number \(F_2\) of relative standard energy deviation \(\sigma_E/E\) via the function \(\varepsilon(\delta)\) by:

\[
F_1 \sqrt{\varepsilon_x} = \varepsilon \left( F_2 \frac{\sigma_E}{E} \right).
\]

It is thus possible to determine the emittance at each energy, but the analytical computation is only possible when the function \(\varepsilon(\delta)\) can be obtained explicitly\(^2\) (this is the case when the function \(\varepsilon(\delta)\) is linearized). If we impose a beam-beam tune shift \((\Delta Q_{BB}, \gamma = 0.03\) in what follows\) and an optimum vertical emittance, this enables to compute the luminosity. Below, this computation is made with the program HARMON\(^7\). It is important to note that the energy defined by eq. (1) has a maximum value for a certain \(J_{x\text{km}}\), and that \(J_x\) must be in the range \(0.5 \leq J_x \leq 2.5\). A certain range of energy standard deviations corresponds to the interval \((0.5, J_{x\text{km}})\) which in turn defines a useful range of the function \(\varepsilon(\delta)\) on the stability limits. Moreover, for energies corresponding to \(J_x\) values smaller than 0.5, \(J_x\) is in fact maintained equal to 0.5 and the luminosity is scaled as \(E^2\), which implies a constant emittance\(^2\).

2. Two non-interleaved schemes for LEP II

(Version 115, 90° per cell, \(\delta x^{1,2} = 0.1 - 0.2\), \(\delta h^1,2 = 1.6 - 3.2\))

2.1 Scheme A

This scheme was so designed\(^6\) as to fill all possible sextupole places starting from the first place close to the dispersion suppressor. A half-octant is shown in Fig. 1 (LEP has four superperiods, each being divided into two symmetrical octants).

![Diagram of Scheme A](image)

F = horizontal focusing
D = vertical focusing

Fig. 1 Non-Interleaved Sextupole Scheme A

The same situation occurs in the other half-octant, which has a different low-B\(^x\) insertion. The sextupole strengths are computed with the program HARMON\(^7\) by half-octant in order to have minimum derivatives of \(B\) with respect to momentum at the middle of the octant and at the crossings. It should be noted that the vertical chromaticity is the most critical one because of the small value of \(\delta x^y\) at the interaction point. As a consequence, the D-sextupoles are at least twice as strong as the F-sextupoles and influence to a great extent the non-linear motion.

For scheme A, the strength of the pair SN2 is large: 1.1 m\(^{-3}\) (length 0.76 m) compared to the value of 0.562 m\(^{-3}\) required for having \(Q_{rh} = 0\) with two sextupole families in total.

This large strength results in a quick decrease of the stable emittance when increasing the amplitude of synchrotron oscillation (Fig. 1). Following the analy-
mix of Chapter 1, the luminosity curve shown in Fig. 4 is obtained. The stability limits and the associated luminosity curve of the standard interleaved scheme are given for comparison in Figs. 3 and 4. It should be noted that for this case, the horizontal non-interleaved scheme is restricted to its useful range as defined in Chapter 1.

2.2 Scheme B

A second non-interleaved sextupole scheme was made by moving the SDI pair close to the middle of the octant by one cell so that it becomes a SDI pair (Fig. 2).

Thus, the phase advance between this new SDI pair and the SDI pair in the middle of the octant is 4x. The SP arrangement has not been modified. The strength of the SDI's becomes 0.43 m-3 which results in stability limits larger than those of scheme A (Fig. 3). The consequence of this increase on the maximum operating energy and on the luminosity is large as can be seen in Fig. 4. Indeed, the maximum energy (Chapter 1) scales as a straight line with a constant slope, and is almost the case at these particular maximum energies.

If a horizontally focusing sextupole component of the same value is introduced inside all the sextupole pairs, the maximum energy is reduced by 1% and the luminosity by 5% in the overlapping energy range (Fig. 7).

3.2 Octupoles

If a horizontally focusing octupole component of integrated value 0.12 m-3 is introduced inside all the sextupole pairs, the maximum energy is reduced by 1% and the luminosity by 5% in the overlapping energy range (Fig. 7).

References

10. G. Guignard, Tolerances for the magnetic elements of the LEP lattice, Paper presented at this Conference.
of Chapter 1, the luminosity curve shown in Fig. 4 is obtained. The stability limits and the associated luminosity curve of the standard interleaved scheme are given for comparison in Figs. 3 and 4. It should be noted that for this case, the tolerances of the function (6) is restricted to its useful range as defined in Chapter 1.

2.2 Scheme B

A second non-interleaved sextupole scheme was made6 by moving the 8D1 pair close to the middle of the octant by one cell so that it becomes a 8D2 pair (Fig. 2).

The SF-arrangement has not been modified. The strength of the maximum increase on the maximum energy (Chapter 1) scales presented as a straight line with a constant slope, and the luminosity scales with $E^4$ for a constant $\propto$, which is almost the case at these particular maximum energies. An example of an SD-pair with the non-excited sextupole, it is very easy to use the sextupoles which are inserted by moving the octant by one cell so that it becomes a 8D2 pair (Fig. 2).

Thus, the phase advance between this new 8D2 pair and the 8D1 pair in the middle of the octant is 45. The 8P-arrangement has not been modified. The strength of the 8P1 octupole becomes 0.63 m x which results in stability limits longer than those of scheme A (Fig. 3). The consequence of this increase on the maximum operating energy and on the luminosity is large as can be seen in Fig. 4. Indeed, the maximum energy (Chapter 1) scales with the abscissa for which $c(\xi) = 0$ if $c(\xi)$ is represented as a straight line with a constant slope, and the luminosity scales with $\propto$ for a constant $\propto$, which is almost the case at those particular maximum energies.

Fig. 7 Non-Interleaved Sextupole Scheme B

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