NUCLEAR SPIN/ISOSPIN RESPONSE FUNCTIONS

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INTRODUCTION

In these lectures I will talk about the response function of infinite nuclear matter to a spin/isospin perturbation.

Why infinite nuclear matter? This is a theoretical concept "invented to escape the difficulties of finite systems". Why not deal directly with finite systems? The main reason is the tremendous simplification introduced by the suppression of the boundaries. But this would not be an argument if we did not have some reasons to believe that this description has something to do with reality. Let me illustrate the motivations of this belief by a specific example.

In Fig. 1 is plotted the experimental value of the magnetic susceptibility per nucleon as a function of the momentum. The susceptibility is the real part of the polarization propagator; we will come back to it. It has been measured in ee' scattering. In the long wavelength limit $q = 0$ the measurements have been performed for a variety of nuclei [1]. This is the region of the discrete states. Their individual contributions have been added to extract the susceptibility. The points fluctuate wildly; the spin-saturated nuclei fall below the cases where the spin-orbit partner is empty. Then, there is a gap in the measurement up to $q = 1 \text{ fm}^{-1}$, for which there exist data [2,3] for two elements, ^{12}C and ^{56}Fe . They have been extracted from the data on the transverse response, itself obtained by a Rosenbluth plot in ee' scattering. The transverse response is essentially magnetic and we have deduced from the data on the quasi-elastic peak the magnetic susceptibility [5]. Remarkably, the points for ^{12}C and Fe fall on the same line.

In the same figure the susceptibilities of infinite nuclear matter are shown, without interactions and with interactions. In the long wavelength limit the measured values have nothing to do with nuclear matter. Indeed at $q = 0$ one probes the surface of the nucleus which is sensitive to the details of the shell structure. As the momentum increases one penetrates more and more deeply inside the system; the surface becomes less important until one probes the bulk properties of nuclear matter. The transition between the two regimes occurs when the product qR of the momentum and the nuclear radius becomes large. This is what is suggested by these curves, although one would like to see the universality established by a few more cases.

Fig. 1. The susceptibility per proton in units of $[2\mu_0^2(\mu_p - \mu_n)^2]^{-1} \text{ MeV}^{-1}$. The experimental points at finite q in ^{56}Fe [bars, from Ref. 3)] and in ^{12}C [crosses, from Ref. 2)] are displayed, as well as the $q = 0$ values for ^6Li (O), ^{28}Si ($\square$), ^{90}Zr ($\bullet$) and ^{208}Pb ($\blacktriangle$) taken from Ref. 1. In the figure $k_F = 1.15 \text{ fm}^{-1}$.

These lectures are organized as follows. Section 1 introduces general features of the response functions and its collective behaviour in the random phase approximation (RPA).

Section 2 is devoted to the particle hole (ph) force, with some interpretations of its short-range phenomenological piece. Its momentum dependence is discussed using an extension to virtual particles of a theorem by Beg.

In Section 3 we describe in the RPA framework the collective features of the response first at low momenta, where a collective state appears. Particular emphasis is given to the role of the nucleonic excitations. Then at moderate momenta for the spin transverse response, we make a comparison with the ee' experimental data.

Section 4 extends the previous RPA description to incorporate the 2p2h excitations. We focus our attention on the case of deep inelastic electron scattering and on the case of pion absorption. We establish a link between these two processes. We show that the existence of the vertex renormalizations of the Lorentz-Lorentz (LL) type makes this link rather subtle. Another link concerns the p-wave π absorption and the Gamow-Teller (GT) strength near the pion threshold.

Section 5 is concerned with the collective aspects of the spin longitudinal response. We discuss the appearance of precursors of pion condensation and we also study briefly the non-static collective state, i.e. the pionic branch. In Section 6 various sum rules are discussed. It is shown how they are affected by the collective features of the response.

Finally, in Section 7 two applications of our results on the response functions are made. One concerns the nucleon structure functions as measured in deep inelastic μ scattering when the nucleon is immersed in the nucleus. The other is the problem of the missing strength in the GT sum rule and the role of the Δ .

1. NUCLEAR RESPONSE

The response of a system to an external potential U acting, not too strongly, at the point x and at the time t , and thus inducing a fluctuation in density, is given, in the frame of the linear approximation, by

$$\delta n = \int_{-\infty}^{\infty} dt' \int dx' \pi(x, t; x', t') U(x', t'), \quad (1)$$

where π is the polarization propagator.

In infinite nuclear matter where the translational invariance of space holds, in addition to that of time,

$$\pi(x, t; x', t') = \pi(x - x', t - t'). \quad (2)$$

It is therefore convenient to work in momentum space dealing with $\pi(q, \omega)$, the Fourier transform of (2). The relation (1) is then expressed as

$$\delta n(\vec{q}, \omega) = \pi(\vec{q}, \omega) U(\vec{q}, \omega), \quad (3)$$

which shows that the system responds at the same wavelength and frequency as the perturbation.

Causality implies an important relation between the real and the imaginary part of π . Indeed if the potential U starts developing at the time t_0 , the requirement that no response should appear before t_0 implies the following dispersion relation

$$\text{Re } \pi(\vec{q}, \omega) = \frac{1}{\pi} P \int d\omega' \frac{\text{Im } \pi(\vec{q}, \omega')}{(\omega' - \omega)}, \quad (4)$$

where P indicates the principal value of the integral.

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Many of the most relevant properties of a many-body system are embodied in the response functions $\pi(\vec{q}, \omega)$ to the different types of excitations. This can be inferred from the spectral representation of π , which is (in symmetric nuclear matter):

$$\pi(\vec{q}, \omega) = \frac{1}{V} \sum_{n \neq 0} \langle 0 | \tilde{n}^+(-q) | n \rangle \langle n | \tilde{n}(-q) | 0 \rangle \left[\frac{1}{\omega - E_n + E_0 + i\epsilon} - \frac{1}{\omega + E_n - E_0 - i\epsilon} \right], \quad (5)$$

where V is a large volume enclosing the system, which has excited states n with an energy E_n and $\tilde{n}(-q)$ is the one-body operator corresponding to the observable l , which in the case that we are interested in is the spin/isospin density fluctuation.

One clearly sees in the expression (5) that the singularities in the complex ω plane are located at the excitation energies of the system. Furthermore, the imaginary part of Π

$$\text{Im } \Pi(\vec{q}, \omega) = -\frac{\pi}{V} \sum_{n \neq 0} \langle 0 | \tilde{n}^+(-q) | n \rangle \langle n | \tilde{n}(-q) | 0 \rangle \delta[\omega - (E_n - E_0)] \quad (6)$$

is manifestly proportional to the inelastic cross-section for the scattering of a probe which couples to these density fluctuations. It is precisely this quantity which, but for some trivial factors, is commonly referred to as the response function or the correlation function of the system.

From now on I will specialize in the spin/isospin density fluctuations. These can be induced by a variety of fields. Some examples are the isovector piece of the electromagnetic (e.m.) current

$$J^{\text{em}} = \mu_0 \mu_V (\vec{\sigma} \times \vec{q}) \frac{\tau_3}{2} \quad \text{with} \quad \mu_0 = \frac{e}{2m_N}, \quad \mu_V = \mu_p + \mu_n = 4.71, \quad (7)$$

the photoproduction process of charged pions

$$T^\pm = \mp ie \frac{g_r}{\sqrt{2} m_N} \vec{\sigma} \cdot \vec{\epsilon} \tau^\pm, \quad (8)$$

and an inelastic $(\pi^\pm, \pi^0)$ process, governed by the spin/isospin-dependent piece of the p-wave πN amplitude

$$d_1 \vec{\sigma} \cdot (\vec{k} \times \vec{k}') \tau^\pm, \quad (9)$$

etc.

What do these excitations have in common? A priori not so much; the first process is purely e.m., the other ones involve one or two strongly interacting probes. Nevertheless all these processes feel in the same way the propagation of the excitation induced by the strong interactions in the nuclear system. In the early sixties, when vector dominance was a fashionable subject, people wondered how a weak or e.m. probe could feel the strong interaction of the nuclear medium and undergo for instance a shadowing effect. Today we are accustomed to this fact: the first step in the excitation is specific to the probe, with a weak or a strong interaction. But the subsequent history of the propagation of the excitation is governed by the strong interactions in the system and is universal, in such a way that there is no fundamental difference between these different probes.

1.1 Response of the Free Fermi Gas

If all the interactions in the system are switched off the nucleons respond individually, and there is no propagation of the excitation. The single-particle excitations are of two kinds.

The first way in which a spin/isospin perturbation can excite the system is to flip the spin and isospin of one nucleon. For this it has to promote it outside the Fermi sea since below the states are blocked.

For a given external momentum q transferred to the nucleon the excitation energy would be $\omega = q^2/2m_N$ if the nucleon is at rest and the response in the ωq plane would be restricted to the line $\delta(\omega - q^2/2m_N)$. The Fermi momentum spreads this distribution. The excitation energies are

$$\omega = (\vec{q} + \vec{k})^2/2m_N - k^2/2m_N = q^2/2m_N + \vec{q} \cdot \vec{k}/m_N \quad (10)$$

with $\vec{k}$ the momentum of the hole satisfying $|\vec{k}| < k_F$. They lie between the two parabolas

$$\frac{q^2}{2m_N} - \frac{qk_F}{m_N} < \omega < \frac{q^2}{2m_N} + \frac{qk_F}{m_N},$$

i.e. inside the shaded region of Fig. 2. Outside this region there are no excitations of the type 1plh and 1m π vanishes. But there exist excitations of the 2p2h or 3p3h type. They cannot, however, be excited with a one-body operator, i.e. some form of interaction is needed. We will postpone their discussion until the introduction of the interactions.

Fig. 2. The region of response of infinite nuclear matter (shaded area) in the ω, q plane.

As an example it is worth noticing that in the absorption of real photons, where energy and momentum are linked by $\omega = q$, the 1plh region is inaccessible and only the 2p2h domain is spanned. In ee' scattering instead both 1plh and 2p2h regions are spanned.

The response function for the spin/isospin fluctuations in symmetric nuclear matter is given by

$$\pi(\vec{q}, \omega) = 4 \int \frac{d\vec{k}}{(2\pi)^3} \theta(k_F - k) \theta[|\vec{q} + \vec{k}| - k_F] \times \left[\frac{1}{\omega - \omega_{q+k} + \omega_k + i\epsilon} - \frac{1}{\omega + \omega_{q+k} - \omega_k - i\epsilon} \right] \quad (11)$$

with $\omega_q = q^2/2m_N$. The factor 4 comes from the spin and isospin averaging.

Notice that the response is the same for density fluctuations or isospin or spin fluctuations. This universality holds for a free gas; it is no longer true when the interactions are introduced.

The imaginary part of π , which is non-vanishing in the shaded domain, behaves as shown in Fig. 3, where it is represented as a function of ω for a fixed q . Its maximum occurs at $\omega = q^2/2m_N$ (but at small momenta where Pauli blocking exerts a great influence). The real part, which is linked to the imaginary part by the dispersion relation, changes sign near the maximum of $\text{Im } \pi$, i.e. around $q^2/2m_N$; this is the usual dispersive behaviour.

Fig. 3. Imaginary and real parts of $\pi(q, \omega)$ at a fixed value $q = 370$ MeV/c as a function of ω .

These excitations would be all for an assembly of point-like Dirac particles but the nucleons do have a structure. In the same way as the field creates a disturbance of the spin distribution of nucleons, it can also create a disturbance inside the nucleon, i.e. it can flip the spin of one of the constituents of the nucleon, leading to an excited nucleonic state.

The excitation spectrum of the nucleon differs markedly according to the type of excitation. For a spin/isospin excitation one state is privileged; it is the Δ_{33} resonance. In the constituent quark model it is the only spin/isospin flip excitation and it is degenerate with the ground state. In reality there is a mass splitting of 300 MeV, which is rather small at the nucleonic scale, and this excitation remains dominant. It is very appealing to keep the picture from the constituent quark model and to retain only this excitation; one reaches then a very symmetric picture where the N and Δ play symmetric roles. To evaluate the magnitude of the error thus introduced we can compare [5] the nucleonic response deduced from the Chew-Low model and the value which is derived from the off-shell extrapolation of the p-wave πN amplitude:

$$a(\omega = 0, q, = 0) = 4\pi c_0(\omega = 0) \quad \text{with} \quad c_0 = 0.19 \, m_\pi^{-3}, \quad (12)$$

while the Chew-Low value is $0.13 \, m_\pi^{-3}$. The Δ gives the major part of the nucleonic response but not all; the error introduced by the neglect of other excitations is of the order of 30%.

For illustrative purposes we will use the symmetric picture of only nucleons and Δ , keeping in mind the existence of other contributions (our Δ is an effective one which may incorporate other excitations). The total polarization propagator is then $\pi^0(q, \omega) = \pi^N(q, \omega) + \pi^\Delta(q, \omega)$ with

$$\pi^\Delta = \frac{64}{9} \int \frac{d\vec{k}}{(2\pi)^3} \theta(k_F - k) \left[\frac{1}{\omega - \omega_{q+k}^\Delta + \omega_k + i\epsilon} - \frac{1}{\omega + \omega_{q+k} - \omega_k - i\epsilon} \right] \quad (13)$$

where $\omega_q^\Delta = \omega^\Delta + (q^2/2m_\Delta)$ with $\omega^\Delta = m_\Delta - m_N = 2.14 \, m_\pi$. The width of the Δ has been ignored, which is a good approximation at low energy.

The consideration of the Δ excitation enlarges the domain of excitation of the free Fermi gas, which acquires an imaginary part in a band around $\omega_q^\Delta = \omega^\Delta + q^2/2m_\Delta$. As for the real part of π^Δ it is negative at low energy and changes sign around $\omega^\Delta \approx 300$ MeV.

Since the ΔN^{-1} excitations are high the energy denominator in the expression of π^Δ [Eq. (13)] can be taken as $\omega_\Delta = m_\Delta - m_N$ independent of q and closure can be performed. In this case π^Δ becomes a local quantity proportional to the nuclear density $\pi^\Delta = \rho$. This approximation may be useful in some cases.

1.2 Fermi Gas with Interactions

When the nucleons interact, the force acting on one nucleon can be transmitted to the neighbours via the interaction. There are many ways in which this transmission can occur. The propagation can take place through a coherent superposition of 1ph excitations, as is indicated below, where the heavy line represents the ph interaction V

or

The summation of this chain of graph is easily performed giving the RPA response

$$\pi(\omega, q) = \frac{\pi^0(\omega, q)}{1 - V(\omega, q)\pi^0(\omega, q)} . \quad (14)$$

More precisely this is the ring approximation which neglects the antisymmetrization. How good this approximation is compared to the exact RPA is very difficult to assert. The hope is that the antisymmetrization which is neglected here can be incorporated in some way, in a phenomenological form. The limitations of this procedure will be discussed.

The RPA approximation is well suited to studying the collective nature of the response. When the RPA denominator deviates appreciably from one the response becomes collective. When it vanishes it is fully collective with all nucleons participating. Then a collective state appears, for which the condition is

$$1 - V(\omega, q)\pi^0(\omega, q) = 0 . \quad (15)$$

When the interaction V is real, as occurs in the low-energy region, this condition requires $\text{Im } \pi(\omega, q) = 0$, i.e. the collective states can exist only outside the shaded region of Fig. 2 where $\text{Im } \pi \neq 0$.

Apart from the collective states which are created outside the region of the 1plh response, the RPA distorts the shape of the response but does not enlarge its domain since $\text{Im } \pi \propto \text{Im } \pi^0$. When $\text{Im } \pi^0 = 0$, $\text{Im } \pi$ also vanishes.

2p2h excitations. The transmission of the force by the coherent superposition of 1plh is not the only possibility. It can also induce 2p2h excitations as shown below

This enlarges the domain of response, which now fills the ωq plane and in particular the region between the nucleon and Δ responses. We will discuss the 2p2h response in Section 4.

2. THE PARTICLE HOLE INTERACTION

In order to study the collective aspects we have made use mostly of the ring approximation. In certain cases we have been able to go beyond it and use the full RPA result, with incorporation of the antisymmetrization and Hartree-Fock effects. The first ones have to be simulated in the ring approximation and in general we have ignored the second ones.

Before entering into the details of the discussion, we have to specify the ph force V . Since we are dealing with spin/isospin excitations, an effective agent for transmitting these excitations is the pion exchange force. This is true only for those excitations which couple longitudinally, probing the component of the magnetization along the momentum. The coupling is of the form $\vec{\sigma} \cdot \vec{q}$ corresponding to the quantum number for the ph pair of $S = T = 1$, $M_S = 0$. The pion exchange force does not act in the transverse channel ($S = T = 1$ with $M_S = \pm 1$), which is explored with a coupling of the type $\vec{\sigma} \times \vec{q}$. In nuclear matter the two types of excitations are completely decoupled. The meson which has the quantum number of the transverse channel is the ρ meson (or two-pion exchange with a p-wave interaction). These exchanges have a higher exchange mass and the interaction that they mediate is quite different. The pion exchange force $-(f^2/m_\pi^2)q^2/(q^2 + m_\pi^2 - \omega^2)$ (where f is the πN coupling constant) vanishes at $q = 0$ but builds up very quickly when q increases owing to the small pion mass. The force mediated by the ρ is instead very slowly varying. As a consequence, the interaction is very dependent on the momentum in the longitudinal channel and much less so in the transverse one.

At short distances it obviously makes no sense to discuss the interaction in terms of π and ρ exchanges (it is not clear even if one should talk at all of ρ exchange); there will be a variety of other mechanisms contributing. A phenomenological approach is indicated. One simulates these short-range effects with a contact interaction governed by the Landau-Migdal parameter g' .

The forces in the longitudinal and transverse channel are, respectively,

$$V_L(q, \omega) = \frac{f^2}{m_\pi^2} \left[g' - \frac{q^2}{q^2 + m_\pi^2 - \omega^2} \right] \quad (16)$$

$$V_T = \frac{f^2}{m_\pi^2} \left[g' + \text{slowly varying force} \right] \approx \frac{f^2}{m_\pi^2} \left[g' - C_\rho \frac{q^2}{q^2 + m_\rho^2 - \omega^2} \right], \quad (17)$$

where C_ρ is related to the ρN coupling constant $C_\rho = (f_\rho^2/m_\rho^2)(m_\pi^2/f_\pi^2) \approx 1$ to 2. In Eq. (17) the model of ρ meson exchange has been taken for the slowly varying force.

In our units, when $m_\pi = 1$, the quantity f^2/m_π^2 is practically one. In order to simplify the formulae it will be often omitted in the following.

How good is the assumption that g' is independent of momentum? Since it simulates short-range effects, no appreciable variation is expected between $q = 0$ and $q \approx$ a few pion masses. We will come back to the question of the momentum dependence. As for the energy dependence it is totally unknown.

The microscopic origin of the short-range repulsion expressed in the Landau-Migdal parameter g' is still unclear; there are in fact too many candidates to explain it. Historically the first suggestions [6] attributed g' to the influence of the short-range repulsion, which prevents a total overlap of two nucleons and therefore cuts the very short range piece of the boson exchange force. An example is provided by the removal of the δ piece of the pion exchange force; this amounts to adding to

the full pion exchange force a contact repulsive force with $g' = 1/3$ (classical Lorentz-Lorenz effect). The effect of the short-range correlations on π and ρ exchanges led to a value of $g' \approx 0.6$ in good agreement with measured values. An alternative suggestion by Dickoff [7] emphasizes the role played by the antisymmetrization which is ignored in the ring approximation. Finally our modern ideas about the structure of the nucleon in terms of quarks confined in a bag suggest that a description of the NN interactions force in terms of meson exchange loses its meaning when the two bags overlap, and should be replaced by the force arising from the merging of quarks into a single bag. It is the most appealing. The short-range force would represent the free flow of spin/isospin by the exchange of the quarks through the direct contact of the bags. If this is the picture, the collective states observed at low energy (the zero sound, for instance) results from the free flow of spin and isospin over the whole nucleus by a direct contact of the bags. However, there is no unambiguous signature for this view since the collective state can also occur otherwise. For the moment this picture remains speculative.

In the model where the Landau-Migdal force arises from the short-range correlations, we expect a characteristic momentum dependence, as described below, which has a connection with Beg's theorem as applied to virtual particles.

2.1 Beg Theorem for Virtual Particles [8]

The theorem of Beg [9] concerns the scattering of physical particles, by scatterers which are correlated in such a way that a total overlap is forbidden. It applies strictly when the range of the interaction between the incident particle and the scatterer is zero. In this case the interaction takes place on a free scatterer and not in a medium and the scattering by the medium does not depend on the off-shell extrapolation of the elementary amplitude.

This theorem holds true as long as the range of the elementary interaction is smaller than the size of the correlation hole.

It is then natural to think of an extension of this theorem to off-shell incident particles (for instance a pion) in the following form. A virtual particle is sustained by a source which for the pion is the nucleonic spin. Being virtual it cannot propagate to infinity and has to remain in the vicinity of the source. The more virtual it is the closer it has to remain. If it becomes so virtual that it cannot leak outside the correlation hole it also cannot feel the presence of the medium. Its propagation has to be the same as in free space.

We will show that this indeed occurs. We take as an example the case of a static pion, $\omega = 0$, of momentum q . We first assume the absence of form factor for the πN vertex. In free space the pion field is

$$\phi(q) = i \frac{g_r}{2m_N} \frac{\vec{\sigma} \cdot \vec{q}}{q^2 + m_\pi^2} . \quad (18)$$

The pion becomes very virtual when q becomes large. In the nuclear medium the new expression of the pion field is, in the RPA approximation,

$$\phi(q) = i \frac{g_r}{2m_N} \frac{\vec{\sigma} \cdot \vec{q}}{q^2 + m_\pi^2 + q^2 f^2 / m_\pi^2 [g' + 2h' - (q^2 / q^2 + m_\pi^2)] \pi^0(q)} , \quad (19)$$

where g' and h' are the scalar and tensor Landau-Midgal parameters. The tensor parameter h' , which vanishes at $q = 0$, is important for the realization of the

theorem. In the RPA denominator there appears the ph force $V_L(q)$ in the longitudinal channel

$$V_L(q) = \frac{f^2}{m_\pi^2} \left[g'(q) + 2h'(q) - \frac{q^2}{q^2 + m_\pi^2} \right]. \quad (20)$$

In a world where the forces are mediated only by pions these parameters are generated by the correlation function which cuts the short-range piece of the pion exchange force. For a point-like pion source

$$g'(q) = - \int d\vec{k} G(|\vec{k} + \vec{q}|) \frac{1}{3} \frac{k^2}{k^2 + m_\pi^2} \quad (21)$$

$$h'(q) = - \frac{1}{2q^2} \int d\vec{k} G(|\vec{k} + \vec{q}|) \frac{\vec{q} \cdot \vec{k} - 1/3 q^2 k^2}{k^2 + m_\pi^2},$$

where $G(q)$ is the Fourier transform of the correlation hole profile which satisfies

$$G(x=0) = \int d\vec{q} G(|\vec{q}|) = -1. \quad (22)$$

The limit of large q implies large k values. It is then clear that $g'(q) \rightarrow 1/3 \int d\vec{k} G(k) = 1/3$. This is also the limit of h' [noticing that $(\vec{q} \cdot \vec{k})^2 \rightarrow q^2 k^2$]. Thus $g' + 2h' \rightarrow 1$ and the ph force

$$V_L = \frac{f^2}{m_\pi^2} \left[g' + 2h' - \frac{q^2}{q^2 + m_\pi^2} \right] \rightarrow 0. \quad (23)$$

No renormalization of the pion field occurs, as expected from the physical argument, and the theorem is proved.

With the introduction of the form factors at the πN vertex $\Gamma(q^2)$ the expressions of g' and h' are modified, but nevertheless the essence of the theorem survives. The ph force goes to zero faster than the form factor, as shown in Fig. 4 which displays

Fig. 4. The quantity $g' + 2h'$ (continuous line) and $(g' + 2h')/\Gamma^2(q^2)$ (dashed line) as a function of the momentum.

the ratio $(g' + 2h')/\Gamma^2(q^2)$ as a function of the momentum. This quantity increases with momentum and goes to unity when q becomes large in such a way that it counterbalances the attraction of the pion for large q values. Here an approximate expression for the correlation function of the Reid potential has been used [10] $G(x) = -j_0(q_c x)$ with $q_c = 5.5 m_\pi$. Notice that the ansatz which had been proposed by Oset and Weise [11] for g' and h' from the same model obeys the theorem.

The scale which governs the variation in q of the Landau-Migdal parameter is set by the range of the correlation hole. For a very small hole, the theorem becomes of academic interest only, the increase occurring at very large q values.

An important application of this theorem concerns the extraction of the πN form factor from the NN scattering data. The correlations introduce a momentum dependence of the force which can be misinterpreted as a genuine form factor at the πN vertex. A reinterpretation of the NN scattering data in that light has been recently performed by Gersten [12].

A similar result applies to the Migdal parameters generated by the Pauli exclusion principle. Their role in the Landau parameter for the mixing force between nucleons and Δ has been repeatedly emphasized [13-15]. The reason is that the quantum numbers of the Δ , $T = 3/2$ and $S = 3/2$, forbid its production from a pair of nucleons in a space symmetric state. It can only be produced from a space antisymmetric pair which exists at large relative distance: $d > k_F^{-1}$. The situation with Δ excitation is then comparable to the previous one. There is a hole of size $d \approx k_F^{-1}$ in the Δ excitation around each nucleon. Since this size is large, the mixing force is then expected to decrease rapidly when the momentum increases. This implies a rapid increase of the Landau parameter with increasing momentum transfer. This variation was indeed found in the calculation of Dickoff et al. [5].

For the nucleon hole force instead their G matrix calculation does not follow the predictions of Beg's theorem. The quantity $(g' + 2h')/\Gamma^2(q^2)$ evaluated with the Bonn potential decreases with increasing momentum. This casts some doubts on the role of the short-range correlation for the generation of this force. Perhaps their role has been overemphasized in the past. This would not question the general validity of the theorem but only its relevance for this particular case. Notice however that the Landau-Migdal force deduced by Speth and co-workers [16] from experimental data indeed increases with momentum. The situation is therefore not completely settled.

2.2 Universality of g'

There is also an important point about the universality of the short-range force. We have NN^{-1} and ΔN^{-1} configurations. The forces which mix the different configurations have no reason to be the same, although in many descriptions they are taken to be equal, essentially for simplicity reasons. However, there are some reasons to question this universality. One has to distinguish between the different forces, i.e. between the different parameters g'_{NN} , $g'_{N\Delta}$, and $g'_{\Delta\Delta}$.

In this case the RPA expressions for π^Δ and π^Δ are the solutions of the two coupled equations [17]

$$\text{Diagram 1} = \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4}$$

where $\bigcirc$ represents the ΔN^{-1} polarization propagator, or equivalently

$$\pi^N = \pi^0 N + \pi^0 N V_{NN} \pi^N + \pi^0 \Delta V_{N\Delta} \pi^\Delta \quad (24)$$

and

or

$$\pi^\Delta = \pi^0 \Delta + \pi^0 \Delta V_{\Delta\Delta} \pi^\Delta + \pi^0 \Delta V_{N\Delta} \pi^N \quad (25)$$

The solution of these two equations gives the RPA expression for the total polarization propagator $\pi = \pi^N + \pi^\Delta$

$$\pi = \tilde{\pi}^\Delta + \pi^0 N [1 + V_{N\Delta} \tilde{\pi}^\Delta] \quad (26)$$

where

$$\tilde{\pi}^\Delta = \frac{\pi^0 \Delta}{1 - V_{\Delta\Delta} \pi^0 \Delta}.$$

I have cast π in a form which is not symmetrical in π^N and π^Δ but which is convenient at low energy where $\text{Im } \pi^\Delta = 0$.

What is known about the various parameters g'_{NN} , $g'_{N\Delta}$, and $g'_{\Delta\Delta}$? The information is quite scarce. The best-known quantity is g'_{NN} . There the information comes from the position of the states of unnatural parity in heavy and light elements [16,17]. It probes the value of g'_{NN} at very low energy and q values ranging from zero to a $q \approx 2 \text{ fm}^{-1}$. The conclusion is that the short-range repulsion is large: a typical value is $g'_{NN} \approx 0.7$.

The existence of this large repulsive force was confirmed when the giant GT resonances were discovered. These collective states are pushed upwards by the repulsive force, in agreement with the previous value of g'_{NN} .

Concerning $g'_{N\Delta}$ the information is scarce. The quenching of the GT strength, if it is interpreted as entirely due to the mixing with Δh configurations, is informative on $g'_{N\Delta}$ ($q = 0, \omega \approx 0$). But even in this interpretation there is some ambiguity, as will be discussed in Section 7.2.

Finally for $g'_{\Delta\Delta}$ the information comes from the data on mesic atoms or on the scattering of low-energy pions, i.e. in an energy domain $\omega \approx 140 \text{ MeV}$. In this energy domain the nuclear contribution to the polarization propagator has faded away (for $\omega \gg \omega_{\text{exc}}$ $\pi^N \rightarrow 0$) and only the Δ contribution and that coming from 2p2h excitations are present. Without consideration of the last contributions, the data on mesic atoms do not indicate the presence of much repulsion and $g'_{\Delta\Delta}(\omega = m_\pi) \approx 0$. However it is not easy to disentangle the effect of the short-range repulsion from the role of the 2p2h excitation in the real part of the polarization propagator. An attractive contribution from π^{2p2h} simulates a decrease in the value of g' [18]. A

careful analysis performed by Tauscher [19] to disentangle the two has given $g'_{\Delta\Delta}(\omega = m_\pi) \approx 0.4 \pm 0.2$, of the same general magnitude as g'_{NN} .

Figure 5 summarizes the regions, in the ωq plane, where some information on the Landau-Migdal parameters exists.

Fig. 5. Region in the ω, q plane where information on the Landau-Migdal parameters exists.

3. COLLECTIVE FEATURES OF THE RESPONSE

Having fixed the ingredients we are now in a situation to proceed and I will start by a discussion of the collective features at very low momentum.

3.1 RPA Response at Low Momentum ($q \leq 0.3 \text{ fm}^{-1}$)

The nuclear-matter description does not apply in this region to a finite system but it is nevertheless instructive as an illustration. The force $V = (f^2/m_\pi^2)/g'$ is strongly repulsive in both channels. On the other hand, $\text{Re } \pi^N$ becomes positive when $\text{Im } \pi^N$ has passed its maximum value; π^Δ instead is negative but the total quantity $\pi^0 = \pi^N + \pi^\Delta$ reaches a positive value. It is then possible to meet the condition for a collective state: $1 - V(q, \omega)\pi^0(q, \omega) = 0$. This collective state is the zero sound predicted by Ikeda and co-workers [20] and beautifully displayed [21,22] in the (p,n) and (pp') reactions. It is pushed above the ph continuum by the repulsive interaction.

The existence of the collective state produces a complete reshaping of the response [23], as is shown in Fig. 6 where the response is given as a function of the energy for a fixed value of the momentum $q = 0.3 \text{ fm}^{-1}$. The dash-dotted line is the response of the free Fermi gas, while the continuous line is the one calculated in the ring approximation for $g' = 0.7$. The collective state depopulates nearly totally the 1ph states.

At low momentum the energy of the collective state ω_c is linear in the momentum, defining the zero sound velocity $v_s = \omega_c/q$. With $g' = 0.7$, $v_s = 0.305 c = 1.07 v_F$,

Fig. 6.

The response of infinite nuclear matter at $q = 0.3 \text{ fm}^{-1}$ in the free case (dash-dotted line) and in the ring approximation (continuous line) for $g' = 0.7$.

i.e. only slightly superior to the Fermi velocity. Without Δ this velocity is only very slightly modified, $v_s = 0.321 \text{ c}$.

Role of the Δ . The Δ has little influence on the position of the giant resonance. However it strongly influences another quantity, the total strength which is the integral of the response over ω . In this definition we include the contribution of the collective state:

$$S(q) = \int_0^{\omega_{\text{coll}}} \text{Im } \pi(q, \omega) d\omega. \quad (27)$$

For the free Fermi gas $S(q)$ has the well-known expression $S(q) = (\frac{3}{2})q/2k_F - \frac{1}{2}(q/2k_F)^3$ valid for $q < 2k_F$.

With the introduction of the repulsive interaction this quantity is reduced as shown in Fig. 7. The quenching exists without the Δ , owing to the effect of the RPA correlations. The Δ introduces a further quenching.

Fig. 7. Total strength (including the collective state) as a function of the momentum in the free case (dash-dotted line), in the ring approximation without (dashed line) and with (continuous line) Δ .

This can be understood from the expression (25). At low energy $\text{Im } \pi^\Delta = 0$ and $\text{Re } \pi^\Delta(\omega)$ is practically independent of ω : $\pi^0 \Delta(\omega) = \text{Re } \pi^0 \Delta(\omega) = \pi^0 \Delta$. Thus

$$\text{Im } \pi(q, \omega) = (1 + g'_{N\Delta} \tilde{\pi}^\Delta)^2 \frac{\text{Im } \pi^0 N(q, \omega)}{[1 - \tilde{g}'_{NN} \text{Re } \pi^0 N(q, \omega)]^2 + [\tilde{g}'_{NN} \text{Im } \pi^0 N(q, \omega)]^2}, \quad (28)$$

where

$$\tilde{g}'_{NN} = g'_{NN} + (g'_{N\Delta})^2 \tilde{\pi}^\Delta$$

with

$$\tilde{\pi}^\Delta = \pi^0 \Delta / (1 - g'_{N\Delta} \pi^0 \Delta).$$

One sees in this expression that the role of the Δ is double.

- i) It renormalizes the interaction which mixes NN^{-1} states $g'_{NN} \rightarrow \tilde{g}'_{NN} = g'_{NN} + (g'_{N\Delta})^2 \tilde{\pi}^\Delta$. Since $\tilde{\pi}^\Delta < 0$, $\tilde{g}'_{NN} < g'_{NN}$. Thus the Δ softens the short-range repulsion. For instance with a typical value $g'_{NN} = 0.7$, $\tilde{g}'_{NN} \approx 0.5$.
- ii) It also produces a uniform modification of the strength by the Lorentz-Lorenz (LL) factor [24] $(1 + g'_{N\Delta} \tilde{\pi}^\Delta)^2$, a quenching effect since $\tilde{\pi}^\Delta < 0$.

Note that in Fig. 7 the difference between the strengths without and with Δ is not the full LL effect since the influence of the Δ results from the combined effect of the softening of the interaction (which increases the strength) and the LL quenching.

Since the Δ modifies so appreciably the interaction you may wonder why it has so little influence on the zero sound velocity. The reason is simple: the collective state is realized for $\pi^0(q, \omega) = 1/\tilde{g}'_{NN}$ (with in addition $\text{Im } \pi^0 = 0$). Defining the variable $x = v_s/v_F$ the sound velocity is defined by the condition [25]

$$\Phi(x) - i \frac{\pi}{2} \theta(1 - |x|) = 2\pi^2 / (m_N k_F \tilde{g}'_{NN}), \quad (29)$$

where $\Phi(x)$ is related to the Lindhard function and is shown in Fig. 8. This relation is always met for $x > 1$ for a repulsive potential. In general the intercept occurs in a region where the curve is very steep, as shown in Fig. 8. The value of x is

Fig. 8. Graphical illustration of the zero sound velocity.

therefore not very sensitive to the precise value of $\tilde{g}'_{NN}$ and consequently to the presence of the Δ . As a corollary, the position of the zero sound does not provide a sensitive measurement of the precise value of $\tilde{g}'_{NN}$.

We will now follow the evolution of collectivity with increasing momentum.

3.2 Response at Moderate Momenta

Beyond values $q \approx 0.3$ fm, the forces in the longitudinal and transverse channels become quite different owing to the growing influence of the pion exchange force (Fig. 9). In the longitudinal channel the force becomes very weak in the vicinity of $q \approx 1$ fm⁻¹ and the response quickly loses its collective character, recovering the single-particle features. In the transverse channel instead the force remains repulsive but more mildly so. It can no longer produce a collective state, but it gives a collective character to the response that we will discuss in detail.

Fig. 9. The ph force in the longitudinal (dash-dotted line) and in the transverse channel (continuous line) for $g' = 0.7$ and $C_0 = 2.18$.

Transverse response [23]. Figure 10 shows the response without and with interaction, at a fixed momentum $q = 1.3$ fm⁻¹, displaying the collective behaviour. The collective state which has entered the ph continuum survives in the form of an appreciable hardening of the response. The low-energy states are depopulated in favour of the high energy ones, and the maximum is strongly shifted towards higher energies. The suppression factor for small ω values is large, 5 or 6. This comes from the quantity $(1 - V \text{Re } \pi^0)$ in the RPA denominator, which is large, π^N and π^Δ adding their effects to produce a large and negative value of π^0 . At larger ω , π^N changes sign and almost cancels π^Δ . Thus the RPA response is close to the free one. Without Δ it would be enhanced over the free value.

Fig. 10. The response function at $q = 1.3 \text{ fm}^{-1}$ in the free case (dash-dotted line) and in the ring approximation (continuous line).

Do the experiments confirm the existence of these collective features? It is easy to find probes which couple to the transverse spin density. The ideal one is obviously the photon, which eliminates the problems linked with strongly interacting probes. The separation between the longitudinal (the charge) and the transverse responses has been performed by a Rosenbluth plot for ^{56}Fe and ^{12}C . The transverse response is dominated by the magnetic coupling of the photon $\mu_0(\mu_s + \mu_v \tau^3) \vec{\sigma} \times \vec{q}$, which is obviously a transverse one. In addition it is essentially isovector owing to the strong cancellation in the isoscalar momentum μ_s between the neutron and the proton magnetic moment

$$\mu_s = \mu_p + \mu_n = 0.9, \quad \mu_v = \mu_p - \mu_n = 4.71.$$

We have selected Fe for a comparison with the theory since it should come closer to a nuclear matter description. The momenta covered ranges from 1 to 2 fm^{-1} , a region where we expect appreciable collective effects. The small orbital contribution has been incorporated with its free Fermi gas value.

Figures 11a and 11b show the experimental cross-sections together with the predicted ones for the free case and with interactions for the two momenta $q = 250$ and 330 MeV/c .

The first striking feature is the total failure of the description at large ω . Here we enter the region where 2p2h excitations are dominant, producing the observed discrepancy. We will come back to that energy region. For the rest the agreement with the interacting Fermi gas is rather good; the slope at small ω values is well reproduced, as is the position of the maximum.

However, we cannot take this as conclusive evidence for collectivity. The response depends on the Fermi momentum, which has been taken as an adjustable parameter. The value found, $k_F \approx 1.15 \text{ fm}^{-1}$, smaller than that for nuclear matter, is reasonable to account for the average density distribution in Fe. But a larger value brings the

Fig. 11. The transverse magnetic response in ^{56}Fe . The dash-dotted line refers to the free Fermi gas with $k_F = 1.15 \text{ fm}^{-1}$. The continuous line is the ring approximation a) at $q = 250 \text{ MeV/c}$, b) at $q = 330 \text{ MeV/c}$.

maximum of the free response closer to the experimental one; the position of the maximum however remains displaced. A more serious problem is that the Hartree-Fock effects have been ignored in our description. If they can be embodied in an effective mass and for instance with $M^* = 0.6 M$ the free response would account rather well for the experimental points, without any need for collectivity.

The real proof of collectivity can only arise from the existence of a contrast between the longitudinal and the transverse response. Indeed in the vicinity of $q \approx 1 \text{ fm}^{-1}$ the longitudinal force vanishes, suppressing collective aspects in the longitudinal response, in contrast with the transverse one. If the observed behaviour in the ee' transverse data is due to the Hartree-Fock effect no contrast should show up. In this case the problem will be a theoretical one: since collectivity has been abundantly demonstrated at small ω , both in (p,n) reactions and in the magnetic transitions, why is it no longer present at these momenta? Why does the pion exchange force, which acts only (for the direct part) in the longitudinal channel, fail to produce the expected contrast?

4. THE 2p2h EXCITATIONS

I come now to the question of the discrepancy at large ω values, observed for all momenta. Here we are getting out of the region of the 1p1h response. It is then natural to assume that the 2p2h type of response takes over. Several attempts have been made to incorporate them, either microscopically [26] or at a phenomenological level, with a quasi deuteron model [27]. I will describe here our recent approach [28].

We have proceeded in two ways. The first one at the microscopic level: in the same way as we calculate π^0 for 1p1h excitations we have evaluated the different contributions to $\text{Im } \pi^{2p2h}$; they are numerous. This approach has considerably improved the fit to the high-energy region of the transverse response. However, there are uncertainties in the calculation which are difficult to estimate. This has been the motivation for an attempt at a phenomenological description of the 2p2h response, linking it to other physical quantities. In order to establish this link the microscopic model has been quite precious. Let us begin by discussing the first description.

4.1 Microscopic Description

The excitation of the 2p2h excitations is induced by a one-body operator through the interaction which creates the dynamical correlations in the system. Since we are probing spin/isospin correlations it is natural that the pion exchange force plays a large role in this problem and, in particular, through its tensor interaction. The ρ meson also contributes to the tensor force. We do not include it explicitly because it produces too much suppression of the tensor force [29]. We do it implicitly by introducing a form factor at the πN vertex chosen in such a way as to reproduce the effective tensor force of Dickoff and co-workers [29]. The corresponding form factor is a monopole with a mass $\Lambda = 800$ MeV. For the scalar piece we have used the same form factor for the pionic part plus the contact interaction with $g' = 0.5$. This gives a reasonable description of the effective scalar ph force in nuclei.

Having specified the interaction there are a variety of ways in which one can attach the photons.

In the graphs of Fig. 12 a small sample of the possibilities is shown. We have drawn the graphs for the scattering of photons. Only the imaginary part of these graphs has been calculated.

In the graphs of Fig. 12a the photon is absorbed by two correlated nucleons. In the graphs of Fig. 12b, a Δ is virtually excited (real Δ excitation occurs at higher energies). The importance of these graphs for the absorption of pions has been pointed out many years ago [30].

Figures 12c and d are of a different nature: they are graphs of the meson exchange type where the photon produces a pion by the contact interaction or interacts with a pion in flight between two nucleons.

Finally, the remaining graphs are the interference terms between all these possibilities: nucleon- Δ , nucleon-contact, nucleon-pion in flight, Δ -contact, etc. In all there are 121 graphs.

In the definition of π^{2p2h} we have deliberately left out the graphs of the meson exchange type (contact and pion in flight). The reason is that they are specific to the probe (γ). If we want to give some universality to π^{2p2h} and make use of it

in other processes we have to leave the meson exchange terms out of the definition (in addition we have also ignored in $\pi^2 p^{2h}$, and for the same reason, the small electric component).

Fig. 12. A small sample of the graphs contributing to the $2p2h$ absorption of photons.

Fig. 13. The transverse magnetic response in ^{56}Fe at $q = 370 \text{ MeV}/c$ including the 2p2h response ($k_F = 1.20 \text{ fm}^{-1}$). The dashed line represents the 2p2h response alone.

The result of this lengthy calculation is shown in Fig. 13, which displays the response at $q = 370 \text{ MeV}$. The 2p2h response is shown, together with the total response in which the RPA 1plh expression is added. The agreement with the experimental points is quite satisfactory; (no new free parameters are introduced, only a slight readjustment of the Fermi momentum to $k_F = 1.20 \text{ fm}^{-1}$ has been performed). It is even surprisingly good in view of the uncertainties inherent in such a calculation where a number of things have to be left out and we do not know how important they are. For instance the Pauli exchange pieces of the graphs of Fig. 11 have been ignored; in the interaction only the spin/isospin part is considered, etc. Although our microscopic description has been quite successful, these limitations have induced us to attempt a phenomenological approach through a link with the absorption of p-wave pions.

4.2 Pion Absorption

Pion absorption takes place along the line $\omega^2 = q^2 + m_\pi^2$, which is outside the region of the 1plh response and it takes place predominantly by 2p2h absorption. On a microscopic level the 2p2h graphs responsible for π absorption are identical to those which build $\text{Im } \pi^{2p2h}$, an external pion line replacing the photon line (in fact our graphs were copied from those of Shimizu and Faessler [31] for pion absorption). However, the coupling is now $\vec{\sigma} \cdot \vec{q}$, a longitudinal one, instead of the transverse magnetic coupling $\vec{\sigma} \times \vec{q}$ of the photon. We have insisted previously on the strong differences induced by the longitudinal or transverse character of the spin coupling. Does it mean that there is no relation between pion and virtual photon absorption?

The answer is that they are *not* related on a global scale, there is no relation between the cross section for ee' scattering and the absorptive one for p-wave pions. The link exists only at a *local* level, as is explained in the following.

The idea is to decompose the system (nucleus) into small subunits (nucleons or nucleon pairs), so small that the difference between a longitudinal and a transverse

coupling becomes insignificant. In this way the link exists between the ee' cross-section and the absorptive optical potential for p-wave pions.

The way to establish this link is to consider the propagator of pions D in nuclear matter. In the framework of lplh it obeys a Dyson equation

or

$$D = D_0 + D_0 \frac{f^2}{m_\pi^2} q^2 \tilde{\pi} D, \quad (30)$$

where D_0 is the free propagator $D_0 = 1/(m_\pi^2 + q^2 - \omega^2)$ and $\tilde{\pi}$ is the lplh polarization propagator irreducible with respect to a pion line and summed to all orders (only the short-range part of the interaction taken for simplicity to be universal can be exchanged)

$$\tilde{\pi} = \frac{\pi^0}{1 - g' \pi^0}. \quad (31)$$

Thus

$$D(q, \omega) = \frac{1}{q^2 + m_\pi^2 - \omega^2 + (f^2/m_\pi^2) q^2 \tilde{\pi}(q, \omega)}, \quad (32)$$

where $(f^2/m_\pi^2) q^2 \tilde{\pi}$ is the pion self energy or optical potential. In the relevant energy and momentum region $\tilde{\pi}$ is purely real and dominated by the Δ piece

$$\tilde{\pi}(q, \omega = m_\pi) = \frac{\pi^0 \Delta(q, \omega = m_\pi)}{1 - g' \pi^0 \Delta(q, \omega = m_\pi)}. \quad (33)$$

This description is insufficient because it ignores the 2p2h excitations. These are responsible for the absorptive piece of the optical potential. Several parametrizations have been given for the p-wave absorptive part. They are deduced from data on π mesic atoms and from the cross-sections for low-energy pions. One of the parametrizations is unambiguous, the other is not. The first one adds to the lplh optical potential $(f^2/m_\pi^2) q^2 \tilde{\pi}$ an absorptive piece parametrized as $i q^2 \text{Im } C_0 \rho^2$. The value of $\text{Im } C_0$ is unambiguously deduced from the data [19,32]: $\text{Im } C_0 \approx 0.05$ to 0.06 m_π^{-6} . Another parametrization incorporates the absorptive piece in $\tilde{\pi}$, i.e. it adds to the bare value π^0 an absorptive piece

$$\pi^0 \rightarrow \pi^0 + i q^2 \text{Im } \mathcal{C}_0 \rho^2. \quad (34)$$

Then the value of $\text{Im } \mathcal{C}_0$ is very sensitive to the value of g' . For instance, the classical LL value $g' = 1/3$ corresponds to $\text{Im } \mathcal{C}_0 = 0.09 \text{ m}_\pi^{-6}$. In this case the fit requires [19] the introduction of $\text{Re } \mathcal{C}_0$, which in this case becomes $\text{Re } \mathcal{C}_0 = 0.12 \text{ m}_\pi^{-6}$.

The value of $\text{Im } \mathcal{C}_0$ is not as well determined as $\text{Im } C_0$ since it depends sensitively on the value of g' . The problem is that it is difficult to disentangle the effect

of g' from that of the $\mathcal{C}_0$. Tauscher has performed [19] a very careful analysis of the π mesic atom data with the special aim of determining separately g' (called $\xi/3$ in his work), $\text{Im } \mathcal{C}_0$, and $\text{Re } \mathcal{C}_0$. His best fit corresponds to $g' = 0.4 \pm 0.17$, $\text{Im } \mathcal{C}_0 = 0.11 \text{ m}_\pi^{-6}$, $\text{Re } \mathcal{C}_0 \approx 0.2 \text{ m}_\pi^{-6}$.

His extreme value $g' = 0.57$ implies large values for both the real and imaginary parts of $\mathcal{C}_0$: $\text{Im } \mathcal{C}_0 = 0.18 \text{ m}_\pi^{-6}$, $\text{Re } \mathcal{C}_0 \approx 0.36 \text{ m}_\pi^{-6}$.

The difference between the two parametrizations C_0 and $\mathcal{C}_0$ reflects the elimination, in the latter, of the vertex renormalization of the type illustrated by Fig. 14, where only the short range Landau-Migdal interaction is exchanged (pion exchange is a distortion effect and does not appear in the optical potential). The relation between C_0 and $\mathcal{C}_0$ is

$$\text{Im } \mathcal{C}_0 \approx \frac{\text{Im } C_0}{(1 - g' \text{Re } \pi^0)^2}. \quad (35)$$

Fig. 14. The vertex renormalization graphs in pion absorption.

These vertex renormalization are not incorporated in our microscopic calculation of $\text{Im } \pi^{2p2h}$, which should then be compared with $\text{Im } \mathcal{C}_0$ and not with $\text{Im } C_0$.

The predicted value from our microscopic calculation,

$$\text{Im } \pi^{2p2h}(\omega = m_\pi, q = 0) \approx 0.20 \text{ m}_\pi^{-6},$$

compares favourably with the value extracted by Tauscher from the analysis of π mesic data only if a large LL effect is present, a factor ≈ 3 (leading to $\text{Im } \mathcal{C}_0 \approx 0.15$ to 0.18 m_π^{-6} , which is just compatible with Tauscher's result). If this is true it has another important implication for the magnitude of $\text{Re } C_0(q = 0, \omega = m_\pi)$ which would have to be attractive and large ($|\text{Re } C_0| \approx 0.3 \text{ m}_\pi^{-6}$) according to Tauscher's analysis. This result is in line with the findings of Oset and co-workers [18], who studied the real part of the π nucleus optical potential, discussing the connection between $\text{Re } C_0$ and the LL correction. They evaluated microscopically $\text{Re } C_0$ from the Δ isobar model and found $|\text{Re } C_0| = 0.1 \text{ m}_\pi^{-6}$, which corresponds to $g' > 1/3$.

With our value $|\text{Re } C_0| \approx 0.3 \text{ m}_\pi^{-6}$ the real part of the total polarization propagator would receive (at $\rho = \rho_0$) a sizeable ($\approx 40\%$) contribution from $2p2h$. It becomes very important to understand if this is still true at $\omega = 0$ since this real part is an essential ingredient in the understanding of the quenching of the GT strength.

4.3 Link between γ and π Absorption

Concerning the relation between γ and π absorption the microscopic model shows that $\text{Im } \pi^{2p2h}$ (therefore $\text{Im } \mathcal{C}_0$) extrapolates smoothly in ω and q ; $\text{Im } \pi^{2p2h}$ depends only on the external momentum through the form factor at the external vertex, as shown below (for a fixed energy $\omega = 140 \text{ MeV}$):

$q(\text{fm}^{-1})$	0.01	1.27	1.87
$\frac{\text{Im } \pi(\omega = m_\pi, q)}{\Gamma^2(q^2)}$	2.1	2.28	1.97

This result suggests the following ansatz for $\text{Im } \pi^{2p2h}$:

$$\text{Im } \pi^{2p2h}(\omega, q) = 4\pi \text{Im } \mathcal{C}_0 \rho^2 \Gamma^2(q^2), \quad (36)$$

which should apply for $100 \lesssim \omega \lesssim 200 \text{ MeV}$ and $q \lesssim 2 \text{ fm}^{-1}$. In order to compare with the experiments in ee' scattering the meson exchange terms (i.e. contact and pions in flight) have to be added. In principle, vertex corrections of Fig. 11 are also present for photons, but the kinematical conditions of Fig. 11 are such that we are in a region where the real part of the total polarization propagator is small, π^N and π^Δ cancelling their effects, and hence these vertex corrections are small.

The presence of the vertex corrections (LL effect) implies that the quantity $\text{Im } C_0$ does not extrapolate smoothly in ω and q , since the vertex corrections are not smooth functions of ω . This is the reason why we have to introduce $\text{Im } \mathcal{C}_0$ which extrapolates smoothly but which however is not unambiguously extracted from the experiments.

With $\text{Im } \mathcal{C}_0 \approx 0.18 \text{ m}_\pi^{-6}$ the ansatz of Eq. (36) gives a satisfactory fit to the deep inelastic scattering experiments similar to that of Fig. 13 (correcting for the pion in flight and contact terms). The philosophy that we can draw from these studies is the following. There exists a link between the magnetic absorption of photons and pion absorption, in spite of the different nature of the couplings. This link is at the *local* scale which means the following. Both the photon and the pion in the nuclear medium do not have their asymptotic momentum but a *local* one (indeed the graph of Fig. 14 applies as well for an external photon line and represents the modification of the momentum by the polarization of the medium. For the pion, in addition, the distortion also modifies the momentum).

The recipe for linking the two processes is to replace at each point the local photon momentum by the local pion momentum. However this local correction, embodied in the LL renormalization, is not a smooth function of the momentum. Since the photon and the pion do not carry the same momentum for a given energy, the link between the two processes is rather subtle.

By linking the two processes we have found evidence for a strong LL quenching in the p-wave π nucleus absorptive potential.

At small q values ($q \lesssim 1 \text{ fm}^{-1}$) photon absorption becomes dominated by the meson exchange terms and its link with π absorption becomes very loose.

4.4 Camow-Teller Strength at High Energy [33]

Quite often we think of the response to a spin/isospin excitation as consisting of two distinct sectors, well separated in energy, at $q = 0$, the low-energy nuclear sector and the high-energy Δ one. This picture is not correct. Between these two sectors the nuclear medium responds as well to the perturbing field. This has been largely demonstrated in the transverse response in ee' scattering. There is no dip between the nuclear and the Δ sector.

A clear theoretical indication of the existence of a nuclear response at high energy came from the study of the energy-weighted sum rule for the magnetic excitations by Zamick and co-workers [34]. Estimating this sum by the double commutator technique and ignoring the Δ excitation they showed that it is quite sensitive to the tensor correlation and they related it to the second-order correlation energy. In ^{16}O , for instance, they found $40 \text{ MeV } \mu_0^2$, while only $13 \text{ MeV } \mu_0^2$ has been detected so far in the low-energy sector. The natural conclusion is that a sizeable amount of magnetic strength lies at high energy. Notice that this strength is not subject to Pauli blocking and increases linearly with the nucleon number A .

It is easy to interpret this result in our evaluation of the $2p2h$ contribution to the nuclear response. We can do this with our microscopic model, but it is even more natural to link it, as we did previously, to the p -wave absorptive potential, i.e. to $\text{Im } C_0$. We have seen that the absorption adds to the $1p1h$ polarization propagator $\tilde{\pi}$ an imaginary piece $i 4\pi \text{Im } C_0 \rho^2$ with $\text{Im } C_0 \approx 0.05 \text{ m}_\pi^{-6}$. This is strictly valid at $\omega = m_\pi$ but it can be safely prolonged in a domain of $\pm 50 \text{ MeV}$ around pion threshold. Translated for the GT strength it means that in symmetric nuclear matter the density of GT strength is

$$G^\pm(\omega = m_\pi) = 4 f_\pi^2 \text{Im } C_0 \rho A, \quad (37)$$

where f_π is the pion decay constant ($f_\pi = 0.94 \text{ m}_\pi$). With $\text{Im } C_0 = 0.05 \text{ m}_\pi^{-6}$ it gives $G^\pm(\omega = m_\pi) = 0.1 A/\text{m}_\pi$. This quantity is deduced from the experiments and hence incorporates the influence of the Δ excitations. It contains also the vertex renormalization (LL effect). Notice that this strength is linear in the nucleon number A , as observed by Zamick and co-workers [34].

Does it have any implications for the question of the missing strength? The answer is clearly no, since there is no sum rule in each individual branch. The only sum rule is for the difference of the two branches $\beta^\pm$. In this case one should consider the difference in the $\pi^\pm$ absorption for a nucleus with a neutron or proton excess. However, the corresponding parameter $\text{Im } C_1$ for the p -wave absorption in the optical potential has been found to be negligibly small from the study of the π mesic data. No visible isovector effects appear in the p -wave absorption. The theoretical interpretation for this fact has been given by Shimizu and Faessler [31]; it results from a destructive interference between the correlation graphs and those with virtual Δ excitation.

One can therefore conclude that the region between 100 and 200 MeV excitations contributes in the same way for the two branches $\beta^\pm$. It is therefore unlikely to produce a removal of strength from the low-energy sector.

The relation that we have established between the GT strength and the parameter for the p -wave absorptive potential also follows from the assumption of axial locality made by T. Ericson and Bernabeu [35]. It was applied by Bernabeu and co-workers [36] to the process of μ capture in the situation of a large energy transferred to the nuclear system.

5. LONGITUDINAL RESPONSE AT MODERATE MOMENTA

The longitudinal force becomes attractive beyond $q \approx 1 \text{ fm}^{-1}$ with the growing influence of pion exchange. The response which had lost the collective features near $q \approx 1 \text{ fm}^{-1}$ that it displayed at small momenta may acquire new ones, of a different type.

5.1 Pion Condensation and Precursors

It has been suggested [37] that the attraction creates a new collective state, named a pion condensate. If it exists, it is pulled below the ph continuum by the attractive nature of the force, i.e. it can occur only at $\omega = 0$ [for which $\text{Im } \pi(q, 0) = 0$]. It is then of a different nature than the zero sound; it corresponds to a phase transition, a reordering of the system.

The condition for its realization is the usual one

$$1 - V(q, 0)\pi^0(q, 0) = 0, \quad (38)$$

which is fulfilled for a finite value q_c of the momentum. The ordered phase has a staggered structure.

The condition (38) cannot be met at the ordinary density; the force is not sufficiently attractive for that. The large amount of short-range repulsion does not allow a strong attraction to develop. The critical density for pion condensation, if it occurs at all, is much larger than the normal nuclear density $\rho_c \geq 3\rho_0$.

However, it is possible that there exist precursors of this phase transition at $\rho = \rho_0$ in the form of collective features with the following characteristics.

It is clear that the pion propagator D and the polarization propagator π have the same singularities. Then D is singular at $q = q_c$, $\omega = 0$, and $\rho = \rho_c$

$$D^{-1}(q_c, 0, \rho_c) = m_\pi^2 + q_c^2 + q_c^2 \tilde{\pi}(q_c, 0, \rho_c) = 0 \quad (39)$$

with

$$\tilde{\pi} = \frac{\pi^0}{1 - g'\pi^0}.$$

Fixing the momentum at $q = q_c$, and for $\omega = 0$ the propagator at the normal density can be written as

$$\begin{aligned} D(q_c, 0, \rho_0) &= \{m_\pi^2 + q_c^2 + q_c^2 [\tilde{\pi}(q_c, 0, \rho_0) - \tilde{\pi}(q_c, 0, \rho_c)] + q_c^2 \tilde{\pi}(q_c, 0, \rho_c)\} \\ &= q_c^2 [\tilde{\pi}(q_c, 0, \rho_0) - \tilde{\pi}(q_c, 0, \rho_c)]. \end{aligned} \quad (40)$$

If the phase transition is not too remote, or more precisely if the difference $|\tilde{\pi}(q_c, 0, \rho_0) - \tilde{\pi}(q_c, 0, \rho_c)|$ is small D is large. The pion propagator is enhanced by the effect of the nuclear medium over its value in free space [38]. To illustrate this statement with some numbers, a value of the critical momentum $q_c = 3 m_\pi \text{ im-}$ plies, from relation (39), $[\tilde{\pi}(q_c, 0, \rho_c)] = 1.1$. The value of $\tilde{\pi}$ at the normal density depends sensitively on the different g' parameters; with $|\tilde{\pi}(q_c, 0, \rho_0)| = 0.8$, $D(q_c, 0, \rho)/D_0(q_c, 0) = 3$. I have cheated somewhat because this enhancement is not physically observable. There are vertex corrections associated with it (the LL effect) which produce a quenching of the pion source besides the enhancement of the propagator. The physically observable quantity results from the two combined effects. But nevertheless some enhancement of the static pion field near $q = q_c \approx 3m_\pi$ is expected to survive.

Fig. 15. Form factor of the longitudinal response in the free case and in the ring approximation at $\omega = 20$ and 40 MeV ($g' = 0.6$).

A similar enhancement is also present in the response [39], for small values of the energy $\omega \lesssim q_c^2/2m_N$, as is shown in Fig. 15. It displays the form factor of the longitudinal response as a function of the momentum for the fixed values of the energy $\omega = 20$ MeV and $\omega = 40$ MeV. The enhancement over the free Fermi gas value is clearly visible.

Notice that the transverse response instead is depressed below the free value, an example of the contrast that we mentioned.

The longitudinal response is not only enhanced near q_c it is also softened [39]. Its maximum when plotted as a function of ω for q fixed near q_c is displaced towards lower energies (Fig. 16). This phenomenon is well known in magnetic

Fig. 16. Response at a fixed momentum $q_c = 2.15 \text{ fm}^{-1}$ (critical value) in the free case (dash-dotted line) and in the ring approximation (continuous line) with $g' = 0.7$.

transitions and it is named thermodynamical braking. It reflects the slowing down of the dynamics in the vicinity of the transition point. At $\rho = \rho_c$ the softening is complete; the fluctuations with the critical momentum become static.

The enhancement of the form factor of the response near q_c and the softening effect are the precursors of pion condensation. Notice that the word precursor should not be taken too literally. If observed, they do not imply that a second-order phase transition has to occur when the density is increased. For instance, it can happen, as suggested in Dickoff and co-workers [40] that the parameter g' increases with density in such a way that no transition occurs. Or it can be that the genuine transition becomes a first-order one [41], in such a way that these precursor or critical effects will not develop further when the density is increased but, nevertheless, the RPA enhancement of the response at q_c or its softening represent the critical phenomena associated with the proximity of a second-order phase transition.

These precursors have been actively looked for in low-lying discrete states and have not been found.

The softening effect translates into the fact that the states of unnatural parity are pulled down by the attractive ph spin/isospin force with respect to the shell-model position [16,17,42]. This is not observed; these states are pushed up or at best at their unperturbed position. You have also heard from Palumbo [41] of the search for precursors in the subtle situation of light deformed nuclei. The attraction from the pion exchange force can be better exploited in such a situation. In principle, it could create collective states, which have not been observed. The conclusion is that too much repulsion is present to allow for these states.

As for the enhancement of the form factor near q_c it has been investigated with pp' scattering for M1 transitions; for example, in the 15.1 MeV 1^+ state in ^{12}C , where no visible enhancement over the shell-model result has been displayed [43].

This is no reason to lose interest in the question. Remember that the critical momentum is large, $q_c \approx 3 m_\pi$. Then the bulk of the strength at this momentum does not lie in the low-lying discrete states which take only a tiny fraction of the total strength, but rather in the quasi-elastic peak. A survey of the over-all response remains therefore of significance even if no spectacular effects are expected. Such a study will elucidate the question of the contrast with the transverse response and thus hopefully display in an indisputable way the influence of the pion exchange force.

Concerning the measurement of the longitudinal response, two interesting possibilities have opened up.

One is the measurement of the polarization transfer in inclusive pp' reactions on nuclei, an experiment in preparation at Los Alamos (I refer to the lectures of Igo on the subject [44]).

The second one is the suggestion made by Chanfray and Delorme [45] for the extraction of the longitudinal response in the pion electroproduction process. They have shown that with a proper solution of the kinematics (the pion along the photon momentum) the transverse and longitudinal spin responses can be separated out with a Rosenbluth type of plot.

These two processes will provide simultaneously the transverse and the longitudinal response, a big asset for the display of the contrast since some systematic uncertainties, such as the distortion effects, will be common to the two responses.

5.2 Pionic or Optical Branch

In the longitudinal channel there exists another collective state, the pionic branch. The polarization propagator and the pion propagator having the same singularities the condition for a collective state can also be written as:

$$m_\pi^2 + q^2 - \omega^2 + q^2 \tilde{\pi}(q, \omega) = 0 . \quad (41)$$

This condition can be met at any momentum and any density near the free pion branch, which itself corresponds to $m_\pi^2 + q^2 - \omega^2 = 0$. The corresponding singularity line (the pionic or optical branch) represents the propagation of a pion in the nuclear medium where it is subject to a strong attraction ($\tilde{\pi} < 0$), the pionic branch is below the free one (Fig. 17).

Fig. 17. Pionic branch (continuous line) and free pion branch (dashed line).

Sawyer has suggested [46] exploring this branch in the following manner. In a scattering process where the scatterer keeps its identity, the momentum transfer is always space-like and cannot therefore probe the free-pion branch. However, in the nuclear medium the pion branch enters the space-like region. In the $\nu\mu$ process, for instance, this would lead to strong peaks for certain values of the μ momentum and for a given scattering angle. Varying the angle the pion branch would be explored.

Notice that this branch is a singularity only in the framework of lplh where $\tilde{\pi}$ is real. In reality, as Sawyer pointed out, the pion self energy has an imaginary piece from the 2p2h absorption and the pion branch in the nuclear medium is not a true singularity. The peaks are broadened by the absorption, but nevertheless taking only absorption into account they should remain clearly visible.

However, there is another essential reason why this branch is not a true singularity, even in the absence of absorption, as pointed out by Delorme and co-workers [47]. The zero of Eq. (41) determines the asymptotic pion field, the one which escapes to infinity. In a finite nucleus this is always the *free pion field* corresponding to the relation $\omega^2 - m_\pi^2 - q^2 = 0$. The pion branch is a pseudo-singularity. It appears only as a genuine one because an infinite nuclear matter treatment ignores the existence of a region external to the nucleus.

Delorme and co-workers [47] have shown, in the static case where $\omega = 0$, that in a model calculation for a finite nucleus, the residue at the pseudo-singularity vanishes, as it should. We plan to extend this treatment to the non-static region $\omega \approx m_\pi$ to see what remains of these peaks in a finite nucleus.

6. SUM RULES [4]

It is sometimes interesting to look at some global properties of the response function and not at their detailed shape. As an example we will study the sum rules $S^n(q)$ defined by

$$S^n(q) = -\frac{1}{\pi\rho} \int_0^{\omega_{\max}} \text{Im } \pi(\omega, q) \omega^n d\omega. \quad (42)$$

The summation over ω is restricted to the nucleonic sector, the Δ one is not included in the sum; but it includes the collective state, the zero sound, when it is present. We have, in particular, been concerned with the cases $n = 1$, $n = -1$, and $n = 0$.

6.1 Energy-Weighted Sum Rule ($n = 1$)

I will be very brief on the subject of S_1 since it is inaccessible experimentally. The reason is that it probes mostly the high-energy region, where the lplh are impossible to disentangle from the 2p2h contributions. However, S_1 is interesting from the theoretical point of view because it is possible to evaluate it exactly in RPA (and in the absence of Δ) by the double commutator technique. For any operator $\hat{O}$

$$S_1 = \frac{1}{2} \langle 0 | [\hat{O}, H] | 0 \rangle. \quad (43)$$

The Thouless theorem tells us that if this expectation value is taken between a Hartree-Fock ground state, then the result is the exact RPA expression. It is then possible to test the ring approximation. This has been applied to the operators $(\vec{\sigma} \times \vec{q})\tau^3$ and $\vec{\sigma} \cdot \vec{q}\tau^3$, giving the following results. Only the spin/isospin-dependent piece of the potential contributes to the double commutator; we have taken π and ρ exchanges (the Landau-Migdal piece, of zero range, gives no contribution). They participate only through antisymmetrization. It is then clear that the ring approximation which ignores it is independent of g' and keeps the free value, as shown in Fig. 18. [Notice, however, that with Δ the ring expression for S_1 depends on g' ,

Fig. 18. S_1/S_1^{free} in the spin longitudinal channel in the exact RPA and in the ring approximation.

Fig. 19. $S_1^{\text{ring}}/S_1^{\text{free}}$ with Δ in the spin longitudinal (dash-dotted line) and in the transverse channel (continuous line) for $g' = 0.7$. The dotted lines correspond to the suppression of the Δ .

Fig. 19]. Here the limitations of the ring approximation are manifest. It is impossible to simulate the Pauli exchange piece of π and ρ exchanges by a constant g' independent of energy, since the ring approximation for S_1 is not sensitive to the value of g' . The antisymmetrization of π and ρ can only be simulated with Landau-Migdal parameters g' and h' which are ω independent (in addition to their q dependence).

6.2 Inverse Energy-Weighted Sum Rule ($n = -1$)

The quantity is of particular interest because it is not sensitive to the high-energy side of the response and hence is the least affected by the 2p2h problem. It is the susceptibility of the system, related to $\text{Re } \pi(q, 0)$ by

$$S_{-1}(q) = -\frac{2}{\rho} \text{Re } \pi(q, 0) . \quad (44)$$

At $q = 0$ it is possible to evaluate S_{-1} exactly (without Δ however) by a resolution of the RPA equations, a method developed by Alberico and co-workers [48]. The result is represented in Fig. 20, where $S_{-1}(q = 0)$ is given as a function of g' in the free case, for the exact RPA and in the ring approximation. S_{-1} is decreased as compared to the free value, reflecting the quenching and hardening of the response. Here it is possible to simulate the exact RPA in the ring approximation with a proper choice of g' . For instance, if there is no genuine Landau-Migdal force the ring approximation gives the proper result for S_{-1} with a value of $g' = 0.4$. However, we have seen that this procedure does not apply for S_1 and cannot be general.

Fig. 20. $1/2 S_{-1}$ at $q = 0$ as a function of the Landau Migdal parameter in the free case, in the exact RPA and in the ring approximation.

Characteristic frequency. It can be defined as $\Omega_c = \sqrt{S_1(q)/S_{-1}(q)}$ (not a convenient definition from the practical viewpoint, but S_1 and S_{-1} can be evaluated exactly at $q = 0$).

An enhancement of Ω_c reflects a hardening of the response. At $q = 0$ the exact RPA evaluation (without Δ) gives a value of Ω_c 1.8 times larger than the free value, reflecting the strong hardening by the collective effects.

Susceptibility. At $q \neq 0$ only the ring approximation is available. It is shown in Fig. 21 as a function of the momentum. In the longitudinal channel it is lowered at low momenta and enhanced at large momenta, the maximum enhancement occurring at q_c , the critical momentum. The quantity S_{-1} is very sensitive to the approach to condensation; at $q = q_c$ it increases rapidly when g' decreases, becoming infinite at the condensation point.

Fig. 21. $1/Z S_{-1}(q)$ in the free case and in the ring approximation for the spin longitudinal channel.

In the transverse channel instead it is always lowered by the repulsive interaction. The Δ which produces a uniform quenching or enhancement (according to the repulsive or attractive character of the force) of the response participates in these features, accentuating them.

No data exist for the longitudinal channel. For the transverse one we have extracted the susceptibility from the electron-scattering data on the quasi-elastic peak. We have eliminated the 2p2h contribution by a somewhat arbitrary procedure (linear cut of the response) but we are not sensitive, anyhow, to the high-energy tail. The small orbital contribution has been removed. It is remarkable that the points for C and Fe fall on the same line, suggesting that the nuclear matter description is applicable at these momenta. However, this coincidence could be accidental; new results for other elements will be welcome. Figure 22 shows the nuclear matter susceptibility

Fig. 22. Susceptibility per proton (in units of $[2\mu_0^2(\mu_p - \mu_n)^2]^{-1} \text{ MeV}^{-1}$) in the free case (dashed line) and in the ring approximation (continuous line). The experimental points are for ^{56}Fe (ϕ) and ^{12}C ($\times$).

in the free case and in the ring approximation, for $k_F = 1.15 \text{ fm}^{-1}$. The experimental points fall close to this last line but this is not conclusive for the same reasons as the response, the theoretical description depending on the Fermi momentum and being sensitive to Hartree-Fock effects.

It would be interesting to find a way to bridge the gap in the measurements between $q = 0$ and $q = 1 \text{ fm}^{-1}$ to follow the transition between the surface and volume regimes. A possible way may be the photoproduction process (or radiative capture in flight). At threshold the momentum transfer is $q \approx 0.6 \text{ fm}^{-1}$ and it can be varied above threshold. Numerous data exist for the radiative capture of pions bound in Bohr orbits, but this binding makes the extraction of the susceptibility from the spectrum of the emitted photon uncertain.

6.3 Total Strength

The total strength $S_0(q)$ is a very interesting quantity. It represents the equal time correlation function. Unfortunately we do not have an exact evaluation in the RPA framework and one has to resort to the ring approximation. For a free Fermi gas the analytical expression is the well-known Pauli correlation function

$$S_0(q) = \frac{3}{2} \frac{q}{2k_F} - \frac{1}{2} \left(\frac{q}{2k_F} \right)^3 \quad \text{for } q < 2k_F$$

$$= 0 \quad \text{for } q > 2k_F .$$
(45)

When the interaction is introduced $S_0(q)$ gets quenched or enhanced depending on the repulsive or attractive character of the force, as is shown in Figs. 23 and 24.

Fig. 23. Total strength in the transverse spin channel in the free case (dash-dotted line) and in the ring approximation without (dashed line) and with (continuous line) Δ .

Fig. 24. Total strength in the longitudinal spin channel in the free case (dash-dotted line) and in the ring approximation without (dashed line) and with (continuous line) Δ ($g' = 0.7$).

In the longitudinal channel there is quenching at low momenta and an enhancement at large momenta. The enhancement becomes infinite when the density reaches the critical value ρ_c . This behaviour contrasts with that observed for the other collective state, the zero sound, which does not modify the strength. I have indicated by an arrow the point at which the collective state enters the ph continuum and disappears. At this point the strength is perfectly continuous. This difference in behaviour characterizes the phase transition as compared to a normal collective state.

The enhancement of the longitudinal response arises mainly from the Δ . The reason is that without Δ the response is enhanced on the low-energy side ($\omega \lesssim q^2/2m_N$) and enhanced beyond, owing to the change in sign of π^N . The two effects partly cancel in the total strength and the over-all enhancement from π^N is small. While instead the Δ produces a uniform enhancement when the force is attractive and gives a larger effect. The enhancement of the strength is therefore mostly sensitive to $g'_{N\Delta}$ and to a lesser extent to $g'_{\Delta\Delta}$, which appears in the effective polarizability [see Eq. (26)]. A universal value of $g' = 0.7$ has been taken for illustrative purposes. In reality, with this value we are likely to underestimate the attraction from the Δ and the softening could be more pronounced than is indicated here -- how much exactly we do not know. One sees here the evolution of the sensitivity to parameters of the different moments of the response: S_1 is sensitive only to the Δ , and hence to $g'_{N\Delta}$, but not at all to g'_{NN} ; S_0 is sensitive mostly to $g'_{N\Delta}$ but only moderately to g'_{NN} ; S_{-1} , which probes the low-energy region of the response, is instead very sensitive to g'_{NN} .

6.4 Characteristic Frequency

A decrease of S_{-1} may be due to a hardening of the response or to a uniform quenching (LL effect). These two effects are disentangled in the characteristic frequency, which is defined as a ratio of two moments that we choose as

$$E_c(q) = S_0(q) / S_{-1}(q) .$$

(46)

Fig. 25. Characteristic frequency in the free case (dash-dotted line) and in the ring approximation, for the spin transverse (continuous line) and spin longitudinal (dashed line) channels.

We could take another definition, for instance S_1/S_0 , but S_{-1} and S_0 are the least sensitive to the high-energy tail. An increase in E_c with respect to the free value indicates a hardening and a decrease reflects the softening of the response. A uniform quenching or enhancement (such as the LL effect) does not affect E_c . In Fig. 25 the ratio $E_c(q)/E_c(q)/q$ in the longitudinal channel shows the hardening at low momentum and the softening at large ones. At the critical density for pion consideration the softening becomes total, $E_c = 0$, which reflects the slowing down of the nuclear dynamics in the vicinity of the transition point.

7. TWO APPLICATIONS

7.1 The EMC (European Muon Collaboration) Effect

Thomas [49] already introduced you to this effect. It concerns the measurement of the structure function of the nucleons through deep inelastic $\mu\mu'$ scattering. These are very high energy experiments designed to probe the distribution of quarks inside the nucleons, in the same way as the inelastic ee' scattering at a few hundred MeV probes the distribution of nucleons inside the nucleus. For reasons of convenience the experiments were performed not on the nucleon but on nuclei. Not much thought was given to it, but the idea was that at these very high energies the nuclear binding is of little importance and that the neutrons and protons interact as if they are free. It came as a surprise that the comparison of measurements on Fe and deuterium revealed an anomaly [50]. The ratio between the structure function F_2 of Fe and D is not unity. It is larger than one at small x ($x \lesssim 0.3$) reaching ≈ 1.15 at $x \approx 0$ and becomes < 1 at large x [50,51]. This result shows that at least one of the two nuclei does not behave as an incoherent sum of protons and neutrons. Since deuterium is a dilute system, with the neutron and proton loosely bound, it is natural to blame the problem on Fe, where the interspacing between the nucleons is smaller and the interactions between nucleons more effective.

Among the interpretations which have been put forward to explain this behaviour, one is of particular interest in the context of the spin isospin excitations. The small- x data show that there is an increase of the sea quarks in Fe as compared to deuterium. Llewellyn Smith [51] has suggested that this increase could take place in the form of pions, and that the observed enhancement of 15% at $x = 0$ can be explained if Fe contains ≈ 8 extra pions.

Fig. 26. Pion contribution to the structure function of the nucleon (a) and in the nucleus (b) to (e).

Indeed it had been shown by Sullivan [52] that the one-pion exchange process plays a role in the deep inelastic process on the nucleon (Fig. 26a). The photon interacts with a pion and hadronization occurs on the pion of the cloud, while the nucleon remains intact or slightly excited (Δ). If this is the case it is natural to expect a modification of this contribution in the nucleus. Indeed the pionic clouds overlap in the nucleus and as a first approximation the nucleus consists of undistorted nucleons with a distorted pion cloud, a picture developed by the Lyon group [47,54]. This picture offers a possible interpretation for the observed anomaly. The estimate made by Thomas and myself [55] goes as follows.

In the nucleon case the one-pion exchange contribution F_2 is expressed in terms of the probability $f(y)$ for the pion to carry a certain fraction of the nucleon momentum (in the infinite momentum frame). Denoting by F_2^π the contribution of the pion exchange graph to the structure function F_2 ,

$$F_2^\pi(x) = \int_0^1 dy F^\pi\left(\frac{x}{y}\right) f(y) \quad (47)$$

where $F^\pi(x)$ is the pion structure function and

$$f(y) = \frac{3g^2}{16\pi^2} y \int_{\frac{m_N^2 y^2}{1-y}}^{\infty} dt \frac{t}{(t + m_\pi^2)^2} \Gamma^2(t), \quad (48)$$

where $-t = \omega^2 - q^2$ is the four-momentum of the pion and Γ the form factor at the πN vertex. This refers to the case where the recoiling particle is a nucleon. Sullivan [52] also gave the expression for a final state with a Δ .

When the nucleon is immersed in the nucleus medium several modifications of the pionic contribution occur. Firstly, the final states below the Fermi sea are blocked. More important, the propagation of the pion is affected by the polarization of the nuclear medium (Fig. 26b-e). There is, in addition, the possibility that the emission of the pion leaves the nucleus in a 2p2h excited state, a process not present for the nucleon.

All these features can be nicely incorporated if the nuclear response function is introduced in the expression of $f(y)$, which becomes

$$f(y) = \frac{3g^2}{16\pi^2} y \int_{m_N^2 y^2}^{\infty} dq^2 \int_0^{\omega - m_N y} d\omega \frac{q^2}{(q^2 + m_\pi^2 - \omega^2)^2} \frac{\text{Im } \pi(\omega, q)}{\pi \rho} . \quad (49)$$

[The form factor is incorporated in the response and t has been replaced by q^2 in the numerator of the integrand ($\omega < q$).] The normalization introduced for $\text{Im } \pi$ is such that the free nucleon expression is recovered in a dilute system.

In the nuclear case there are two variables of integration, ω and q , instead of one, t , for the free case. The reason is that, in the nucleon case, the momentum and energy of the virtual pion are linked by the condition that the final nucleon is on shell. Denoting the energy and momentum of the final nucleon by $(m_N + \omega)$ and $\vec{q}$, respectively, we get

$$(m_N + \omega)^2 - q^2 = m_N^2 \quad \text{or} \quad 2m_N \omega + \omega^2 - q^2 = 0 . \quad (50)$$

In the infinite momentum frame the nucleon momentum is $p_N = \gamma m_N$ where γ is the boost factor, while the pion one is $p_\pi = \gamma(-q_3 - \omega)$ where q_3 is the component of q along the photon momentum. The fraction y of the nucleon momentum carried by the pion is $y = (-q_3 - \omega)/m_N$ giving

$$-q_3 = \omega + m_N y . \quad (51)$$

The condition $q^2 > q_3^2$ can thus be written as

$$q^2 > \omega^2 + 2m_N \omega y + m_N^2 y^2$$

or, according to Eq. (50),

$$(q^2 - \omega^2)(1 - y) > m_N^2 y^2 \quad \text{or} \quad t > \frac{m_N^2 y^2}{1 - y} .$$

Hence the lower bound for the t integration in the free nucleon expression, Eq. (48).

In the nuclear case the introduction of the response function $\text{Im } \pi(\omega, q)$ guarantees that the final nuclear states are on shell (whatever their type NN^{-1} , $N^{-1}\Delta$, or 2p2h).

The only condition then is written as $q^2 > q_3^2$ or $q > \omega + m_N y$. In the ωq plane the boundaries of integration are straight lines, shown in Fig. 27 for some values of y . When y is large (≥ 0.3) the integration is pushed into the region of high momentum, where the form factor Γ cuts the response. The small-momentum region ($q < 1 \text{ fm}^{-1}$) contributes only for very small y values $y \leq 0.1$; but because $f(y)$ contains an explicit factor y [Eq. (49)] it is small for small y values. Thus $f(y)$ has a maximum for $0.2 \leq y \leq 0.3$. The region of momentum relevant for the structure function F_2^T is then from Eq. (47) the region $q \approx 300$ to 400 MeV/c where an enhancement of the

Fig. 27. Boundaries of integration of Eq. (49) in the ω, q plane for different y values.

response is expected, owing to the attractive nature of the ph interaction. The low-momentum region, where the response is suppressed both by the Pauli blocking and by the repulsive nature of the interaction, plays only a minor role in F_2^π .

If the full nuclear response is introduced in Eq. (49), the final ΔN^{-1} and $2p2h$ states are taken into account. It should be noted, however, that the Δ contribution (for which $\omega \approx \omega_\Delta$) is pushed towards high momenta by the condition $q > m_N y + \omega$. Therefore its contribution to $f(y)$ is restricted to small y values, or equivalently its contribution to F_2^π is restricted to small x values ($x \lesssim 0.05$). In our analysis we have omitted it; in principle it is also subject to a nuclear renormalization, but this should show only at small x values.

A similar consideration applies in part to the $2p2h$ states. The high-energy part of the $2p2h$ response, which fills the dip between the NN^{-1} and ΔN^{-1} regions of response, is partly cut by the limits of integration.

In a first stage of the analysis we have kept only the NN^{-1} response with the following inputs.

- i) The response was taken to be that of infinite nuclear matter, which fails at low momentum but should be good at the most important momenta $q \approx 300$ MeV/c.
- ii) For $\pi^\Delta(q, \omega)$ we have used the following ansatz.

$$\pi^\Delta(q, \omega) = -4\pi c_0 \rho \Gamma^2(q^2)$$

$$\Gamma = \left(\frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + q^2} \right) \quad \text{with} \quad \Lambda = 1.67 \text{ GeV} . \quad (52)$$

The value for c_0 is taken from the π mesic data $c_0 = 0.22 \text{ m}_\pi^{-3}$. With this expression for π^Δ which is purely real, the Δ region of response is automatically eliminated.

- iii) We have used a softer form factor at the last vertex, namely the form factor of the cloudy bag [56]

$$F(q) = e^{-\lambda(q^2/m_\pi^2)} \quad \text{where} \quad \lambda = 0.106 \text{ m}_\pi^2 R^2 \quad \text{with} \quad R = 0.7 \text{ fm} . \quad (53)$$

You may be shocked by this procedure, but the form factor at the πN vertex is not necessarily the one that appears in a two-body interaction. Indeed T. Ericson has presented [57] the evidence for a hard form factor in this last case, while Thomas [49] has shown the lower limit for the bag radius from the deep inelastic data; it does not correspond to the form factor of the NN interactions. If you are not convinced by these arguments I see another reason for putting a cut-off in the integrals of $f(y)$ of Eq. (49). The form factor $F(q)$ governs the regions of momenta which participate in $f(y)$; a harder form factor means that very large momenta ($q \approx 10 \text{ m}_\pi$) contribute to $f(y)$. However, if we need to invoke phenomena at these large momenta where we do not know what happens, our interpretation would lose its credibility. (Remember that a soft form factor restricts the region of integration and therefore makes it more difficult to build up an effect of the observed magnitude.) We have shown in Section 2 (Beg theorem) that we have reasons to believe that the nuclear renormalization disappears at large q . The soft form factor $F(q)$ is a way of getting rid of it.

- iv) As regards the Landau-Migdal parameters we have assumed them to be equal and have taken conservative values $g'_{NN} = g'_{N\Delta} = 0.7$. Notice that these values apply to the region of momentum $q \approx 400$ MeV/c, the important one.

With these inputs we obtain a result of the right order of magnitude represented in Fig. 28 together with the experimental points.

We have explored the sensitivity to the input parameters. As expected it is large. The enhancement of the nuclear response depends sensitively on the form factors, and on the values of the g' parameters. For instance in Fig. 28 is shown the result for a bag radius of $R = 0.8$ fm. Concerning the values of g' , we have introduced different values for g'_{NN} , $g'_{N\Delta}$, and $g'_{\Delta\Delta}$ and we have found that our results are mostly sensitive to $g'_{N\Delta}$. This is to be expected. In Eq. (49) the integration over ω , which runs practically over the full NN^{-1} region of response for most values of the allowed momenta, gives as a result the total strength $S_0(q)$ (see Fig. 27). We have seen that the enhancement of this quantity arises essentially from the Δ mixing. Indeed a lowering of $g'_{N\Delta}$ from 0.7 to 0.6 enhances the effect by 30% (see Fig. 28), while the enhancement is only of 10% if the same modification is applied to g'_{NN} or $g'_{\Delta\Delta}$. For $g' = 0.8$ the effect disappears.

Fig. 28. The fractional increase in the ratio of the structure function in Fe compared with D, as a function of $x (= Q^2/2m_N\nu)$. The data are from the EMC Collaboration, and the shaded area indicates possible systematic errors. The standard input (solid curve) for Fe is $k_F = 1.30 \text{ fm}^{-1}$, $g_{NN} = g_{N\Delta} = g_{\Delta\Delta} = 0.7$, a bag radius of 0.7 fm in $F(q^2)$, and $\Gamma(q^2)$ is a dipole of mass 1.67 GeV . In the other curves the effect of altering any single one of these parameters is shown.

Expressed in different terms, in order to reach an effect of the observed magnitude we require an enhancement of the total strength $S_0(q)$ by a factor of about 2 in the momentum region near $q = 300$ to $400 \text{ MeV}/c$ (for a value of the bag radius $R = 0.7 \text{ fm}$).

We have also explored in a preliminary way the influence of the 2p2h excitations. Since this process does not exist for the free nucleon it is bound to lead to an enhancement of the nuclear structure function. We have used the ansatz of Eq. (36) above 100 MeV : $\text{Im } \pi^2 p^2 h(q, \omega) = 4\pi \text{Im } \phi_0 \rho^2 \Gamma^2(q^2)$. Below a linear extrapolation was performed, reaching zero at $\omega = 40 \text{ MeV}$. The region of 2p2h response is for the moment taken to coincide with the 1p1h one, i.e. we have totally ignored the filling of the dip. The motivation for this restriction (not a good one) is the existence of the pionic branch which enters the allowed region of response and for which a finite nucleus description is needed. With this assumption we find that the 2p2h contribution is appreciable; by itself it leads to an enhancement of F_2 of the order of 30% of the observed one.

One can then ask the question. Is it possible that all the observed enhancement arises from the 2p2h excitations, without any need for an enhancement of the 1p1h response? We do not think so. The ansatz of Eq. (36) has been tested in ee' scattering. We have neglected the region of the 2p2h response above the NN^{-1} continuum. But this region is partly cut by the boundaries of integration in such a way that it influences the structure function F_2 only at small x values. We hope to reach a better description of the 2p2h contribution by incorporating an understanding of the role of the pionic branch.

An experimental indication of the role of the 2p2h could be obtained from the following argument. The 2p2h response varies as ρ^2 , while the enhancement of the 1p1h response is essentially linear in the density. In light nuclei like ^{12}C , where most of the nucleons sit on the surface, the 2p2h contribution should be considerably lowered (a factor of 3 as compared to Fe).

The A dependence of the effect may be a way of distinguishing between different interpretations of the EMC effect. In our interpretation the enhancement should go as $A^{1/3}$, i.e. approximately as the density. For Fe we have used $k_F = 1.3 \text{ fm}^{-1}$. With a value $k_F = 1.1 \text{ fm}^{-1}$ (applicable to a lighter nucleus such as ^{12}C) the effect is nearly a factor of 2 smaller (see Fig. 22). In heavy nuclei it should saturate.

The type of final nuclear states is also specific to our model. In spite of the large photon energy the pionic events leave the nucleus in a low excited state (one nucleon ejected with a distribution in energy of a quasi-elastic peak around $\omega \approx 50$ to 80 MeV; in certain cases with two nucleons ejected).

7.2 Quenching of the Gamow-Teller Sum Rule. Role of the Δ

We have seen that in symmetric nuclear matter the quenching of the total strength $S_0(q)$ by the Δ is not the full LL effect. The reason is that there is a parallel modification of the force which mixes the NN^{-1} configurations and produces by itself a quenching that gets reduced by the presence of the Δ .

In unsymmetrical nuclei instead, with $N \neq Z$, there is one quantity which is subject to the full LL modification. It is the GT sum rule, which applies to the difference in the strengths for $\beta^\pm$.

Consider the sum

$$S = S^- - S^+ = \sum_n \left\{ |\langle 0 | A^{3+}(q=0) | n \rangle|^2 - |\langle 0 | A^{3-}(q=0) | n \rangle|^2 \right\}. \quad (54)$$

In the absence of Δ excitations

$$A^{3\pm}(x) = g_A \sum_i \sigma_i^3 \tau_i^\pm \delta(x - x_i) \quad (55)$$

and

$$S = \sum_n \left\{ \left| \langle 0 | \sum_i \sigma_i \tau_i^+ | n \rangle \right|^2 - \left| \langle 0 | \sum_i \sigma_i \tau_i^- | n \rangle \right|^2 \right\}. \quad (56)$$

Performing closure one gets

$$S = \langle 0 | \sum_i \tau_i^3 | 0 \rangle = (N - Z). \quad (57)$$

The sum S depends only on the neutron excess and is totally independent of the force that mixes the different NN^{-1} configurations.

In practice, only the β^- branch (corresponding to the τ^+ operator) has been measured, through the p,n reactions. In nuclei with $N > Z$ the β^+ branch is strongly suppressed by the Pauli blocking. In general its contribution was evaluated. If it is totally ignored the sum rule (57) provides a lower limit for the β^- branch.

The measured value of the sum is a factor of 2 smaller [58] than the calculated one. The Δ excitation provides a natural mechanism of quenching. Here the full LL effect should apply since the sum S is totally independent of the mixing force. Is a factor of 2 reasonable?

An exact RPA calculation is not available for the GT sum rule. Delorme and co-workers have utilized the approximation where π^Δ becomes a local quantity proportional to the local density $\rho(x)$. The LL effect amounts to the following replacement:

$$\begin{aligned} A_3^\pm(x) &= g_A \sum_i \sigma_i^3 \tau_i^\pm \delta(x - x_i) \\ &\rightarrow g_A \left[1 + g'_{N\Delta} \tilde{\pi}^{0\Delta}(x) \right] \sum_i \sigma_i^3 \tau_i^\pm \delta(x - x_i) \end{aligned} \quad (58)$$

with

$$\tilde{\pi}^{0\Delta}(x) = \frac{a\rho(x)}{1 - g_{\Delta\Delta} a\rho(x)}$$

where a is the nucleonic polarizability and the sum rule S becomes

$$\begin{aligned} S &= g_A^2 \langle 0 | \left[1 + g'_{N\Delta} \tilde{\pi}^{0\Delta}(x) \right]^2 \sum_i \tau_i^3 | 0 \rangle \\ &= g_A^2 \int d\vec{x} \left[\rho_n(x) - \rho_p(x) \right] \left[1 + \tilde{\pi}^{0\Delta}(x) \right] \end{aligned} \quad (59)$$

The sum rule S has been evaluated for a number of nuclei with realistic density distributions.

The result is shown in Fig. 29 where the ratio S/S_{free} is plotted for two values of a parameter P that we define as the product of $g'_{N\Delta}$ and the value of $\tilde{\pi}^\Delta$ taken at the nuclear matter density

$$P = g'_{N\Delta} \tilde{\pi}^\Delta(\rho = \rho_0) .$$

In Fig. 29 the two curves give a quenching in the observed range; they are drawn for $P = -0.43$ and -0.35 . In Fig. 29 the two arrows represent the corresponding

Fig. 29. Quenching factor of the G-T sum rule as a function of the nuclear number A .

nuclear matter limits. Notice that even in the heaviest nuclei these are not reached; a fraction of nucleons are in the surface region where the density is lower.

The corresponding values of the mixing force $g'_{N\Delta}$, which is needed in order to reach these values of P , depends on π^{Δ} , i.e., on $g'_{\Delta\Delta}$ and on π^{Δ} ($\omega = 0$, $q = 0$). Assuming $g'_{\Delta\Delta} = g'_{N\Delta}$ and $\pi^{\Delta} = a \rho_0$, where a is taken from the extrapolation of the p-wave πN amplitude ($a = 2.39 \text{ m}_{\pi}^{-3}$), the corresponding values of $g'_{N\Delta}$ are 0.63 and 0.45, respectively. [With the same assumptions the Chew-Low value leads to larger values, $g'_{N\Delta} = 0.8$ and $g'_{NN} = 0.67$.]

It has been argued that the Pauli principle makes large values of $g'_{N\Delta}$ implausible. The values that we find are moderately large and not incompatible with the microscopic evaluation of Ref. 15. They are also compatible with the values that we have used in the EMC effect, allowing for some q dependence. It is necessary to discuss also another uncertainty, which is the role of the 2p2h excitations in the LL quenching. In this problem, we can draw the analogy with the GT strength at $\omega = m_{\pi}$, which we have linked to the p-wave optical potential. We have seen in Section 4 that we have every reason to believe that a strong quenching applies also there (a factor ≈ 3). It is due to a LL effect, but in order to reach such large quenching the real part of the polarization propagator should include not only the Δ piece but also the 2p2h one, $\text{Re } \pi^{2p2h}$, which plays a very significant role.

One may wonder if such a situation also occurs at $\omega = 0$. In fact, Desplanques [60] has suggested that *all* the GT quenching arises from the 2p2h contribution. This picture is extreme, but it may be that $\text{Re } \pi^{2p2h}$ plays a role at $\omega = 0$, as it does at $\omega = m_{\pi}$. For this we need an evaluation of $\text{Re } \pi^{2p2h}(q = 0, \omega = 0)$ which we plan to carry out in the near future. Our conjecture at the moment is that the nucleonic excitations (embodied in our effective " π^{Δ} ") play a significant role in the quenching but do not exclude the possibility of other contributions (2p2h, etc.).

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