QUANTITATIVE TESTS OF PION PHYSICS IN SIMPLE NUCLEAR SYSTEMS

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1. THE NEED FOR QUANTITATIVE TESTS OF PION PHYSICS

Frequent questions from particle physicists in the last few years (and in particular from younger particle physicists) have been: is there any real evidence that pions exist inside nuclei? Does not the "evidence" simply reflect evidence for quark-antiquark pairs or for gluon exchanges? Even with pions present, will not their interaction properties and their propagation be radically modified by the nuclear environment?

The first reaction to such a challenge to a nuclear theorist is one of shock and indignation. After all whole meetings are held on mesonic and isobaric effects in nuclei. How can we even contemplate finding evidence for quarks in nuclei, if we do not have clear evidence for pions? This must be elementary textbook material. In fact, the textbooks are singularly evasive on this issue, although it clearly concerns a very central and basic issue in nuclear physics. We are not given clear answers to the following three questions:

- i) How do we know experimentally that virtual pions exist in nuclear systems?
- ii) How accurately do we know that they exist and how well do we know their interactions?
- iii) What evidence is so clear that it is instantly acceptable to a non-specialist? Which are the decisive experimental data?

It is interesting to reflect on the historical reasons for the absence of answers to these questions in the textbooks. In my opinion this is due to the lack of alternative descriptions in the fifties and sixties, when the meson-exchange theories of nuclear forces had their foundations established. The natural long-range force was the one-pion exchange potential (OPEP), and it was just enough to establish and impose the πN coupling. As for the intermediate range, bosonic and isobaric models worked well in practice, although they were a bit complicated. Nucleons were natural sources for all these fields, although the short-range description was phenomenological. The emergence of confined quarks has changed all that. It is no longer evident that baryons exist in the normal sense all the time in nuclei. The forces may be transmitted by quite different mechanisms as compared to meson exchanges.

The consequence of the new situation is that it has become urgent to isolate well-defined, quantitative test cases for our knowledge of nuclear pionic interactions. Contrary to the circumstantial evidence frequent in nuclear physics, we need clear-cut evidence permitting us to reason with a rigour similar to the one met in weak interactions. In other words, we need evidence so compelling that the alternative explanations can be kept to an absolute minimum. Otherwise, it is all too easy to escape the constraints. This condition limits one in practice to considering simple nuclear systems, for otherwise wave-function uncertainties are too great. In addition, it will mostly be practical to consider large effects, rather than small perturbations, since accuracy is best tested when relative errors are small.

By achieving a very quantitative control of the situation and by clearly establishing at what distances the pionic interactions are still well-described, we should obtain accurate and quantitative constraints on quark-bag models, for example. Such bags will overlap at shorter distances. This will have two effects. Firstly, the pion source function due to the bag will be spread out with a form factor. This is no different from the standard πNN form factor, but clearly it is important to establish experimentally the range (and shape) of the form factor. Secondly, the interpenetration of the two bags will cause quarks to flow from one bag to another in quite a different way from other exchange mechanisms. Clearly, it is crucial to establish what is the distance compatible with observable effects of this kind.

With this motivation let us now consider some possible test cases for pionic interactions.

- i) The range of the NN force. This is a classical answer, but it must be made much more precise. In my opinion the three good pieces of evidence (to be discussed below) are a) the P-wave pp scattering lengths, b) the np charge-exchange cross-section, near the forward direction over a wide range of energies and c) the $\pi^0 NN$ coupling constant from the pp forward dispersion relations. All of these are good pionic tests, but none is completely ideal. I would only give these tests a two-star rating.
- ii) The deuteron D-state. Both the quadrupole moment and the asymptotic D/S ratio are pionic to very high accuracy. This is the only evidence I know of which merits a three-star rating. In addition, the deuteron binding is heavily influenced by the pionic tensor interaction, but the exact relationships are difficult to disentangle.
- iii) Various meson-exchange effects, mostly corrections. None of these merit more than a one-star rating as tests, although several are very provocative. In particular, one notes i) the radiative np capture anomaly at threshold [a correction only], ii) backward deuteron electro-disintegration [a large effect mainly due the pionic pair term, but the explanation is rather involved, and the absolute normalization is not very precise], iii) renormalization of g_ρ in nuclear matter [not so accurate with possible alternative explanations, but otherwise striking], etc. We will not discuss these further here. However, one should recognize that indeed such effects provide important secondary tests, although more circumstantial ones.

2. THE ONE-PION EXCHANGE POTENTIAL [STATIC LIMIT]

The potential arising from pion exchange is usually derived as the non-relativistic limit of a $\gamma_5^{(1)} \cdot \gamma_5^{(2)}$ coupling of nucleons 1 and 2. The disadvantage of such a formal derivation is that it obscures totally the close analogy between the OPEP interaction and the classical interaction between two electric dipoles. Let us, therefore, make an explicit derivation of the potential starting from a non-relativistic $(\vec{\sigma} \cdot \vec{\nabla})$ coupling.

For a point source at the origin of the pion field the source function $j_{\pi}^{\alpha}(\vec{x})$ is

$$j_{\pi}^{\alpha}(\vec{x}) = \frac{f}{\mu} \tau^{\alpha} (\vec{\sigma} \cdot \vec{\nabla}) \delta^{(3)}(\vec{x}). \quad (1)$$

We note immediately that this is a dipole source emitting p-wave pions.

The corresponding pion field $\phi^{\alpha}(\vec{x})$ is

$$\phi^{\alpha}(\vec{x}) = -\frac{f}{\mu} \tau^{\alpha} (\vec{\sigma} \cdot \vec{\nabla}) \frac{e^{-\mu r}}{r}. \quad (2)$$

This pion field looks very much like the electromagnetic (e.m.) dipole field $\phi(\vec{x})$ outside a point dipole with dipole moment $\vec{d}$ at the origin:

$$\phi(\vec{x}) = -(\vec{d} \cdot \vec{\nabla}) \frac{1}{4\pi r}. \quad (3)$$

Clearly it is useful to associate an axial dipole moment $\vec{m}^{\alpha} = (f/\mu) \tau^{\alpha} \vec{\sigma}$ with the nucleon creating the pion field.

The OPEP interaction between two point nucleons 1 and 2 is therefore a dipole-dipole interaction in complete analogy with the e.m. dipole-dipole interaction (see Fig. 1).

Fig. 1. The analogy between the electric dipole-dipole interaction and the pionic axial dipole-axial dipole interaction.

The corresponding potential energy is the usual source-field coupling

$$V_{\pi} = \int d^3x j_2^{\alpha}(\vec{x}) \phi_1^{\alpha}(\vec{x}), \quad (4)$$

which gives

$$V_{\pi} = (\vec{\tau}_1 \cdot \vec{\tau}_2) \frac{f^2}{\mu^2} (\vec{\sigma}_1 \cdot \vec{\nabla}) (\vec{\sigma}_2 \cdot \vec{\nabla}) \frac{e^{-\mu r}}{4\pi r} \quad (5)$$

where $r \equiv |\vec{x}_1 - \vec{x}_2|$.

The identical reasoning can be applied to the e.m. dipole-dipole interaction, for which the potential energy is

$$V_{em} = (\vec{d}_1 \cdot \vec{\nabla}) \phi_2(\vec{x}) = (\vec{d}_1 \cdot \vec{\nabla}) (\vec{d}_2 \cdot \vec{\nabla}) \frac{1}{4\pi r}. \quad (6)$$

Clearly Eqs. (5) and (6) have identical structure but for the substitution $1/r \rightarrow e^{-\mu r}/r$.

The potentials can easily be separated into a scalar potential and a tensor potential, using the identity

$$\vec{\nabla}^2 \frac{e^{-\lambda r}}{r} = \lambda^2 \frac{e^{-\lambda r}}{r} - 4\pi \delta^{(3)}(\vec{x}) . \quad (7)$$

The e.m. potential is then

$$V_{em} = -\frac{1}{3} (\vec{d}_1 \cdot \vec{d}_2) \delta^{(3)}(\vec{x}) + \frac{d_1 d_2}{4\pi} \frac{S_{12}(\hat{x})}{r^3} . \quad (8)$$

The δ -function term is well-known from the magnetic dipole-dipole interactions in the theory of atomic hyperfine structure ("Fermi contact interaction"). It is very real in spite of its singular behaviour at $r = 0$. The second term is the usual dipole tensor interaction, varying like r^{-3} at the origin.

The OPEP interaction has much the same structure, apart from the slight complications due to the Yukawa behaviour:

$$\begin{aligned} V_{\pi} &= \frac{1}{3} \frac{f^2}{4\pi} \left[\frac{e^{-\mu r}}{r} - \frac{4\pi}{\mu^2} \delta^{(3)}(\vec{x}) \right] (\vec{\sigma}_1 \cdot \vec{\sigma}_2) (\vec{\tau}_1 \cdot \vec{\tau}_2) \\ &+ \frac{1}{3} \frac{f^2}{4\pi} \left[1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2} \right] \frac{e^{-\mu r}}{r} S_{12}(\hat{x}) (\vec{\tau}_1 \cdot \vec{\tau}_2) \equiv \\ &\equiv V_{\pi}^{\text{scalar}} + V_{\pi}^{\text{tensor}} . * \end{aligned} \quad (9)$$

The main new feature is the appearance of a long-range scalar Yukawa interaction, absent in the e.m. case.

The OPEP has some general outstanding features:

- i) The central OPEP is basically very weak. Indeed, if one considers only the Yukawa part of the central OPEP, it is attractive for $L = 0$. While the deuteron is bound and the singlet NN state is nearly bound, it would require $f^2 > 1.680 \mu/M = 0.222$ to bind a state by this interaction instead of the experimental value $f^2 \approx 0.08$. The central OPEP is three times too weak to bind the deuteron. In addition, the substantial additional short-range repulsion must be compensated for, which requires a further increase of coupling.
- ii) The tensor OPEP, on the other hand, is very strong. It is crucial to the deuteron binding for which it is mainly responsible. Further, in the absence of this interaction, the deuteron has no D-state. The quadrupole moment requires therefore a tensor interaction, and it is the signature of it. The crucial rôle of the tensor OPEP for the description of the quadrupole moment will be discussed in detail in Section 8.

3. THE CASE OF THE P-WAVE NN SCATTERING LENGTHS

The low-energy S-wave NN scattering is not such an easy testing ground for pion physics, for important and uncontrollable contributions can occur from short-range

* For convenience of notation we will in the following write f^2 instead of $f^2/4\pi$ and g^2 instead of $g^2/4\pi$ for the π NN coupling constants.

phenomena. Higher partial waves are much better in this respect, since the centrifugal barrier suppresses the inner region and makes the interaction more peripheral. The practical difficulty is that phase shifts with high L cannot be determined with precision at low energy. A compromise is the P-waves with moderate centrifugal barriers. The P-wave scattering lengths (1P_1 , $^3P_{0,1,2}$) have in the last years been determined with rather good accuracy (Dumbrajs et al., 1983).

Let us investigate the contributions in the Born approximation. The Born contribution to a scattering amplitude of phase shift δ_L in angular momentum state L is for potential scattering of spinless objects

$$\left[\frac{e^{i\delta_L} \sin \delta_L}{k'} \right]_{\text{Born}} = -M \int_0^\infty r^2 \left[j_L(k'r) \right]^2 V(r) dr . \quad (10)$$

Here k' is the c.m. momentum and $j_L(x)$ is a regular spherical Bessel function. As a simple example consider the 1P_1 state with $L = J = 1$, $S = T = 0$. The OPEP in this channel is purely central and repulsive with $V(r) = 3f^2 \exp(-\mu r)/r$. The corresponding Born amplitude is

$$\left[^1P_1 \right]_{\text{Born}}^{\text{OPEP}} = -3f^2 \frac{M}{2k'} Q_1 (1 + \mu^2/2k'^2) , \quad (11)$$

where Q_1 is the Legendre function of the second kind

$$Q_1(x) = \frac{1}{2} x \ln \left[\frac{x+1}{x-1} \right] - 1 . \quad (12)$$

The reason I gave this expression is that you can immediately see the logarithmic singularity due to the pion pole. The singularity occurs extremely close to the physical region at a negative c.m. kinetic energy of $E' = -\mu^2/4M = -5.3$ MeV or at a lab energy of -10.5 MeV. The Born scattering length a_L is much simpler in structure than the general Born amplitude.

$$a_L^B = \frac{M}{[(2L+1)!!]^2} \int_0^\infty r^{2L+2} V(r) dr . \quad (13)$$

It is straightforward to evaluate the OPEP Born terms for the P states. One finds, using the appropriate pion masses, that

{central} + {tensor}			
$a^B(^1P_1):$			3.07 fm^3
$a^B(^3P_0):$	$\frac{2M}{\mu} f^2 \mu^{-3}$	$\left\{ \begin{pmatrix} 1 \\ 1/9 \end{pmatrix} + \begin{pmatrix} 0 \\ -10/9 \end{pmatrix} \right\}$	-3.28 fm^3
$a^B(^3P_1):$		$\left\{ \begin{pmatrix} 1/9 \end{pmatrix} + \begin{pmatrix} 5/9 \end{pmatrix} \right\}$	2.19 fm^3
$a^B(^3P_2):$		$\left\{ \begin{pmatrix} 1/9 \end{pmatrix} + \begin{pmatrix} -1/9 \end{pmatrix} \right\}$	0

(14)

The contributions to the singlet P_1 state and to the triplet states are very different in character.

The singlet state is purely central and dominated by the weak central OPEP, that is usually difficult to isolate. The triplet states have a nine times weaker contribution from the central OPEP, and are dominated by the OPEP tensor interaction in both the 3P_0 and 3P_1 states. In the 3P_2 state the OPEP tensor contribution is much weaker and the central and tensor OPEP Born terms cancel exactly.

It is very instructive to display the contributions to the Born term in coordinate space. In Fig. 2a we show the integrand for the 1P_1 Born term, which for OPEP is simply $r^3 e^{-\mu r}$. One notes the very long range with a maximum at 4.2 fm. For comparison, we have indicated the modifications when the Born term for the Paris potential (Lacombe and co-workers, 1980) is used. One sees immediately that these are likely to have little effect on the calculated amplitude. In Fig. 2b the corresponding interaction density for the 3P_1 state is given for the Born OPEP case, for

Fig. 2a. The Born integrand in coordinate space for the 1P_1 scattering length for pure OPEP and for the Paris potential. Rescattering corrections amount to a few per cent only.

Fig. 2b. The 3P_1 interaction density for OPEP and the Paris potential as compared to the OPEP Born term. Note the rescattering corrections of about -15%. The long-range additional reduction for the Paris potential (-6%) is due to the long-range 2π exchange.

OPEP with rescattering, and for the Paris potential with rescattering. In this case rescattering effects are more important than for the 1P_1 case, but the integrands are very similar in their general character. The numerical comparison of the OPEP Born values for the scattering lengths and measured or deduced values are given in Table 1.

TABLE 1 Comparison of OPEP Born terms with Measured and Calculated Values for the P-Wave Scattering Lengths

Scattering lengths (fm ³)	Born	Barker ^{a)}	Van der Sanden ^{b)}	Nagels ^{c)}
$a(^1P_1)$	3.07	-	-	3.02
$a(^3P_0)$	-3.28	-4.82 ± 1.11	-2.71 ± 0.34	-2.84
$a(^3P_1)$	2.19	1.78 ± 0.10	1.97 ± 0.09	1.99
$a(^3P_2)$	0	-0.317 ± 0.033	-0.316 ± 0.016	-0.29

a) M.D. Barker and co-workers, *Phys. Rev. Lett.* 48, 918 (1982), (measured).

b) W. van der Sanden and co-workers, Nijmegen preprint 1982 (analysis).

c) M.M. Nagels and co-workers, in *Few Body Systems and Nuclear Forces* (ed. H. Zingl), Springer-Verlag, Berlin, 1978, p. 17 (semi phenomenological calculation).

The close agreement between the Born results and the experimental or deduced values is very striking. The most accurate test of OPEP is given by the 3P_1 scattering length, which is known experimentally to about 5%. The OPEP Born prediction of 2.19 fm is reduced by the rescattering in the repulsive interaction to 1.88 fm for a pure OPEP. This agrees clearly with the directly measured value 1.78(10) fm. One sees from Fig. 2 that the test mainly concerns the very long part of the interaction with only about a 10% contribution inside 2 fm. The 3P_1 state tests primarily the OPEP tensor interaction. A direct measurement of the 1P_1 scattering length would be of considerable interest as a test of the central part of OPEP. To my knowledge there does not exist a pedagogical and transparent discussion of the detailed model dependence of the P-wave scattering lengths in the literature, but there can be no doubt that they give a very direct test of the long-range OPEP interaction.

4. THE CASE OF THE OPEP POLE IN FORWARD NN DISPERSION RELATIONS

The low-energy NN scattering has clearly very important OPEP contributions, as we noted in the previous section. The P-wave scattering lengths mainly originate at very large distances and are therefore primarily governed by the πN coupling constant. The strength (i.e. the coupling constant) of the OPEP potential can be determined directly from low-energy NN data using forward dispersion relations. In fact, the pion term in the dispersion relation is exactly a Born contribution. For an ordinary potential the Born term is the Fourier transform $\tilde{V}(\vec{q})$ of the potential. For forward scattering this is simply a constant $\tilde{V}(\vec{q}=0)$. This is not yet a very useful statement. The OPEP has exchange character, however. Exchange potentials are important for identical particles, since the full scattering amplitude is the coherent superposition of scattering through θ and $\pi - \theta$, where θ is the scattering angle. Forward scattering has then contributions both from $\vec{q}=0$ and $\vec{q}=2\vec{k}$ (i.e. from the u-channel). Consequently, the Born term contains also $\tilde{V}(2\vec{k})$. For a pionic Yukawa potential this gives a pole term $[(2k)^2 + \mu^2]^{-1}$. In kinetic energy the pole position is at $E_{cm} = -\mu^2/4M \approx -5.3$ MeV, or at lab. energy $E_{lab} = -\mu^2/2M = -10.5$ MeV. The occurrence of this pole is, therefore, a non-relativistic potential problem,

although it is of course easy to express it in a formally field theoretic way. An analysis of pp data according to these lines determines the π^0 NN coupling constant. When high precision is required, data must be used in the first few hundreds of MeV of kinetic energy. An improvement of a previous analysis by Viollier and co-workers (1973) has recently given the value $g_0^2 = 14.52 \pm 0.45$ (Kroll, 1981). The extrapolation of the modified discrepancy function is shown in Fig. 3. The very short region of extrapolation should be noted, as well as the smoothness of the extrapolated function. While this dispersion approach yields the coupling strength, it tells strictly nothing about the practical validity of OPEP in the short-range region. The approach is therefore not very convincing as a demonstration of the importance of OPEP in nuclear interactions. The fact that the coupling is given to high accuracy by low-energy NN data emphasizes, however, that OPEP has a large influence on them.

Fig. 3. The modified discrepancy function $\tilde{\Delta}_S^{pp}(T)$ versus lab. kinetic energy T . Note the suppressed origin. The figure is by courtesy of P. Kroll.

5. THE CASE OF $np \rightarrow pn$ NEAR THE FORWARD DIRECTION

The np charge-exchange reaction near 0° has played a central role in the discovery and exploration of nuclear Gamow-Teller (GT) states. Everybody is now acutely aware of the close connection of the GT resonances and nuclear pion physics. What is the situation for this reaction for the nucleon itself? The empirical situation is very remarkable. Consider first the 0° differential cross-section in the lab. system (see Fig. 4).

Fig. 4. Differential $np \rightarrow pn$ lab. cross-section at 0° as a function of lab. energy (Hürster and co-workers, 1980). The cross-section has almost the same value even at 10 GeV/c.

In the region from about 100 to 600 MeV, in which accurate data are available, the cross-section is remarkably constant. This constancy persists up to beyond 10 GeV. Indeed, there are few quantities in nuclear physics having such a constant behaviour with energy.

The variation with momentum transfer is also universal. In Fig. 5 I show the relative $d\sigma/dt$ versus t at 360 and at 580 MeV. The curves are indistinguishable. In the

Fig. 5. The relative differential $np \rightarrow pn$ cross-section as a function of t for lab. energies 360 and 580 MeV. The shape is nearly the same at other energies including the multi-GeV region. [From Hürster and co-workers (1980).]

multi-GeV region similar differential cross-sections exist, but with less accuracy. These have very similar shape. The characteristic region of variation of t is $\mu^2 = 0.02 \text{ (GeV/c)}^2$, the pion mass squared. It is, therefore, clear that the pion interaction is crucial in the simple and universal behaviour of the charge-exchange cross-section. Potentially this reaction is therefore a good test case.

The constancy of the forward cross-section would be a natural consequence of the Born approximation. Indeed, for a potential $V(\vec{x})$ the forward Born amplitude in the lab. system is

$$(0^\circ \text{ Born amplitude}) \propto \int V(\vec{x}) d\vec{x} = \text{const.} \quad (15)$$

The cross-section can be directly calculated in one of the current NN models. It is found for example in the Paris model (Lacombe and co-workers, 1981), that the data are described very well with no special assumption (see Fig. 6). Let us therefore try the Born approach. The charge-exchange reaction has a t -channel contribution due to $\pi^\pm$ exchange, and a u -channel contribution due to π^0 exchange (Fig. 7).

The cross-section in the helicity representation is described by the five amplitudes Φ_i with

$$\frac{d\sigma}{dt} = \frac{1}{(Mk_{\text{lab}})^2} \frac{\pi}{2} \left(\sum_{i=1}^4 |\Phi_i|^2 + 4|\Phi_5|^2 \right). \quad (16)$$

Fig. 6. The experimented charge-exchange cross-section at $T_{\text{lab}} = 324$ MeV compared with theoretical values from the Paris potential. [From Lacombe and co-workers (1981).]

Fig. 7. The t-channel and u-channel pion Born contributions to charge-exchange.

The pionic contributions are the following:

$$\left. \begin{aligned}
 &\underline{\pi^\pm \text{ t channel}} \\
 &\Phi_1 = -\Phi_2 = -g^2 \frac{(-t)}{(\mu^2 - t)} ; \quad \Phi_2 \equiv \Phi_4 \equiv \Phi_5 = 0 \\
 &\underline{\pi^0 \text{ u channel}} \\
 &\Phi_1 \equiv \Phi_2 \equiv \Phi_5 = 0 ; \quad \Phi_2 = \Phi_4 = -\frac{1}{2} g^2 \frac{(-u)}{(\mu^2 - u)} \approx -\frac{1}{2} g^2 .
 \end{aligned} \right\} \quad (17)$$

This approach fails, however, and gives a wrong angular distribution of approximately the shape of Fig. 8.

A major reason is that the Born approximation cannot be applied to the point OPEP of Eq. (9). The central part of the potential contains a δ function. As a consequence, the S-wave Born amplitude will violate unitarity. In fact, the Born amplitude gives for $V(\vec{r}) \propto \delta^{(3)}(\vec{r})$

$$\left[e^{i\delta_0} \sin \delta_0 \right]_{\text{Born}} \propto k' \int_0^\infty \delta^{(3)}(\vec{x}) \left[j_0(k' r) \right]^2 d\vec{x} \propto k' . \quad (18)$$

Fig. 8. The qualitative shape of pure pionic Born contributions to the relative differential $np \rightarrow pn$ cross-section versus t compared with the experimental shape. The need for modifications is obvious.

With increasing k' the r.h.s. will grow indefinitely, while the l.h.s. stays finite. This is a sure sign that perturbation theory is invalid for $L = 0$. We must therefore be prepared to modify at least the S-wave. The problem occurs already in the region of a few hundred MeV, where the S-wave phase shift is small. The simplest approach is to remove the δ function by hand. This gives an approximate forward cross-section

$$\pi \frac{d\sigma^{\text{lab}}}{d\Omega} (0^\circ) \approx 2\pi \left(\frac{g^2}{2M} \right)^2 \approx 141 \text{ mb} . \quad (19)$$

This value is very close to the observed cross-section ($158 \text{ mb} \pm \text{a few \%}$).

A detailed understanding of the physics of this process does not exist at present. At high energy various approaches have been used involving removal of the δ function (Poor Man's Absorption Model), conspiracies between Regge trajectories etc. (Jones, 1980). On the other hand, no such assumptions are needed for the apparently identical physics at low energies. A very plausible suggestion by M. Ericson (1983), explored further by Gersten (1983), is that the most central waves are suppressed by the nucleon pair correlations, so that the pionic character comes from the higher partial waves. A preliminary analysis along these lines gives excellent results (Gersten, 1983). Awaiting a better understanding of the detailed physics, the np charge exchange can only be noted down as an example of a pion-dominated process, which as yet lacks a quantitatively and pedagogically satisfactory description.

6. ACCURATE TEST CASES OF PIONIC INTERACTIONS

While the previous cases doubtlessly are very pionic, they are unsuitable as test cases in a wider sense for several reasons. Both the P-wave scattering lengths and the dispersion relation sample the asymptotic aspect of the interaction only. The scattering lengths have, in addition, only moderate precision. They say little about the pionic interaction at intermediate range. The charge-exchange interaction is in addition only partially understood in its details. Far more accurate quantitative understanding can be obtained from the deuteron D/S ratio and quadrupole moment which are related. The reason why these can be discussed to extreme accuracy is, on the one hand, the very high experimental accuracy (the D/S ratio is $0.0271 \pm \text{a few per cent}$; the quadrupole moment is $0.2859 \text{ fm}^2 \pm 0.1\%$), and on the other hand the well-understood general properties of the deuteron. In view of the high accuracy of the following discussion it will be necessary to examine both the analysis and the consistency to a higher degree of stringency than usual.

7. THE CASE OF THE DEUTERON D/S RATIO η

i) This quantity has recently been carefully analysed (Ericson and Rosa-Clot, 1982, 1983). Consider a potential approach describing the deuteron S- and D-wave functions $u(r)$ and $w(r)$. For large r these have the form

$$\left. \begin{aligned} u(r) &\xrightarrow{r \rightarrow \infty} A_S e^{-\alpha r} \\ w(r) &\xrightarrow{r \rightarrow \infty} \eta A_S e^{-\alpha r} \end{aligned} \right\} . \quad (20)$$

The two wave functions are tied together by a pair of coupled differential equations. The D-wave has the equation

$$w'' - [\alpha^2 + 6/r^2 + U_{22}(r)] w = \sqrt{8} U_T(r) u(r) . \quad (21)$$

Here, $U_{22} \equiv MV_{22}$ is the $L = 2$ rescattering potential (heavily repulsive because of OPEP), and $U_T = MV_T$ is the tensor interaction, which couples S and D states. Equation (21) for $w(r)$ has a driving term on its r.h.s. owing to the coupling to the S-state. As a consequence the D-state originates from this source term with propagation in the repulsive $L = 2$ potential (see Fig. 9).

Fig. 9. The D-state centrifugal and OPEP potential. Note the strongly repulsive character.

The normal approach is to solve straightforwardly for $u(r)$ and $w(r)$. The problem with this approach is that it is not very clear which properties are model-dependent and which are not. We have therefore adopted a different philosophy. Our approach (Ericson and Rosa-Clot, 1982) involves the following steps:

- assume that $u(r)$ is reasonably well-known, in particular for larger r .
- generate the D-state wave function $w(r)$ by explicit construction from the source term.
- use the fact that the repulsion in the D-state eliminates the short-range contribution controllably.

The technical means by which this is achieved is a Green function approach, which permits an exact formal solution for η .

Consider the $L = 2$ Green function $G_2(r, r')$ of Eq. (21). It obeys the equation

$$G_2''(r, r') - [\alpha^2 + 6/r^2 + U_{22}(r)] G_2(r, r') = \delta(r-r') . \quad (22)$$

As a consequence, the exact formal solution for $w(r)$ is

$$w(r) = \int G_2(r, r') \sqrt{8} U_T(r') u(r') dr' . \quad (23)$$

From the knowledge of the two exact solutions $J_2(r)$ and $H_2(r)$ of the homogenous equation with the properties that $J_2(r)$ is regular at the origin and $H_2(r)$ vanishes exponentially at infinity, we have

$$G_2(r, r) = \alpha r_{<} r_{>} J_2(r_{<}) H_2(r_{>}) . \quad (24)$$

The normalizations are chosen so that

$$\begin{aligned} r J_2(r) &\xrightarrow{r \rightarrow \infty} -\frac{1}{2} \alpha^{-1} e^{\alpha r} \\ r H_2(r) &\xrightarrow{r \rightarrow \infty} \alpha^{-1} e^{-\alpha r} . \end{aligned} \quad (25)$$

If the rescattering potential $U_{22}(r)$ in the D-state is neglected [$U_{22} = 0$], these functions are the usual spherical Bessel functions

$$\left. \begin{aligned} J_2(r) &\equiv j_2(i\alpha r) \\ H_2(r) &\equiv h_2^{(1)}(i\alpha r) \end{aligned} \right\} \text{ for } U_{22} = 0 . \quad (26)$$

We can then consider $w(r)$ for large r and find

$$w(r) \xrightarrow{r \rightarrow \infty} \alpha r H_2 \int_0^\infty r' J_2(r') \sqrt{8} U_T(r') u(r') dr' . \quad (27)$$

It is immediately seen that there is an exact expression for the D/S ratio

$$\eta \equiv \int_0^\infty \eta(r) dr = \sqrt{8} \int_0^\infty r J_2(r) U_T(r) \tilde{u}(r) dr . \quad (28)$$

In the basic relation (28) we have normalized the S-wave function at ∞ , so that $\tilde{u}(r) \xrightarrow{r \rightarrow \infty} e^{-\alpha r}$. The importance of the formulation (28) is that it is easy to control uncertainties and model dependence of η in this form. In fact, as a consequence of the repulsion from the centrifugal barrier as well as from the OPEP in the D-state, the regular solution J_2 describes barrier penetration of a wave coming in from infinity with an exponential fall-off, as it approaches the origin. This is the mechanism which eliminates the inner region efficiently and controllably.

ii) Let us now approach the problem stepwise, starting from crude approximations. Our purpose is to separate the irrelevant from the essential features.

A very crude approximation is the Born approximation [$U_{22} = 0$]. This neglects rescattering, so that $J_2(r) \approx j_2(i\alpha r)$, which gives approximate proportionality to r^2 in practice. The integrand $\eta^{\text{Born}}(r)$ is therefore proportional to $r^3 U_T(r) \tilde{u}(r)$. For small r , $U_T^{\text{OPEP}} \propto r^{-3}$. There is then no singularity in the integrand, for $r^3 U_T^{\text{OPEP}} = \text{constant}$. Straightforward evaluation of the OPEP Born term, using any reasonable S-wave function, gives

$$\eta_{\text{Born}}^{\text{OPEP}} = 0.033 . \quad (29)$$

This is an excellent first approximation, although it is 20% larger than experiments. The spatial contributions to the Born integrand come mainly from outside 1 fm (see Fig. 10).

Fig. 10. A comparison of the shape of the integrand $\eta(r)$ of Eq. (28) for a) OPEP Born term, b) OPEP with rescattering, and c) Paris potential. Note the small difference (-4%) between the curves b) and c). [From Ericson and Rosa-Clot (1982).]

As a second approximation let us assume a pure point OPEP interaction and solve for $J_2(r)$. The change in the shape of the integrand is seen in Fig. 10: the repulsion discriminates heavily against the region inside 1.5 fm. The resulting OPEP value of η is in comparison with the Paris potential value (using the identical πN coupling constant for comparison)

$$\begin{aligned}\eta^{\text{OPEP}} &= 0.02762 \\ \eta_{\text{Paris}} &= 0.02633 \\ \eta_{\text{exp}} &= 0.0271 \pm \text{few \%} .\end{aligned}\tag{30}$$

Clearly this is a tremendous success, since we have hardly started the discussion yet. The entire sophistication of the Paris potential is only a -4% correction. The problem in the following is that corrections could easily make the situation worse, not better.

iii) The next question is how sensitive the result is to the input S-wave function. The answer is: extremely insensitive provided the correct experimental effective range is used. Very little model dependence is introduced in this way, and the absolute normalization of $u(r)$ is nearly irrelevant. In Fig. 11 is shown the different results for the Paris (Lacombe and co-workers, 1980) and the Reid Soft Core (RSC) wave functions $\tilde{u}$ (Reid, 1968). Here it should be noted that the RSC wave function is inconsistent with the experimental effective range, and it is not a very good wave function. In spite of that, the difference in η is only about 0.5% with the main contribution coming from the region of the peak of the integrand at $r \approx 2$ fm. A useful empirical relation of the change η to the S-wave probability is

$$\frac{\Delta\eta}{\eta} \approx \frac{\Delta P_S}{3} .\tag{31}$$

Fig. 11. Dependence of the integrand η_π on the choice of S-wave functions $\tilde{u}(r)$ normalized to $e^{-\alpha r}$ at infinity. Note the very weak dependence on the details of $\tilde{u}(r)$. [From Ericson and Rosa-Clot (1983).]

Therefore a 1% change in P_S leads to only a 0.3% change in η .

iv) Let us now turn to more important corrections and uncertainties (Ericson and Rosa-Clot, 1983). First consider the 2π exchange contributions, which are not simply iterations of OPEP. These are essentially those of the theoretical part of the Paris potential (Lacombe and co-workers, 1980) or any of the other more or less equivalent theoretical dispersive approaches. One may consider these contributions as basically consisting of two ingredients (see Fig. 12), the influence of πN elastic

Fig. 12. The main physical dispersive contributions to the 2π exchange potential in a schematic representation. The " σ " and " ρ " contributions refer to S- and P-wave $\pi\pi$ phase shifts.

scattering in exchange processes and the $\pi\pi$ interaction as described by the known phase shifts in S-wave and P-wave interactions.

Using this dispersive formulation it is a technically complicated, but in essence a relatively direct procedure to derive an equivalent potential interaction for 2π exchange (Cottingham and co-workers, 1973).

$$V_i^{2\pi}(r) = -\frac{1}{2\pi^2} \int_{4\mu^2}^{\infty} \eta_i(t) \frac{e^{-\sqrt{t}r}}{r} \phi_i(\sqrt{t}r, t) dt \quad (32)$$

Here t is (exchanged mass)² and $\phi_i(\sqrt{t}r, t)$ tells us, whether we have a scalar interaction [$\phi_i = 1$], tensor interaction [$\phi_T = (1 + 3/\sqrt{t}r + 3/tr)t$], etc. The important quantity is the weight function $\eta_i(t)$, which contains all the dynamics. In the present case the important interaction is the $T = 0$ tensor potential. This contribution is heavily dominated by P-wave $\pi\pi$ exchange, and there is little theoretical uncertainty (see Fig. 13). We will now show how these contributions show up in practice in coordinate space.

Fig. 13. The isospin 0 2π tensor weight-function $\eta_T(t)$ as a function of the square of the exchanged mass. The isoscalar and isovector contributions $3\eta_T^{(+)}$ and $-6\eta_T^{(-)}$ are shown separately. [From Ericson and Rosa-Clot (1983).]

Let us first see how different regions of masses contribute to the 2π tensor potential. This is shown in Fig. 14, where the units are chosen, for convenience, relative to the tensor OPEP at the same r . The outstanding feature is the heavy discrimination of masses heavier than $\sqrt{t} = 6.5 \mu$. This is because of the exponential function in Eq. (32), which cuts out such contributions very effectively. As a consequence, the 2π exchange is described extremely well even at distances $r \approx 0.5$ fm by masses no heavier than the ρ mass. One also notes that the ρ -meson does not dominate 2π exchange at distances larger than 1 fm, where the exchange is mainly non-resonant.

In order to transform this information more concretely into the contribution to η , let us now display the -4% correction due to 2π exchange explicitly as a function of

the maximum exchanged mass μ_0 . This is shown in Fig. 15. One sees clearly that nearly half the contribution comes from exchanges of less than five pion masses. Near saturation is achieved at six pion masses. The Paris potential (Lacombe and co-workers, 1980) conforms closely to the theoretical value with no apparent influence of its phenomenological components. What is the theoretical uncertainty in the 2π term? The experts are unanimous that the weight function $\eta_T(t)$ is well known

Fig. 14. The contribution of different mass regions to the 2π tensor potential for fixed r . For convenience the contributions are measured in units of the tensor OPEP. Note the important contributions of low masses and the small contributions of masses above 6μ . [From Ericson and Rosa-Clot (1983).]

Fig. 15. The theoretical correction to the OPEP $\eta(r)$ owing to dispersive 2π exchange as a function of r . The 2π exchange has been summed for all masses smaller than μ_0 . The curves are based on the weight function $\eta_T(t)$ of Fig. 13, while the Paris potential is obtained from Lacombe and co-workers (1980). Note the important contribution for masses $\mu < 5$, as well as the well-determined shape in the region below 1 fm. [From Ericson and Rosa-Clot (1983).]

at the relevant masses as far as its shape is concerned. On the other hand, its normalization may be uncertain by $\pm 15\%$ (Loiseau, 1983). The conclusion of all this is that the 2π correction is well in hand to a precision of $0.15 \cdot 4\% = 0.6\%$.

v) An experimental uncertainty is introduced into the description of η by the experimental limits to our knowledge of the πNN coupling constants (Ericson and Rosa-Clot, 1983). An accurate and careful description of the OPEP potential must take into account the small corrections due to the exchange of charged and neutral pions. The correct potential is

$$V_{\pi} = \frac{1}{3} \left[2 V_{\pi c} + V_{\pi 0} \right] . \quad (33)$$

To a few parts in a thousand it is an excellent approximation to use the average masses and coupling constants

$$\begin{aligned} \mu &= \frac{1}{3} [2 \mu_c + \mu_0] \\ f^2 &= \frac{1}{3} [2 f_c^2 + f_0^2] . \end{aligned} \quad (34)$$

The experimental uncertainty in f_c^2 is presently $\pm 1.3\%$ from πN scattering which gives $f_c^2 = 0.0790(10)$, while $f_0^2 = 0.0749(20)$, consistent with pseudoscalar coupling (Dumbrajs and co-workers, 1983). The average experimental coupling constant $f^2 = 0.0776(9)$ is thus presently uncertain by $\pm 1.2\%$ (Ericson and Rosa-Clot, 1983), which is a quite significant uncertainty in the further discussion.

vi) The form factor of the πNN interaction is associated with at least 3π exchange because of quantum numbers. The previous investigation of 2π contributions gives no information on the importance of the form factor. To assess its effects let us first take an empirical approach (Ericson and Rosa-Clot, 1983). Assume the πNN form factor has a monopole form

$$K(t) = \frac{\Lambda^2 - \mu^2}{\Lambda^2 + t} . \quad (35)$$

The equivalent interaction radius is

$$\langle r^2 \rangle_{\pi NN} = 6 \Lambda^{-2} . \quad (36)$$

Our results concerning the form factor are described nearly uniquely by the interaction radius parameter and are nearly independent of further details on the shape of the form factor. Before continuing, it is well worth noting that at this point other effects than the form factor are under tight quantitative control on the per cent level. We will now see that form factors with $\Lambda \lesssim 1$ GeV/c cause very serious problems [and even worse ones for the quadrupole moment (Ericson and Rosa-Clot, 1983) discussed in the next section].

In order to gauge the effect of the form factor on the important tensor potential consider first OPEP with different form factors as compared to the Paris potential (Fig. 16). The general shape of the Paris potential corresponds effectively to $\Lambda_{\text{eff}} \approx 800$ MeV/c.

This is not a form factor effect of course, since we noticed previously that most of this effect is due to 2π exchange. It should be noted in particular that a form factor with $\Lambda \approx 500$ MeV/c modifies the interaction very far out to nearly 2.5 fm, while $\Lambda \approx 800$ MeV/c gives important effects even at 1.5 fm. These effects are seen more clearly by considering $\eta(r)$ for different form factors (Fig. 17). It is use-

Fig. 16. The tensor OPEP for various πNN form factors compared with the Paris potential. The corresponding source radii are $R = 0.63$ fm, 0.84 fm, and 1.26 fm. [From Ericson and Rosa-Clot (1983).]

Fig. 17. The variation of $\eta_\pi(r)$ with various form factors for the πNN vertex. The integral decreases by 3.1%, 8.5%, and 19% in the three cases, which correspond to $R = 0.63$ fm, 0.84 fm, and 1.26 fm. Note the rapid modification of intermediate-range contributions for $\Lambda \lesssim 1000$ MeV/c. [From Ericson and Rosa-Clot (1983).]

ful to think of the cut-off in terms of an equivalent uniform interaction distribution with radius R

$$R = \frac{\sqrt{10}}{\Lambda} \quad (37)$$

From the figure it is apparent that even $\Lambda \approx 800$ MeV/c provokes an 8% decrease in η , which is a very large effect [$R(\Lambda = 800 \text{ MeV/c}) = 0.84$ fm]. If the agreement with the theoretical description is to be preserved Λ must be about 1 GeV/c or larger, i.e. $R \lesssim 0.65$ fm.

This discussion demonstrates the magnitude and the structure of the form factor effects. What is experimentally and theoretically known about Λ ? The answer is: amazingly little. The principal evidence is the following:

- a) np and $\bar{p}p$ charge exchange at 8 GeV/c. Such reactions are frequently believed to give a form factor with $\Lambda \approx 700$ MeV/c (Bongardt and co-workers, 1974). Later analysis by Dominguez and VerWest (1980) have shown that an analysis in a similar spirit gives $\Lambda \approx 900$ MeV/c. As we have discussed previously (Section 5) the np charge-exchange low-energy data look identical or nearly so with no apparent form factor effect. This evidence is therefore probably irrelevant and establishes no more than a lower limit for Λ .
- b) The Goldberger-Treiman discrepancy. The small difference of 6% in the Goldberger-Treiman relation has had only 2% explained by $\Lambda \geq 1100$ MeV/c, which is the best that a number of theoretical efforts have produced. A smaller Λ could fit the data. However, the discrepancy is a small one and other explanations have been suggested (Pagels, 1975).
- c) NN phase shifts. The NN interaction analysis tends to require an effectively point-like π NN interaction. In particular, the 3D_1 phase shift, which depends heavily on the $T = 0$ tensor interaction, behaves in a grossly different way from experimental values for $\Lambda \lesssim 1400$ MeV/c (Holinde, 1981). This evidence is indirect, however, and would require detailed discussion to be compelling.
- d) Theoretical dispersion results. These descriptions of the π NN in terms of 3π contributions give large values of Λ , which are very difficult to push down even to 1 GeV/c, and which tend to favour values of $\Lambda \approx 1.4$ GeV/c or similar (Jones and Scadron, 1975; Nutt and Loiseau, 1976; Loiseau, 1983).
- e) The axial form factor. This form factor is not really related to the π NN form factor. In view of the close relation of many axial and pionic phenomena, it has some relevance, however, that the experimental axial form factor is well known to have $\Lambda_{\text{axial}} \approx 700$ MeV/c (Amaldi and co-workers, 1979).
- f) Quark bag models. Form factors from quark-bag models, which describe the axial form factor, tend to give $\Lambda < \Lambda_{\text{axial}}$, i.e. quite soft π NN interactions (Thomas, 1983).

The upshot of this is that the best evidence (the dispersive theoretical approach and the NN scattering analysis) favours rather hard Λ , but the indications are far from clear cut. It is therefore of some interest to reverse the process and use η to put a limit on Λ . While this will be done more accurately in Section 9 using simultaneous information from the quadrupole moment, we can in fact give the theoretically predicted η as

$$\eta = 0.02608 \left[1 + \frac{(f^2 - 0.0780)}{0.0780} - \epsilon_{\eta}(\Lambda) \right], \quad (38)$$

where $\epsilon_{\eta}(\Lambda)$ is the modification of η due to the form factor. Its value is displayed in Fig. 18. Since the uncertainty in the theoretically predicted η is mainly due to the experimental uncertainty in f^2 [$\pm 1.2\%$] with little model dependence other than that due to Λ , we can make a comparison with the experimentally observed value $\eta_{\text{exp}} = 0.0271(4)$. The predicted and experimental values differ by 2 standard deviations in the opposite direction from a form factor effect. It would be quite awkward to have the form factor decrease the predicted value of η to 0.0251, i.e. by -4%. The effective value of Λ as manifest in η must therefore be about 1 GeV/c or larger or, in terms of an equivalent radius, $R \lesssim 0.65$ fm.

Fig. 18. The modification function $\epsilon_\eta(\Lambda)$ and $\epsilon_Q(\Lambda)$ for η [according to Eq. (38)] and for Q [according to Eq. (43)]. Note the rapidly increasing effect with decreasing Λ . [From Ericson and Rosa-Clot (1983).]

From the present discussion of η we can conclude that whatever the small final corrections, the OPEP interaction describes the D/S ratio accurately, nearly model-independently, and non-trivially.

8. THE CASE OF THE DEUTERON QUADRUPOLE MOMENT

One of the most accurately known nuclear quantities is the deuteron quadrupole moment Q . It has recently been deduced from experiment to a precision of $\pm 0.1\%$. Its value is $Q = 0.2859(3) \text{ fm}^2$ (Ericson and Rosa-Clot, 1983). The quadrupole moment cannot exist in the absence of the nucleonic tensor force, which is pion dominated. As a consequence, it is an excellent candidate for a test. Since the quadrupole moment depends considerably on the asymptotic part of the wave function, the information from this source is similar to, but not identical with, that of the D/S parameter. The discussion will therefore be quite brief on analogous points with emphasis on the specific features involving Q .

Let us first take the potential approach discussed previously. The quadrupole moment can then be defined in terms of the S- and D-wave functions. It is usually broken up into two terms Q_1 and Q_2 :

$$Q = Q_1 + Q_2$$

$$Q_1 = \frac{1}{\sqrt{50}} \int_0^\infty r^2 u w \, dr \quad (39)$$

$$Q_2 = -\frac{1}{20} \int_0^\infty r^2 w^2 \, dr .$$

The dominant term is Q_1 , which contributes 106%, while Q_2 is a -6% correction. This last term is numerically $Q_2 = -0.0182 \text{ fm}^2$. It is known quite accurately ($\pm 0.3\%$) as compared to the total quadrupole moment so we will not discuss it further.

The important term is the interference term Q_1 . This can be rewritten identically using the Green-function technique and related to $\eta(r)$.

Straightforward algebra and the use of Eqs. (20), (23), and (28) gives

$$Q_1 = \frac{A_S^2}{\sqrt{50}} \alpha^{-3} \int_0^\infty dr \eta(r) \left\{ \frac{\alpha^3}{r J_2(r)} \int_0^\infty G_2(r, r') r'^2 \tilde{u}(r') dr' \right\} \quad (40)$$

Denoting the brace in Eq. (40) by $F(r)$, we can express Q_1 as

$$Q_1 = \text{const} \times \int_0^\infty dr \eta(r) F(r) \quad (41)$$

Consequently the dominant part of the quadrupole contribution is simply $\eta(r)$ modified by a weight function $F(r)$. The shape of the weight function is nearly model independent apart from small modifications inside 1 fm. This is displayed in Fig. 19 for several rather extreme assumptions.

Fig. 19. Model dependence of the quadrupole weight function $F(r)$ using a) the asymptotic wave function $\tilde{u}(r)$ and no interaction, b) a realistic wave function $\tilde{u}(r)$ with no interaction compared with c) a realistic $\tilde{u}(r)$ and OPEP and d) a realistic $\tilde{u}(r)$ and full Paris potential. For comparison the shape of $\eta(r)$ using OPEP is given. Note that the folded value $F(r) \eta(r)$ is nearly model independent. [From Ericson and Rosa-Clot (1983).]

In the figure is also shown the shape of $\eta(r)$. It is clear that the product $\eta(r) \times F(r)$ is extremely stable, and that it is difficult to introduce errors from this source. The uncertainties in Q_1 are essentially those of $\eta(r)$. This is seen in Fig. 20, which gives the integrand of Q_1 comparing the pure OPEP and the Paris model. Its behaviour is of the same nature as the corresponding curve for $\eta(r)$ in Fig. 10.

In order to keep the sources of uncertainty in mind let us simply briefly list them.

Fig. 20. Dependence of the integrand for Q_1 on the interaction. The difference occurs nearly entirely due to changes in $\eta(r)$. The difference between the OPEP and Paris potential value is -6.3%. Compare the corresponding information for $\eta(r)$ in Fig. 10. [From Ericson and Rosa-Clot (1983).]

i) The weight function $F(r)$.

Its effect is negligible ($\leq 0.2\%$). Even free propagation with 60% violation of unitarity in $u(r)$ gives the quadrupole moment to a precision of 5%! With a more realistic approach, the uncertainty is nearly eliminated.

ii) Dependence on $\eta(r)$.

a) Since the integrand at intermediate range is very similar to that of $\eta(r)$ with somewhat different weighting, all intermediate range effects [like $\pi\pi\pi$ form factor, etc.] scale as follows:

$$\frac{\Delta Q}{Q} \approx 1.7 \frac{\Delta \eta}{\eta}. \quad (42)$$

b) The long-range proportionality of OPEP to f^2 gives a direct linear relation between η and Q due to changes in f^2 .

iii) The quadratic D-state term Q_2 .

This introduces a $\pm 0.3\%$ uncertainty.

iv) The proportionality of Q to A_S^2

The experimental uncertainty in the asymptotic S-wave amplitude gives a $\pm 0.2\%$ uncertainty in Q . Many of the theoretical approaches in the literature are not accurate enough in the value they use for this number.

v) The $\pi\pi\pi$ form factor

Since the weight function $F(r)$ emphasizes the inner regions more than for $\eta(r)$ the sensitivity to Λ is greater in Q than in η (see Fig. 18). This will be further discussed.

vi) Non-potential effects such as meson-exchange currents

These will be discussed later on.

We can therefore write the theoretical potential model prediction of Q as

$$Q_{\text{pot}} = 0.2785 \left[1 + \frac{(f^2 - 0.0780)}{0.0780} - \epsilon_Q(\Lambda) \right], \quad (43)$$

where $\epsilon_Q(\Lambda)$ is the form factor correction of Fig. 18. The practical uncertainty in this expression is about 1.2%, nearly entirely due to the experimental uncertainty in f^2 , if Q is taken independent of η . If η is known, the uncertainty in Q is reduced to less than 0.5%. As compared to the experimental value for Q , the value Q_{pot} from Eq. (43) is 3% lower using $\Lambda = \infty$ and the experimented value of f^2 . Such a comparison cannot be made validly, however, before a correction has been made for isobar, exchange-current and relativistic effects, which now will be discussed.

The meson-exchange current correction (MEC) to the deuteron quadrupole moment concerns the isoscalar electric part. There is no consensus on how to treat this contribution. The basic calculation is straightforward as far as pure evaluation is concerned; the problem is conceptual (and controversial). It deals with the terms to be retained (Friar, 1980). Still there is a consensus up to a point: there is agreement that the pionic terms are dominant. There is also agreement that the final quadrupole moment will be identical to leading order in these corrections whether a pseudoscalar, a pseudovector, or a mixed pion-exchange interaction is used consistently. The basic problem is that all practical approaches using potentials are mixtures of OPEP and some phenomenology. In addition, the energy- and velocity-dependence is usually not included consistently.

Let us briefly see what happens. The main starting point is either pseudoscalar (PS or $\mu = 0$) coupling, favoured in NN potential approaches, or pseudovector (PV or $\mu = 1$) coupling, preferred in soft pion approaches. An energy-independent OPEP requires the combination ($-PV + 2$ PS or $\mu = -1$). This is in a sense the most static potential possible, but it has also very large MEC terms (+4%), so that it is by no means obvious that this is the most convenient formulation.

As a concrete example consider the Paris potential. In its original formulation it is based on PS coupling. However, the energy dependence in the central potential is later transformed into a velocity dependence proportional to $\vec{p}^2$. This has no practical effect in our case. In addition, the energy dependence of the tensor potential (numerically small) is simply neglected. If the energy dependence is simply ignored, the Paris potential should have $\mu = 0$ (PS) MEC corrections. These give $\Delta Q = +0.006 \text{ fm}^2$ (+2%). While the treatment is not fully consistent, this is still the reasonable correction to make, since the inconsistency is a small one.

What are the dangers of using energy-dependent potentials indiscriminately? They lead to changed orthogonality relations and to a changed definition of the probability and charge density!

Consider the potential $V_E = V_0 + EV_1$ with the Schroedinger equation

$$(T + V_0 + EV_1)\Phi = E\Phi. \quad (44)$$

By the usual procedure of multiplying two solutions Φ_1 and Φ_2 of energy E_1 and E_2 , we have

$$(E_1 - E_2) \int \Phi_1 V_1 \Phi_2 d\vec{x} = (E_1 - E_2) \int \Phi_1 \Phi_2 d\vec{x}. \quad (45)$$

Therefore $\int \phi_1 \phi_2 d\vec{x} \neq 0$, but instead we have the orthogonality relation

$$\int \phi_1 (1 - V_1) \phi_2 d\vec{x} = 0. \quad (46)$$

For the same reason the probability and charge densities are now defined by

$$\rho = (1 - V_1) \phi^2. \quad (47)$$

Such a charge density becomes particularly important in our case if the energy-dependent potential V_1 is a tensor potential. A part of the quadrupole charge density is then generated by u^2 . A consequence of this phenomenon is that the D-state probability P_D is no longer well defined.

The reason is obvious: the energy-dependent tensor potential introduces a "twisted metric" in the system, although the physics has not changed. It is just as if we had chosen a deformed coordinate system. Different mixtures of PS-PV approaches lead to a different degree of energy dependence in the OPEP, and hence to different ϕ^2 . This is an important reason for the difference in MEC contributions. In addition to the choice of the PS-PV mixture, the choice of gauge is also important. Once more physics has not changed, but full consistency is necessary. The most common choice is $v = \frac{1}{2}$ (no recoil). Typical corrections are given in Table 2.

TABLE 2 ΔQ for Various Gauges and OPEP
Choices according to Kohno (1983)

μ	v		$\Delta Q \times 10^3$ (fm ²)	$\Delta Q/Q$ (%)
0	$\frac{1}{2}$	PS	6	+2
1	$\frac{1}{2}$	PV	3	1
-1	0	static	8	3
-1	$\frac{1}{2}$	static	10	+4

The most plausible value in the present case is $\mu = 0$, $v = \frac{1}{2}$, which is a +2% contribution. This contribution should be subtracted from Q_{exp} for a comparison with the predicted value (43). It is noteworthy that whatever version is chosen in Table 2, the MEC correction is only a few per cent. Therefore, no major uncertainty is introduced by this correction. We will take it to be $\Delta Q_{\text{MEC}} = (0.006 \pm 0.003) \text{ fm}^2$ ($2 \pm 1\%$), where the uncertainty reflects the uncertainty in the choice of μ and v .

An additional contribution to Q may come from a non-nucleonic content of the deuteron such as isobars. This is a short-ranged effect, which basically is suppressed because of the r^2 weighting of the integrand (39). Indeed, explicit investigations (Hadjimichael, 1978; Vassanji and co-workers, 1981) show that the modifications are not due the isobar contributions themselves, but indirectly to the depletion of the nucleon probability by the isobars. Since the entire method we use is based on the insensitivity to exact normalization to unity, the effect is quite small. In fact, a 1% change in nucleon probability implies only as $\frac{1}{3}\%$ change in η and a 0.5% change in Q according to Eqs. (31) and (42).

Hence we can conclude that the MEC and isobar corrected experimented quadrupole moment is $(0.2859 - 0.006) = (0.280 \pm 0.003) \text{ fm}^2$. The potential quadrupole moment

with point-like OPEP coupling gives, according to Eq. (43), a predicted value $(0.277 \pm 0.003) \text{ fm}^2$ with $f^2 = 0.0776(9)$, the uncertainty being due to the coupling constant. There is agreement between these numbers within experimental errors. The quadrupole moment is more restrictive than η for deviations from point-like πNN coupling and indicates $\Lambda \gtrsim 1400 \text{ MeV/c}$.

9. THE JOINT CASE OF η AND Q

While the analysis of the D/S and quadrupole moment gives strong evidence for accurate pionic dominance, the data can be better visualized in a more general way when analysed together (Ericson and Rosa-Clot, 1983). Using Eq. (42), we have an approximately linear relation between η and Q owing to intermediate-range effects in different theoretical approaches. This relation should be observed by all previous potential models. These cannot be immediately compared, however, for they do not use the same πN coupling constants, and they do not conform to the experimental value for A_S . These effects can be eliminated by considering $Q/A_S^2 f^2$ and η/f^2 instead. Indeed, Fig. 21 shows that all the major approaches neatly correlate on a universal straight line to high precision. When the experimental values for Q/A_S^2 and η are introduced, however, the corresponding (accurate) point, deviates from this universal line by about 2 standard deviations. This is mainly due to experimental uncertainty in η . In order to understand the importance of form factor effects, it should be noted that a πNN vertex function with $\Lambda \approx \Lambda_{\text{axial}} \approx 700 \text{ MeV/c}$ is far off scale to the left of the figure with $\eta \approx 0.0230$. This is a difficulty for various quark-bag models, for which $\Lambda \approx \Lambda_{\text{axial}}$ (Thomas, 1983).

The implications of the data can also be visualized as a plot of η versus f^2 with the quadrupole moment as a constraint (Fig. 22). Just as Fig. 21 it shows that present experimental data are slightly inconsistent with any value of η and Q , but not big enough to cause serious worry. The figure emphasizes the great difficulty in describing these data by πNN couplings with $\Lambda \lesssim 1000 \text{ MeV/c}$.

Fig. 21. The calculated values for Q_{pot}/A_S^2 in various standard theoretical approaches. Both Q_{pot} and η have been reduced to a "standard" value f_0^2 for the πN coupling constant by multiplication by f_0^2/f^2 . Note the high accuracy in the correlation on the universal line. [From Ericson and Rosa-Clot (1983).]

Fig. 22. The η - f^2 plane. The diagonal lines are constant Λ contours with the area to the left of $\Lambda = \infty$ unphysical. The more horizontal straight lines correspond to constant Q_{pot} . The values Q_a , Q_b , and Q_c correspond to the following MEC assumptions:
a) $\delta Q = 0$; $Q_a = Q_{\text{exp}} = 0.2859 \text{ fm}^2$;
b) $\delta Q = 0.009 \text{ fm}^2$; $Q_b = 0.277 \text{ fm}^2$ (Vassanji and co-workers, 1981);
c) $\delta Q = 0.013 \text{ fm}^2$; $Q_c = 0.273 \text{ fm}^2$ (Hadjimichael, 1978). Note that the MEC corrections above are larger than expected. [From Ericson and Rosa-Clot (1983).]

10. CONCLUSION

The previous examination of various test cases for pionic interactions shows that OPEP appears naturally in the qualitative description of several directly measured quantities in the NN systems. There are notably the P-wave scattering lengths, the np nearforward charge exchange, the D/S ratio and quadrupole moment. To make a high accuracy check it is natural to require that the intermediate-range region as well as non-trivial rescattering effects should be severely tested. Only the D/S ratio and quadrupole moment provide accurate tests at present in this case. On the other hand, the theoretical uncertainties in the analysis are by now so small that the experimental uncertainty in the πN coupling constant is a major limitation.

The present results are not only a test of pionic nuclear interactions, but in addition the extremely high accuracy of the description provides an important challenge in the present efforts to describe nuclear forces using a quark picture. There is no doubt that an important necessary requirement of such models is that they should be able to explain quantitatively why the apparent πNN form factor is point like.

In the present discussion we deliberately eliminated the pionic evidence from complex nuclei. The reason for this was twofold. Firstly, the experimental accuracy of the known cases are far from that attainable in the case of the deuteron. Secondly, the large number of variables increases the number of possible "escape-

clauses" in the case of disagreements, so that a non-controversial discussion is more difficult. This is unfortunately a fact of life. While I am personally convinced by other nuclear evidence, although it is less compelling than the one discussed here, many of my particle friends find it much less acceptable. The case of the deuteron alone is, however, quite sufficient as a clear-cut quantitative test.

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APPENDIX: EXPERIMENTAL METHODS FOR DETERMINATION OF η AND Q

It is interesting to examine briefly the principles of the experimental techniques so as to appreciate properly the model dependence.

A. The Subcoulomb Method

Stripping reactions below the Coulomb barrier in heavy nuclei can in selected cases provide η very accurately. At first this seems an unlikely source for high precision, but on closer examination it is a natural consequence. Consider a deuteron below the Coulomb barrier incident on a high Z element (see Fig. 23). The strongly repulsive Coulomb field keeps the deuteron far away from the nuclear interaction. In a stripping process like $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ a neutron must tunnel from the deuteron orbit to the Pb nucleus. The tunnelling will be greatly facilitated if the deuteron "cigar" is oriented, so that the neutron is as close to the Pb nucleus as possible (see Fig. 23). Since this orientation as well as the deformation of the deuteron can only be achieved by its D-state component, it is not surprising that the process is sensitive to this amplitude. The orientation of the deuteron is obtained by using a polarized beam. In order to increase the sensitivity it is judicious to choose the Q -value of the reaction as close to $Q = 0$ as possible. This is a clever choice, since it corresponds to tunnelling even at a distance $r = \infty$ (resonance condition). It has, in addition, the consequence that the outgoing proton follows exactly the same classical Coulomb trajectory as the deuteron with no mismatch. There exist only a small number of good cases with $Q \approx 0$. Using the reaction $^{208}\text{Pb}(d,p)^{209}\text{Pb}$ with $Q = 0.3$ MeV, Goddard and co-workers (1982) have found

$$\eta = 0.0271(8) [\pm 3\%] . \quad (48)$$

Fig. 23. Schematic picture of the classical orbit of a deuteron in a subcoulomb stripping reaction. [From Knutson (1977).]

The most dangerous correction is the small long-range tail of the nuclear tensor potential (a few per cent $\pm$ few per cent). The interaction region in such experiments is quite far outside the nucleus, with very little nuclear overlap as seen in Fig. 24 for deuterons of 7 MeV energy.

Fig. 24. Overlap integral for subcoulomb stripping for polarized deuterons at $E_d = 7$ MeV in the $^{208}\text{Pb}(d, p)^{209}\text{Pb}$ reaction. Note the peak of the interaction far outside the nuclear region. [From Goldfarb (1966).]

B. $(\vec{d}, p)$ Elastic Scattering

Consider the exchange graph of the (d, p) elastic scattering (Fig. 25). This process has a pole very close to the physical region corresponding to $\vec{k}_n^2 + \alpha^2 = 0$ in the c.m. system. The position of the pole corresponds to

$$\cos \theta_{\text{CM}}^{\text{pole}} = -1.25 - 2.25 B/E_d. \quad (49)$$

Fig. 25. Diagrammatic representation of the (d, p) elastic scattering exchange. The pole position occurs for $k_n^2 + \alpha^2 = 0$.

At the pole the process has infinite range, so that the amplitude measures the asymptotic S- and D-wavefunction directly. In fact, the amplitude has the structure

$$\text{Amplitude} \propto (A_s + \dots \eta A_s) \frac{1}{(\vec{k}_n^2 + \alpha^2)} (A_s + \dots \eta A_s) . \quad (50)$$

A polarization experiment extrapolated to the pole depends on η to at least leading order. It is therefore proportional to

$$\frac{d\sigma^{\text{pol}}}{d\Omega} \propto \eta A_s^4 \quad (\text{at the pole}) . \quad (51)$$

A typical experimental angular distribution is seen in Fig. 26 with its extrapolation.

Fig. 26. Tensor polarization cross-section for $(\vec{d}, p)$ elastic scattering at $E_d = 28$ MeV versus $z = \cos \theta$ and its extrapolation to the pole at $z_{\text{pole}} = -1.43$. [From Gruebler and co-workers (1980).] The pole divergence has been removed.

The main problems in the analysis are the following:

- i) Use of correct A_s^4 . This depends on the effective range. This correction can and must be made correctly conforming to the accurately known experimental value for A_s .
- ii) Coulomb effects. These are probably taken into account correctly to a precision of a few per cent (Santos and Colby, 1981; Londergan and co-workers, 1983).
- iii) Stability and uniqueness of the extrapolation procedure. This is at present the main source of uncertainty and debate. A precision of the order of 3% seems likely.

A recent evaluation of all experiments has been made by Londergan and co-workers (1983). The experimental value found is

$$\eta^{\text{elastic}} = 0.0263(9) [\pm 3\%] . \quad (52)$$

It is interesting to note that the pole extrapolation is physically very similar to $Q = 0$ subcoulomb stripping (infinite range), although the technical evaluation is quite different.

C. Pole Extrapolation in Stripping

The basic physics is very similar to the previous elastic scattering case. The process used in practice is ($dd \rightarrow pT$) using tensor polarized deuterons. Since there is no Coulomb singularity the analysis is simpler than for the elastic scattering case. The main uncertainty in the extrapolation to the pole is the same stability and uniqueness problem as in the elastic-scattering case. Borbély and co-workers (1982) find the value

$$\eta^{\text{inelastic}} = 0.0272(4) [\pm 1.5\%] . \quad (53)$$

This is the most accurately quoted value for the η parameter. There is no doubt, that the determination by Borbély and co-workers (1982) is very precise, but it is not clear at present whether or not systematic errors are larger than those quoted in Eq. (53).

On averaging the different methods one finds

$$\eta_{\text{exp}} = 0.0271(4) [\pm 1.5\%] . \quad (54)$$

D. The Deuteron Quadrupole Moment

The basic measurement on which quadrupole determinations are based is the molecular-beam resonance experiments by Ramsey and co-workers (Code and Ramsey, 1971). These determine quadrupole splittings to an accuracy of 0.01% in the $J = 1$ state of the D_2 molecule and also in the HD molecule. The actual quadrupole moment is deduced from these data by a theoretical analysis using variational wave functions. The standard literature value is that of Reid and Vaida (1973, 1975): $Q = 0.2860 \text{ fm}^2 (\pm 0.5\%)$. Since variational techniques are well-known to give accurate eigenvalues, while they often are far less precise for matrix elements, one must obviously question the basic uncertainty in such a number. One can visualize the likelihood of high accuracy as follows. The electron feels the quadrupole interaction $Q_{ik}x_i x_k / r^5$ from each of the two deuterons. For a single atom, the quadrupole splittings can be observed in P-states or higher L-states. The interaction contributes then from regions where the wave function is large (i.e., with large contributions to the binding energy). In the present case the leading wave-function is that of two independent atoms (see Fig. 27). An important contribution to the quadrupole splitting comes already from this simplest approximation, which contributes importantly to the total binding energy as well.

Fig. 27. Schematic picture of the D_2 molecule.

The variational approach, particularly the recent one of Bishop and Cheung (1979), uses very sophisticated, state-of-the-art wave functions. There is no reason to believe that any significant contributions to quadrupole splittings come from regions with small contributions to the total energy. Therefore, the variational technique should be exceptionally reliable. From the Bishop and Cheung analysis (1979) one obtains the value (Ericson and Rosa-Clot, 1983) (not stated previously in the literature):

$$Q = 0.2859(3) \text{ fm}^2 [\pm 0.1\%] .$$

This value includes a small correction for non adiabatic motion of the electrons. It agrees with the value of Reid and Vaida (1975) but the uncertainty has been reduced by a factor of five. Relativistic effects, finite deuteron size corrections, and polarizability corrections are all very small (Ericson and Rosa-Clot, 1983). The conclusion is therefore that the quadrupole moment is experimentally known very precisely. In fact, the present theoretical analysis based on the current state of nuclear and particle physics cannot do full justice to this number, since uncertainties in the e.m. meson-exchange corrections as well as the experimental uncertainty in the πNN coupling constant are a magnitude larger.

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