EXPERIMENTAL DETERMINATION OF WEAK MIXING ANGLES

IN THE SIX-QUARK SCHEME

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ABSTRACT

Experimental results on weak decays of hyperons and B mesons, on neutrino production of charm quarks, and on the B lifetime are used to obtain, in a combined fit, values for the three mixing angles in the Kobayashi-Maskawa scheme: $\sin \theta_1 = 0.231 \pm 0.003$, $0.015 < \sin \theta_2 < 0.09$, and $\sin \theta_3 < 0.04$. In the Maiani parametrization, the angles obtained are: $\sin \theta = 0.231 \pm 0.003$, $|\sin \gamma| = 0.052^{+0.015}_{-0.010}$ and $\sin \beta < 0.008$.

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1. INTRODUCTION

The six-quark mixing scheme proposed by Kobayashi and Maskawa [1] serves as a useful parametrization of the connection between generations of quarks. The elements U_{ik} of the quark mixing matrix ($i = u, c, t$; $k = d, s, b$) are parametrized in terms of three angles θ_1 , θ_2 , and θ_3 , and one phase δ , possibly related to CP violation (Table 1a). If CP violation is due to quark mixing, then this phase δ is related to the parameter ϵ , describing the admixture of wrong CP parity in the long- and short-lived neutral K-meson states, measured to be $\epsilon = (2.28 \pm 0.05) \times 10^{-3} \times \exp(i\pi/4)$ [2]. An approximate relation derived by Pakvasa and Sugawara [3] is $|\epsilon| = |(m_t - m_c)/m_c| \sin 2\theta_2 \tan \theta_3 \sin \delta / (2\sqrt{2} \cos \theta_1)$ where m_t and m_c are the top- and charm-quark masses. An alternative parametrization of the matrix U has been given by Maiani [4] in terms of angles θ , γ , and β (see Table 1b).

Experimentally, information on the weak mixing angles comes from measurements of weak decays of light and heavy quarks and from neutrino production of charm quarks as observed in dimuon events, as summarized in previous papers [5-8]. Recently, new results on the B-meson lifetime [9,10] and on hyperon semileptonic decays [11] have been obtained. In this paper, we summarize the impact of all these constraints on the weak mixing angles. We first go through the constraints on the coupling parameters U_{ik} , and then proceed to derive bounds on the mixing angles.

2. CONSTRAINTS ON MATRIX ELEMENTS

2.1 Light-quark couplings

2.1.1 Coupling U_{ud}

This coupling parameter has been determined from a comparison of measured rates of nuclear beta decays with that of muon decay. Two different evaluations of this quantity have been made and their results are $U_{ud} = 0.9730 \pm 0.0024$ [6] and $U_{ud} = 0.9737 \pm 0.0025$ [12]. Combining these two, one obtains

$$U_{ud} = 0.9733 \pm 0.0024 . \quad (1)$$

2.1.2 Coupling U_{us}

In a series of experiments, the WA2 Collaboration have studied five different hyperon semileptonic decays, i.e. the electronic weak decays $\Sigma^- \rightarrow n e^- \bar{\nu}$, $\Sigma^- \rightarrow \Lambda e^- \bar{\nu}$, $\Xi^- \rightarrow \Lambda e^- \bar{\nu}$, $\Xi^- \rightarrow \Sigma^0 e^- \bar{\nu}$, and $\Lambda \rightarrow p e^- \bar{\nu}$. Including radiative corrections and using in addition the neutron lifetime [13], this experiment gives [11]

$$U_{us} = 0.231 \pm 0.003 . \quad (2)$$

This represents a substantial improvement over former analyses [6,12].

2.2 Charm-quark couplings

2.2.1 Coupling U_{cd}

We take the value

$$U_{cd} = 0.24 \pm 0.03 \quad (3)$$

from the CDHS Collaboration [14]. This value is obtained by measuring the charm production by neutrinos and by antineutrinos from nuclei and by subtracting out the contribution of charm production from strange quarks, leaving the one from d quarks.

2.2.2 Coupling U_{cs}

In charged-current reactions this coupling appears always together with the strange-sea structure function $xs(x)$ or its integral $S = \int xs(x) dx$, where we use the following notation: $s(x)$ is the quark density distribution of strange quarks in the proton in the Bjorken scaling variable x , $u(x)$ and $d(x)$ are the distributions of up and down quarks, with $U = \int x u(x) dx$ and $D = \int x d(x) dx$; and similarly for antiquarks. The quantity measured is $|U_{cs}|^2 \cdot 2S$ [5]. In the absence of an independent determination of S , only the upper limit for $2S$ given by SU(3) symmetry, $2S \leq \bar{U} + \bar{D}$, and a corresponding lower limit on $|U_{cs}|$ can be obtained. The product $|U_{cs}|^2 \cdot 2S$ can be extracted in three ways from the neutrino and antineutrino dimuon production data [8]. We use here the results from the x distribution of neutrino dimuons [8,14]

$$\frac{|U_{cs}|^2}{U_{cd}^2} = (6.26 \pm 0.73) \frac{1 + \alpha^*}{\alpha} \quad (4)$$

and the one from the cross-sections of neutrino- and antineutrino-induced dimuon production using the semileptonic branching ratio of D mesons:

$$|U_{cs}|^2 = (0.41 \pm 0.09) \frac{1 + \alpha^*}{\alpha}, \quad (5)$$

where $\alpha = 2S/(\bar{U} + \bar{D})$ is the ratio of momentum fractions carried by strange and non-strange sea quarks in the nucleon, and $\alpha^* = 2S(U_{us}^2 + U_{cs}^2/r_s)/(\bar{U} + \bar{D})$ is the same ratio modified by the threshold suppression factor r_s for the charm-quark mass, which is $r_s = 1.5$ for the experiment considered [14,15].

2.3 Bottom-quark couplings

2.3.1 Ratio $|U_{ub}|/|U_{cb}|$

We use the value

$$|U_{ub}|/|U_{cb}| < 0.2 \quad (90\% \text{ C.L.}) \quad (6)$$

obtained by the CLEO [16] and CUSB [17] Collaborations at the Cornell storage ring CESR.

2.3.2 Coupling U_{ub}

From the unitarity relation $|U_{ub}|^2 = 1 - |U_{ud}|^2 - |U_{us}|^2$, and eqs. (1) and (2), we obtain

$$|U_{ub}|^2 = -0.0006 \pm 0.0060. \quad (7)$$

2.2.3 B lifetime

The lifetime τ_B of b flavoured hadrons, apart from phase-space factors, depends on the magnitude of the couplings U_{cb} and U_{ub} . In fact [18]

$$\tau_B = 0.93 \times 10^{-14} \text{ s} / (2.75 |U_{cb}|^2 + 7.7 |U_{ub}|^2). \quad (8),$$

The previous upper limit obtained by the JADE Collaboration [19], $\tau_B < 1.4 \times 10^{-12} \text{ s}$ at 95% C.L., is significantly improved by the recent measurements of the Mark II and MAC Collaborations [9,10]. Using their high-precision vertex chambers and tagging b decays by their semileptonic decays into muons or electrons, they obtain

$$\begin{aligned} \tau_B &= (10.4^{+4.9}_{-3.2} \pm 2.6) \times 10^{-13} \text{ s} \quad [10] \\ \tau_B &= (18 \pm 6 \pm 4) \times 10^{-13} \text{ s}, \quad [9] \end{aligned} \quad (9)$$

which in turn gives the following constraint

$$(|U_{cb}|^2 + 2.8 |U_{ub}|^2)^{0.5} = 0.052 \begin{matrix} + 0.009 \\ - 0.007 \end{matrix} . \quad (10)$$

2.4 Combined fit

Using the constraints of eqs. (1) to (7) and (9), we obtain for a minimum $\chi^2 = 1.2/5$ D.F. the values $\sin \theta_1 = 0.231 \pm 0.003$, $\alpha = 2S/(\bar{U} + \bar{D}) = 0.49 \pm 0.07$, and values of $\sin \theta_2$ and $\sin \theta_3$ with the error contours in the $(\sin \theta_2, \sin \theta_3)$ plane given in fig. 1. These contours vary slightly with the value of the phase δ . From the resulting error contours, we obtain a finite value of $0.015 < \sin \theta_2 < 0.09$, and an upper limit for $\sin \theta_3 < 0.04$.

We conclude from this analysis that the second mixing angle θ_2 is smaller than the first one, θ_1 , i.e. $\sin \theta_2 / \sin \theta_1 < 0.4$. The third angle, θ_3 , is still compatible with zero, with the upper limit $\sin \theta_3 < 0.04$ at the 67% C.L. This pattern of decreasing mixing angles means that weak transitions between members of different quark families are suppressed more for heavy quarks than for light ones.

On the other hand, the curves given in fig. 1 from a phenomenological analysis [20] of the parameters of the neutral kaon system can be compared with the experimentally allowed range of angles. Along these curves the phase δ varies from 0 to $\pi/2$ on the lower branch as $\sin \theta_3$ goes from 1 to 0, and from $\pi/2$ to π on the upper branch as $\sin \theta_3$ goes from 0 to 1. The constraint $\sin \theta_3 < 0.04$ can be used to set the limit $2^\circ < \delta < 178^\circ$. A determination of δ requires specific experiments on CP violation in the $\bar{K}^0 - K^0$ or $\bar{B}^0 - B^0$ systems.

Analogously for the Maiani parametrization [4], the error contours in the plane of the parameters $\sin \gamma$ (corresponding approximately to $\sin \theta_2$) and $\sin \beta / \tan \theta$ (corresponding to $\sin \theta_3$) are given in fig. 2. Here error contours at the one (1σ) and two (2σ) standard deviation level are given. These contours are nearly independent of the phase angle δ' in the Maiani parametrization. The values for the angles are $\sin \theta = 0.231 \pm 0.003$, $|\sin \gamma| = 0.052 \begin{matrix} + 0.015 \\ - 0.010 \end{matrix}$ and $\sin \beta < 0.0077$. From the range of values of the mixing angles in either parametrization, the values of the Kobayashi-Maskawa *matrix elements* can be obtained.

These are given in table 2. It is evident that the error on these matrix elements from the common fit is, for most of them, much smaller than the one obtained from individual experimental bounds on one matrix element.

It also appears from the values in table 2 that, apart from the diagonal elements, and the three off-diagonal elements observed directly up to now, (U_{us} , U_{cs} , and U_{cb}), there is only one other non-diagonal element (U_{ts}) whose magnitude is such as to allow direct observation, possibly in the reaction $\nu + s \rightarrow \mu^- + t$. The other two elements, connecting the first and third families, are of a magnitude which makes their detection very difficult, and therefore represent a challenge to future experimentation.

This analysis is done in the framework of a six-quark model. If the number of quark flavours is larger than six, the unitarity condition on the elements of the 3×3 matrix U_{ik} is replaced by $\sum_{i=1}^3 |U_{ik}|^2 \leq 1$ and $\sum_{k=1}^3 |U_{ik}|^2 \leq 1$. In this generalized case, the ranges of values for U_{ik} are given in table 3.

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REFERENCES

- [1] M. Kobayashi and K. Maskawa, *Progr. Theor. Phys.* 49 (1973) 652.
- [2] K. Kleinknecht, *Ann. Rev. Nucl. Sci.* 26 (1976) 1.
- [3] S. Pakvasa and H. Sugawara, *Phys. Rev. D* 14 (1976) 305.
- [4] L. Maiani, *Int. Symp. on Lepton and Photon Interactions at High Energies*, Hamburg, 1977 (DESY, Hamburg, 1977), p. 877.
- [5] K. Kleinknecht, *Proc. 12th Int. Neutrino Conf., Balatonfüred, 1982* (Central Res. Inst. Physics, Budapest, 1982), Vol. 1, p. 115.
- [6] E.A. Paschos and U. Türke, *Phys. Lett.* 116B (1982) 360.
- [7] S. Pakvasa, *Proc. 21st Int. Conf. on High Energy Physics, Paris, July 26-31, 1982*, *J. Phys.* 43, Suppl. 12 (1982) C3-234.
- [8] K. Kleinknecht and B. Renk, *Z. Phys.* C15 (1982) 19; and Univ. Dortmund preprint 83-276, to appear in *Z. Phys. C*.
- [9] MAC Collaboration, paper given at the EPS Europhysics Conference on High Energy Physics, Brighton, July 1983, by N. Chadwick.
- [10] Mark II Collaboration, paper given at the EPS Europhysics Conference on High Energy Physics, Brighton, July 1983, by G. Hanson.
- [11] M. Bourquin et al., WA2 Collaboration, preprint July 1983, and private communication by J.M. Gaillard.
- [12] R.E. Shrock and L.L. Wang, *Phys. Rev. Lett.* 41 (1978) 1692 and 42 (1979) 1589.
- [13] C.H. Christensen et al., *Phys. Rev. D* 5 (1972) 1628.
J. Byrne et al., *Phys. Lett.* 92B (1980) 274.
- [14] H. Abramowicz et al., *Z. Phys.* C15 (1982) 19.
- [15] H.G.J. de Groot et al., *Z. Phys.* C1 (1979) 143.
- [16] B. Gittelman, CLEO Collaboration, *Proc. 21st Int. Conf. on High Energy Physics, Paris, 1982*, *J. Phys.* 43, Suppl. 12 (1982) C3-110.
- [17] P. Franzini, CUSB Collaboration, *Proc. 21st Int. Conf. on High Energy Physics, Paris, 1982*, *J. Phys.* 43, Suppl. 12 (1982) C3-114.
- [18] M.K. Gaillard and L. Maiani, *Proc. Summer Institute on Quarks and Leptons, Cargèse, 1979* (Plenum Press, New York, 1980), p. 433.

- [19] W. Bartel et al., Phys. Lett. 114B (1982) 71.
- [20] L.L. Chau, W.-Y. Keung and M.D. Tran, BNL preprint (Aug. 1982).
L.L. Chau, Phys. Rep. 95 (1983) 1.

Table 1

Parametrizations of the quark mixing matrix

a) Kobayashi-Maskawa parametrization [1]

$$U = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 + e^{i\delta} s_2 s_3 & c_1 c_2 s_3 - e^{i\delta} s_2 c_3 \\ -s_1 s_2 & c_1 s_2 c_3 - e^{i\delta} c_2 s_3 & c_1 s_2 s_3 + e^{i\delta} c_2 c_3 \end{pmatrix}$$

b) Maiani parametrization [4]

$$U \equiv \begin{pmatrix} c_\beta c_\theta & c_\beta s_\theta & s_\beta \\ -s_\gamma c_\theta s_\beta e^{i\delta'} - s_\theta c_\gamma & c_\gamma c_\theta - s_\gamma s_\beta s_\theta e^{i\delta'} & s_\gamma c_\beta e^{i\delta'} \\ -s_\beta c_\gamma c_\theta + s_\gamma s_\theta e^{-i\delta'} & -c_\gamma s_\beta s_\theta - s_\gamma c_\theta e^{-i\delta'} & c_\gamma c_\beta \end{pmatrix}$$

Table 2

Elements of quark mixing matrix U_{ik} from fit of experimental constraints (1 standard deviation range)

	d	s	b
u	0.9723 - 0.9737	0.228 - 0.234	0.000 - 0.008
c	0.228 - 0.234	0.9704 - 0.9726	0.042 - 0.067
t	0.003 - 0.016	0.041 - 0.066	0.9977 - 0.9991

Table 3

Elements of quark mixing matrix U_{ik} from experimental constraints if number of quark flavours is larger than 6

0.9709 - 0.9757	0.228 - 0.234	0.000 - 0.013
0.21 - 0.27	0.78 - 1.00	0.042 - 0.067
0.00 - 0.12	0.00 - 0.58	0.000 - 0.999

Figure captions

- Fig. 1 : Error contours in the $(\sin \theta_2, \sin \theta_3)$ plane (solid lines) for three values of the phase δ (10° , 90° , and 170°). Also shown (dashed lines) are results from a phenomenological analysis [20] of the $K_L - K_S$ mass difference and the CP violation parameter ϵ , for two values of the top quark mass, m_t . Along those curves, the phase δ varies from 0° at $\sin \theta_3 = 1$ to 90° at $\sin \theta_3$ on the lower branch, and from 90° at $\sin \theta_3 = 0$ to 180° at $\sin \theta_3$ on the upper branch.
- Fig. 2 : Error contours in the plane of mixing parameters $\sin \gamma$ and $\sin \beta/\tan \theta$ in the Maiani parametrization. One standard deviation (1σ) and two standard deviation (2σ) contours are given.

Fig. 1

Fig. 2