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ACOUSTIC DETECTION OF SUPERHEAVY MONOPOLES IN
GRAVITATIONAL ANTENNAS.

**ACOUSTIC DETECTION OF SUPERHEAVY MONOPOLES IN
GRAVITATIONAL ANTENNAS**

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Grand Unified theories predict the existence of superheavy magnetic monopoles of the 't Hooft-Polyakov type /1/.

These particles would be too heavy to be produced in laboratory experiments ($M \sim 10^{16}$ GeV) but some of them could have been created in the very early times of the Big Bang, as a consequence of the time of the phase transition which led to the breaking of grand symmetry /2/. The estimates of the monopole density vary from values which exceed the present limits, to a single monopole in the visible universe, or none at all.

The detection of relic monopoles would give important clues both on the structure of physics at very short distances and on the very early history of the Universe.

In the present note we present an evaluation of the signal produced by a monopole traversing a Weber-type tuned gravitational antenna /3/. In its simplest form this device is a large cylinder of Al, instrumented with transducers which allow the continuous monitoring of the amplitude of its lowest longitudinal axisymmetric vibration mode, the one best suited for the detection of gravitational radiation.

To be concrete we will consider the case of the Rome-CERN cryogenic antenna /4/ whose basic data are:

$$L = 3 \text{ m} , \quad R = 30 \text{ cm} , \quad \nu_1 = 858 \text{ Hz} \quad (1)$$

where ν_1 is the frequency of the aforementioned vibration mode.

A relic superheavy monopole, moving with $\beta \approx 10^{-3} - 10^{-4}$ would traverse the antenna in a time which is negligible with respect to the oscillation period of one of the lowest vibration modes (see eq. (1)). This event can then be considered as instantaneous.

The mechanical interactions of the monopole with the antenna are of two different types:

- 1) If the antenna is superconducting, the monopole leaves a tube of trapped magnetic flux along its trajectory.

We recall that Dirac's condition /5/

$$eq = \frac{n}{2} (hc) \quad (n = 1, 2, 3 \dots) \quad (2)$$

implies that the trapped flux is a multiple of the unit flux /6/

$$\phi = n \phi_0 = n \left(\frac{hc}{2e} \right) = n \left(2,07 \cdot 10^{-7} \text{ gauss} \cdot \text{cm}^2 \right)$$

For the 't Hooft-Polyakov monopole /1/ one would expect $n = 2$.

- 2) The monopole will heat the metal along its trajectory. This can happen through ohmic losses or atomic excitation.

Ohmic losses have been recently evaluated by G.W.Ford /7/, who obtains for Aluminium and monopoles with $n = 2$ a stopping power:

$$\left| \frac{dE}{dx} \right| = 540 \left(\frac{\beta}{10^{-3}} \right) \frac{\text{MeV}}{\text{cm}} \quad (3)$$

when the monopole velocity v is much less than the Fermi velocity v_F ($v_F/c = 6,8 \cdot 10^{-3}$ in Aluminium), a condition satisfied by the range of monopole velocities of interest.

A recent paper /8/ indicates that a large contribution to the energy loss of a monopole could be given by the non-adiabatic deformation of electron orbits leading to electron excitation. The effect proposed by Drell et al. has only been computed for very light atoms, and it is not at present known what its contribution would be in Al.

We will neglect this effect in our numerical evaluation keeping in mind that this could represent an underestimation, possibly by a substantial factor.

Our evaluation of the energy deposited in the lowest longitudinal axisymmetric mode by an $n = 2$ monopole traversing the antenna through its center in a direction perpendicular to the axis is

$$\left(\frac{\delta E_1}{K_B} \right)_{d.} = 3,4 \cdot 10^{-8} \text{ } ^\circ K \quad (4)$$

from tube formation, and

$$\left(\frac{\delta E_1}{K_B} \right)_{th.} = 1,6 \cdot 10^{-5} \left(\frac{\beta}{10^{-3}} \right)^2 \text{ } ^\circ K \quad (5)$$

from the thermal effect.

Our evaluation of the thermal effect is pessimistic since, as mentioned above, we neglect the possible contribution of the atomic excitation effect of Ref. /8/. In any case we expect the contribution of the thermal effect to dominate over that due to flux-tube formation for the interesting range of monopole velocity, $\beta = 10^{-3} + 10^{-4}$.

The contribution of tube formation has been evaluated, with a simplified model of the flux tube, for the case of pure Al. The study /9/ of the superconductive behaviour of the 5056 alloy used in the Rome-CERN antenna indicates that its behaviour is different from that of pure Al, but not radically different: both the critical temperature and the $T = 0$ critical field H_0 are close (within ~50%) to those for pure Al.

In any case we expect the contribution of the thermal effect to dominate that due to flux tube formation.

We can expect that improvements in antenna technology will put a signal of $10^{-5} \text{ } ^\circ K$ above the level of thermal noise, but this would not by itself transform the device into a good monopole detector.

In order to do this one would require one of the following, possibly both:

- a) improve the device by adding multimode detection capability; an improvement useful also for discriminating gravitational signals from those of different origin /10/;
- b) Combine the gravitational antenna with other monopole sensitive devices. A good choice would be a combination of the antenna with Cabrera style^(*) superconductive loops /11/ which can coexist in the same cryogenic environment.

A.De Rujula, who has been engaged in similar calculations with C.Bernard and B.Lautrup, has pointed out^(**) that a good design for a monopole detector could be based on an three dimensional array of small antennas, to act as an hodoscope. The use of small antennas would improve the signal to thermal noise ratio while the ability to detect "tracks" would improve the rejection of non-monopole (e.g. gravitational) signals.

In the rest of this note we will outline the methods used in our evaluations.

2 Small vibrations of the antenna and the monopole signal, general formalism.

The state of the vibrating antenna can be described by a vector field $\vec{u}(x,t)$ representing the local displacement from equilibrium. Free vibrations of an isotropic solid can be described by a lagrangian /12/

(*) G.Pizzella and I.Modena, private communication.

(**) A.De Rujula, private communication.

$$\mathcal{L}_0 = \frac{1}{2} \int d^3x \left[\rho (\dot{\vec{u}} \cdot \dot{\vec{u}}) - \lambda (\text{div} \vec{u})^2 - \frac{\mu}{2} \sum_{i,k} \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right)^2 \right] \quad (6)$$

A monopole which pierces the antenna leaves along its path a region of different temperature (thermal effect) or in a different phase (flux tube effect).

These effects can be simulated by introducing an "effective pressure" term in the lagrangian:

$$\mathcal{L}' = \int d^3x \quad P(x,t) \cdot \text{div} \vec{u} \quad (7)$$

The equation of motion is then

$$\frac{\partial^2 \vec{u}}{\partial t^2} = \frac{1}{\rho} \left\{ (\lambda + \mu) \vec{\nabla} \cdot (\vec{\nabla} \vec{u}) + \mu \Delta \vec{u} - \vec{\nabla} P(x,t) \right\} \quad (8)$$

It is convenient to develop $\vec{u}(x,t)$ in terms of the normal modes

$$\vec{u}(x,t) = \sum_k \vec{u}_k(x) B_k(t) \quad (9)$$

where the $\vec{u}_k$ are normalized as

$$\int (\vec{u}_k \cdot \vec{u}_m) d^3x = \delta_{km} \quad (10)$$

The equation of motion for $B_k(t)$ results then, using equations (8), (9)

$$\ddot{B}_k + \omega_k^2 B_k = g_k(t) \quad (11)$$

where, with eq. (10)

$$g_k(t) = \frac{1}{\rho} \int d^3x \quad P(x,t) (\vec{\nabla} \cdot \vec{u}_k(x)) \quad (12)$$

The physical effects associated with the passage of the monopole develop in a time which is very short with respect to the oscillation period $\tau_1 = \gamma_1^{-1}$, and remain essentially unchanged for a time which is long compared with τ_1 , so that for a monopole hitting at $t = 0$ we can assume:

$$g_k(t) = g_k \theta(t) \quad (13)$$

Using this form for $g_k(t)$, the general solution to eq.(11) can be written as

$$B_k(t) = B_k^{(0)} \cos(\omega_k t + \varphi_k) + \theta(t) \frac{g_k}{\omega_k^2} \cos \omega_k t \quad (14)$$

In recent gravitational experiments the amplitude of the choosen mode is monitored continuously and the data are reduced to yield an amplitude change. The data is then expressed as an energy, δE_k , defined as the energy that the choosen mode would have acquired if it had started at rest ($B_k^{(0)} = 0$). According to this definition we would then have

$$\delta E_k = \rho \frac{\delta_k^2}{\omega_k^2} \quad (15)$$

In the calculation of both magnetic and thermal effects, the effective pressure can be considered as a constant P inside a cylinder of small cross section S centered on the monopole path. The integral in eq. (12) can then be reduced to a line integral over the monopole path:

$$g_k = \frac{\Delta P}{\rho} S \int dl (\vec{\nabla} \cdot \vec{u}_k(x)) \quad (16)$$

Except for special cases /13/, the exact form of the normal modes $\vec{u}_k(x)$ is not known analytically. However, for the lowest longitudinal axisymmetric mode of a "long" antenna ($R \ll L$) a suitable approximate expression is given in cylindrical coordinates /14/ by:

$$u_1^r = A_1 \frac{\pi}{L} \sigma r \sin\left(\frac{\pi}{L} z\right)$$

$$u_1^z = A_1 \left(1 - \frac{\sigma}{2} \frac{\pi^2}{L^2} r^2\right) \cos\left(\frac{\pi}{L} z\right)$$

with

$$A_1 = \left(\frac{V}{2}\right)^{-\frac{1}{2}} \left[1 + \frac{\sigma(1-\sigma)}{4} \frac{\pi^2}{L^2} R^2\right]$$

$$\omega_1^z = \frac{\pi^2}{L^2} \frac{E}{\rho} \left(1 - \frac{\sigma^2}{2} \frac{\pi^2}{L^2} R^2\right)$$

where E is the Young's modulus ($E = 7,0 \cdot 10^{11}$ (dine/cm²) for Al)
 σ is the Poisson's ratio ($\sigma = 0.3$ for Al), V is the volume of
the cylinder and ρ is the density ($\rho = 2,7$ gr/cm³ for Al).

We have then:

$$\text{div } \vec{u}_1 = \frac{\pi}{L} A_1 \sin\left(\frac{\pi}{L} z\right) \left[(2\sigma-1) + \left(\frac{\sigma}{2} \frac{\pi^2}{L^2}\right) r^2 \right] \quad (17)$$

In the case of a monopole crossing the center of the antenna at
right angles to its axis we thus find for eqs. (16), (17)

$$g_1 = - \frac{\Delta P}{\rho} S \cdot \left(3,0 \cdot 10^{-4} \text{ cm}^{-\frac{3}{2}} \right) \quad (18)$$

For different orientations of the monopole trajectory one
would obtain a result which differs from this by an $O(1)$ factor.

3. The thermal effect.

The heat released by the monopole along its path will spread in a very short time ($\ll \tau_1$) over a cylindrical region of transverse dimensions which are small compared with the dimension of the antenna.

If we assume for simplicity that the heat is uniformly spread over a cylinder of cross section S , the temperature inside the cylinder would change by

$$\Delta T = \left(\frac{V}{c_v} \right) \cdot \left| \frac{dE}{dx} \right| \cdot \frac{1}{S}$$

where dE/dx is the stopping power for monopoles. This corresponds to an effective pressure

$$\begin{aligned} \Delta P &= \left[V \left(\frac{\partial P}{\partial V} \right)_T \right] \left[\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P \right] \Delta T = \\ &= \gamma \cdot \left| \frac{dE}{dx} \right| \cdot \frac{1}{S} \end{aligned} \quad (19)$$

where γ is the Grüneisen "constant" /15/ (for Al at $T = 0$, $\gamma = 1.6$). Our evaluation (eq. (5)) is obtained by substituting (19) into eq. (18) and using the value in eq. (3) for dE/dx .

We note that g_k does not depend from the value of S (compare eqs. (19) and (16)) a fact which justifies a posteriori eq. (13) where g_k is assumed to be a constant for $t > 0$. The value of g_k will show significant variations only when the heat is diffused over a substantial portion of the antenna, but this will only happen for $t \gg \tau_1$.

4. Flux-tube effects.

A simple treatment of the structure of the flux tube can be given if we neglect the energetic contribution of the tube surface. This means that the tube is pictured as a region of normal conductor with a sharp boundary. Since the difference in free energy density between the normal and superconducting phase is, for type I superconductors, equal to $H_c^2/8\pi$, free energy is minimized if H inside the tube equals the critical field.

The cross section of the tube generated by an $n = 2$ monopole is then given by

$$S = \frac{2\phi_0}{H_0} = 4.0 \cdot 10^{-9} \text{ cm}^2 \quad (20)$$

where we have used $H_0 = 104$ Gauss, the value appropriate for pure Al at $T = 0$ /16/. Although rough this treatment is sufficient for our evaluation.

Mechanical effects of the tube would then arise from the different specific volume in the two phases. This difference can be measured, either directly, or through the relation /16/

$$\frac{\Delta V}{V} = \frac{V_n - V_s}{V_s} = \frac{H_0}{4\pi} \left(\frac{\partial H_0}{\partial P} \right)_T \quad (21)$$

for Al we have a value /16/ at $T = 0$

$$\frac{\Delta V}{V} = -2.2 \cdot 10^{-8} \quad (22)$$

This can be translated into an effective pressure through:

$$\Delta P = V \left(\frac{\partial P}{\partial V} \right)_T \frac{\Delta V}{V} = K \cdot \frac{\Delta V}{V} = -1.6 \cdot 10^4 \frac{\text{dine}}{\text{cm}^2} \quad (23)$$

Where K is the modulus of compressibility ($K = 7.3 \cdot 10^{11}$ (dine/cm²) for Al).

Substituting this in the general expressions of section II we are led for the case of the Rome-CERN antenna to the evaluation in eq. (4).

5. Concluding remarks.

We have presented an evaluation of the possible signal generated in a Weber-type resonant gravitational antenna by the passage of a monopole through it.

The treatment has been kept at a very simple level, which we considered sufficient for a first evaluation.

The main uncertainty in our estimate originates from the insufficient knowledge of the short-distance contribution of the stopping power of A1 for monopoles, in particular of the contribution of effects of the kind proposed in Ref. /8/.

We note that in a correct treatment one would have to add coherently the vibration amplitude contributed by each effect; the vibration energy would then include, in addition to the terms in eqs. (4) and (5), an interference term which, according to our estimate (eqs. (19) and (23)) is negative, but still (for the interesting range of monopole velocities, $\beta \approx 10^{-4} - 10^{-3}$) smaller than the thermal contribution.

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