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FERMION MASS HIERARCHY AND GLOBAL
HORIZONTAL SYMMETRIES

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A B S T R A C T

We present a mechanism for quark mass generation in zeroth order using induced representations rather than the minimization of the horizontal potential. Using a simplicity criterion, we derive a realistic mass matrix. We also discuss a possible application of the mechanism to various models.

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The replication of the fermion generations still raises many questions. The possibility that local horizontal broken symmetries relate generations has been considered in the last years ¹⁾⁻⁴⁾. A large global symmetry, reflecting the freedom of rotating fermions of the same charge into each other (generation mixing) is associated to any vertical gauge symmetry when there is no Higgs field in the theory. A part of the global symmetry could remain when Higgs fields are added. One of the main reasons ^{*}) for gauging this remaining global symmetry was the fear that the Goldstone bosons, associated to each spontaneously broken generator, if not "eaten up" by gauge bosons, could produce non-phenomenologically acceptable features, such as new long-range interactions.

Recently, we have learnt that Goldstone bosons could easily be "invisible" enough as not to invalidate a model predicting their existence ⁵⁾. Here we shall concentrate on global horizontal symmetries ⁶⁾. There is another independent motivation to care about them which is related to the existence of a different global symmetry already proposed some years ago, the Peccei-Quinn, $U_{PQ}(1)$, symmetry ⁷⁾. It has been introduced to explain the smallness of the value of θ , the strong CP violation parameter. Since this symmetry must be broken at a certain scale different from those associated to GUTs [between 10^9 GeV ⁸⁾ and 10^{12} GeV ⁹⁾], in order to obtain an "invisible" axion ¹⁰⁾, this new scale could be associated to the breaking of a larger global symmetry. The $U_{PQ}(1)$ could form part of a larger horizontal group, a $U_H(n)$ for example.

The game to be played with the vacuum expectation values (VEVs) of the Higgs fields to get fermion mass matrices (that is, the fermion masses and mixing angles) is, in principle, the same either with local or global horizontal symmetries. There are, however, a few essential differences :

- the lower bound for the breaking scale is much higher for global ($W \sim 10^{10}$ GeV) ⁷⁾ than for gauge (10^4 - 10^5 GeV) ²⁾ horizontal symmetries. The reason is that the coupling of the Goldstone bosons to usual fermions is proportional to W^{-1} and must be small enough to have escaped detection so far.

^{*}) There are also specific motivations to introduce horizontal vector bosons as, for example, to explain flavour mixing through radiative corrections ³⁾ or to produce CP violation ^{1),4)}.

-- the assignment of particles to various representations of a global group is completely free, because one does not need to worry about anomalies, with one exception, a horizontal $U_H(1)$ group. A chiral $U_H(1)$ is actually broken by the strong anomaly into a Z_n group, and the cosmological problem of avoiding domains appears, as for the $U_{PQ}(1)$, the Peccei-Quinn group¹¹⁾.

The group $SU(3)$ has been considered many times as the horizontal symmetry, always with anomaly-free assignments. However, the assignment of all the left- and all the right-handed fermions, f_L and f_R [(u_L, c_L, t_L) and (u_R, c_R, t_R) for example], to conjugate fundamental representations, say $\bar{3}$ and 3 respectively, deserves particular attention. Without introducing new heavy fermions ad hoc, this assignment is not anomaly-free. Allowed representations for Higgs fields coupled to $\bar{f}_R f_L$ are, then, $\bar{3}$ and 6 . The second rank symmetric representation 6 is not traceless when written as a 3×3 matrix. Therefore the VEV

$$\langle [6] \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} v \quad (1)$$

is possible and one Higgs field alone could account for the zero order approximation to the fermion mass matrix, in which only the 3rd generation is massive. This feature is attractive because, usually, a magic cancellation between two independent Higgs fields [for example between the representations 1 and 3 or 3 and 5 of a horizontal $SU(2)$] is necessary to obtain the same result. If the Higgs field in the representation 6 were the only one, the VEV of Eq. (1) would correspond to a minimum of the Higgs potential. This cannot be the case. We need other scalar fields to break the global symmetry at a scale W higher than v , the Glashow-Weinberg-Salam¹²⁾ scale, and to produce the masses of the first two generations. Using the freedom to make an arbitrary global $SU(3)$ rotation, the $\langle [6] \rangle$ could lead to the form (a, b, c) , but not $(0, 0, 1)$, in the diagonal.

Here we present an idea to obtain the VEV of Eq. (1) and, as we will see later, to avoid flavour changing neutral currents in a natural way, at the price of introducing new heavy fermions in a hidden sector of the theory. Our group structure is $G_V \times G_H$, the product of a vertical and horizontal group. Let us take the $G-W-S$ as the vertical group. The extension to any other is immediate. Classified under the representations of $(SU_V(2), SU_H(3))$, the f_L are $(2, \bar{3})$ while the f_R are $(1, 3)$ and the simplest possibility for the usual doublet φ is $(2, 1)$. Notice that a direct Yukawa coupling $\bar{f}_L f_R \varphi$ is forbidden in this way.

These assignments solve automatically a problem which involves, in general, unnatural fine tuning : there is no flavour changing Higgs interaction. There is also no direct mass term, but we want to form mass terms combining the VEV $\langle \varphi \rangle \sim v$ with two other VEVs of one Higgs field, call it χ , transforming as a triplet under $SU_H(3)$ (see Fig.1). Following the simplicity criterion, let χ be in the representation $(1,3)$. This is, $\langle \chi \rangle \neq 0$ only breaks the horizontal and $\langle \varphi \rangle \neq 0$ only the vertical group. Why do we want to take twice a representation $[3]$? The advantage is that it is always possible, through a global $SU_H(3)$ rotation, to carry $\langle \chi \rangle$ to the form $\langle \chi \rangle = (0,0,1)W$. Moreover, two identical triplets can only sum up into a symmetric combination, $3 \times 3 \supset 6$. Therefore, two VEVs $\langle \chi \rangle$ produce an "effective" $[6]$ Higgs VEV as in Eq. (1). The matrix in Eq. (1) does not result from the minimization of a potential, but it can be chosen to be so because of symmetry reasons. Some fermions, call them generically F , are necessary to complete the mass diagram. They are taken to belong to a heavy hidden sector of the theory. The simplest choice requires two types of F 's : a singlet $(1,1)$ (with the hypercharge Y of the right-handed standard fermions : $-1, 2/3, -1/3$) and a doublet of the G-W-S group, $(2,1)$ (with the Y of the left-handed standard fermions $-1/2, 1/6$). Both the right and left components of the F fermions are taken in the same representations. Therefore invariant mass terms are possible for them. Let us consider briefly other vertical groups.

The same model applied to an $SU_V(5)$ vertical group implies simply the assignment of the usual fermions to the $(10, \bar{3})$ and $(\bar{5}, \bar{3})$ representations, of φ to the $[5,1]$ and of χ , for example, to the $[1,3]$ representations. In this way, the good predictions for the third generation of a Higgs field in the representation $[5]$ of $SU(5)$, $m_b = 3m_\tau$, would be kept, while the bad ones, $m_e/m_u \sim m_d/m_s$ would be avoided ¹³⁾. Heavy fermions F in the representations $[10,1]$ and $[5,1]$ are necessary. For $G_V = SO(10)$ with fermions f_L in the representation $(16, \bar{3})$, and φ in $(10,1)$, the assignment of χ to $(1,3)$ would lead to the bad relation $m_t = m_b$.

Up to now we have seen how the 3rd generation can be given a mass leaving untouched an unbroken $SU_H(2)$ horizontal subgroup, which ensures that the first two generations are massless. There are many open ways of completing a realistic mass matrix. But it is in any case necessary to break the remaining $SU_H(2)$. To accomplish this task we choose the simplest possibility: another horizontal triplet Higgs field, call it χ' . Through a global $SU_H(3)$ rotation (which does not alter $\langle \chi \rangle$) the VEV of χ' can always be chosen as $\langle \chi' \rangle = (a,0,b)w$. We would like that $\langle \chi' \rangle \sim w \ll \langle \chi \rangle \sim W$, and that χ' would be the sole responsible for the masses of the 1st and 2nd generations. In

order to avoid that χ could replace χ' in some of the mass diagrams, these two fields have to differ in some quantum number. The idea of using the $U_{PQ}(1)$ charge for this purpose comes very naturally. Thus, the whole horizontal group could be taken as $G_H = SU_H(3) \times U_{PQ}(1)$. The PQ charge of every particle is fixed by the requirement of avoiding a domain structure in the universe ¹¹⁾.

The $U_{PQ}(1)$ group is broken to a discrete group by the gauge anomalies. When a transformation of the PQ group with parameter α is applied to the fermions, the value of θ changes to $\theta \rightarrow N\alpha + \theta$. We see that values $\alpha = (2\pi/N)n$, $n=0,1,\dots,N-1$, leave invariant the angle θ . Therefore a group Z_N^{PQ} (central group of order N) is still a symmetry of the Lagrangian. This is the symmetry which is spontaneously broken by the VEVs of some Higgs fields. The vacuum could fall in any of the N different values leading, in principle, to the formation of a domain in each causally disconnected region of the universe. This is incompatible with the observed homogeneity of the universe. N is a function of the number and representations of the fermions f , as well as their PQ charges, Q_f . One solution to the problem is to obtain that Z_N^{PQ} coincide with the centre of the continuous group. In such a way, continuous transformations change any of the N vacua into each other, and the distinction between them is unphysical. For the group $SU_L(2) \times U_Y(1) \times SU_H(3) \times U_{PQ}(1)$ and the standard fermions this is possible by taking the left-handed field charges $Q_L = -1$ and the right-handed field charges $Q_R = -2$. Actually

$$N = n_f [2 Q_L - 2 Q_R] = 6 \quad (2)$$

where $n_f = 3$ is the number of families. The Yukawa couplings fix the values of the PQ charges for the Higgs fields. For $SU_V(5) \times SU_H(3)$, $N=15$ corresponds to $Q_{[10]} = -1$, $Q_{[\bar{5}]} = 8$. The equation for N is

$$N = n_f [3 Q_{[10]} + Q_{[\bar{5}]}] \quad (3)$$

Solutions for $G_V = SO(10)$ do also exist ¹¹⁾.

Actually, in the case of $G_V = SU(5)$, it may be more convenient to take χ in $[1,3]$, as we said, but χ' in $[24,3]$. Therefore in a diagram as the one in Fig. 2, we would have f_R in $[10,3]$, ϕ in $[5,1]$, the intermediate heavy fermion in $[10,\bar{3}]$, χ' in $[24,3]$ and f_L in $[\bar{10},\bar{3}]$ or $[5,\bar{3}]$. The effective Higgs field is in the representation $[5+45+70,3]$. A complex vertical Higgs structure seems necessary in this case in order to get realistic masses (for example for a $[45,3]$ a too small m_s arises ¹³⁾).

Let us present a realistic model along the previous lines, keeping $G_Y = SU(2) \times U_Y(1)$. The fermions and Higgs bosons are presented in the Table. Beside the standard fields, there are two horizontal triplets χ , a $[1,3]$ and χ' , a $[1,\bar{3}]$ and the necessary heavy fields called F, F', F'' ($[1,1]$, $[2,1]$, $[1,\bar{3}]$ respectively). Only the following Yukawa vertices are allowed :

$$\bar{F}_R f_L \chi, \quad \bar{f}_R F_L \chi, \quad \bar{F}_L F_R'' \chi \quad (4)$$

and

$$\bar{f}_R F_L'' \chi' \quad (5)$$

Here f_L stands for $\ell_L = (\nu_L, e_L)$ or $q_L = (u_L, d_L)$, f_R stands for e_R, u_R, d_R and in each case the heavy fermion F, F' or F'' with the appropriate hypercharge Y has to be understood. At very high energy, when all VEVs are zero, only F, F' and F'' have invariant mass terms of order M . The origin of those masses is beyond our scope. At the $SU(3)_H \times U_{PQ}(1)$ breaking scale the fields χ and χ' acquire VEVs $\langle \chi \rangle = (0,0,1)W$ and $\langle \chi' \rangle = (a,0,b)\omega$. An obvious way to implement this symmetry breaking pattern consists in introducing an extra singlet field σ such that the Higgs potential contains an additional non-trivial coupling between χ and χ'

$$(\chi \chi') \sigma \quad (6)$$

It is straightforward to show that this trilinear coupling is indeed needed in order to drive a non-vanishing b . Two things happen: 1) the F 's fermions get mixed among them and with the f 's, leaving certain massless combinations which are the usual fermions ^{*}) (if $M \gg W, \omega$ the F component of the light fermions is small, such as W/M or ω/M); 2) Nambu-Goldstone bosons (G.b) associated to the horizontal breaking appear. For every horizontal generator λ^a $a=(1,\dots,8)$ of the $SU_H(3)$, there will be a G.b

$$(G.b)^a \sim \text{Im} [W \lambda_{3j}^a \chi_j + (a\omega \lambda_{1j}^a + b\omega \lambda_{3j}^a) \chi'_j] \quad (7)$$

^{*}) It is the conservation of $SU(2)_L$ that ensures the absence of mass matrix elements between the doublets and singlets. Then, the combination of f_L with other F_L doublets and of f_R with the F_R singlets which should form "doublet-singlet" mass terms remains massless. These combinations are the current fermion states of the G-W-S model.

The coupling of the G.b's with fermions is derivative. Thus it is of the order of m/W or m/ω , where m is the order of the masses of the fermions. The experimental bounds on $\mu \rightarrow e(G.b)$ and $k \rightarrow \pi(G.b)$ imply $\omega, W \gtrsim 10^{10}$ GeV⁶⁾. Moreover, a pseudo-Goldstone boson, the axion, appears due to the breaking of the axial $U_{PQ}(1)$ group^{*)}. From the astrophysical and astronomical constraints on axions, we know that 10^9 GeV $\leq W, \omega \leq 10^{12}$ GeV^{8),9)}, that is, also an upper bound on the scales. (Let us mention that an interesting relation between an upper bound on the horizontal breaking¹⁴⁾ scale and the possible mass of decaying neutrinos has been studied recently.) At the G-W-S breaking scale, the VEV of φ becomes $\langle \varphi \rangle = (0, v)$ and the light fermions acquire masses through the only two types of diagrams presented in the Figs. 1 and 2. They are responsible for the masses of the 3rd and of the 1st two generations, respectively. The diagram in Fig. 1 shows an "effective" symmetric $[2,6]$ Higgs field, while in Fig. 2 appears an "effective" anti-symmetric $[2, \bar{3}]$. The mass matrix of each charge is of the form

$$m = \begin{pmatrix} 0 & b_f & 0 \\ -b_f & 0 & a_f \\ 0 & -a_f & \mu_3 \end{pmatrix} \quad (8)$$

where $\mu_3 \sim W^2 v / M^2$ (Fig.1) and $\mu \sim \omega v / M$ (Fig.2). The requirement $\mu_3 \gg \mu$ implies the hierarchy $W \gtrsim \sqrt{\omega} M$, that is, $M > W > \omega$ (we could take $M \sim 10^{14}$ GeV, $W \sim 10^{12}$ GeV, $\omega \sim 10^{10}$ GeV around). The matrix in Eq. (8) has the same eigenvalues and produces the same mixing angles (i.e., $m_m^\dagger$ and $m_m^\dagger$ coincide respectively in both cases) as a realistic matrix already studied at length in the literature [e.g., Ref. 15]

$$m = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix} \quad (9)$$

*) Notice that the G-W-S doublet φ does not carry PQ charge, therefore the axion is formed by X and X' only. However, it is coupled to light fermions through the same diagrams of Figs. 1 and 2, but where one X or X' is taken as a shifted field.

where $C = \mu_3$, $B = -a^* \mu$ and $A = (a/a^*)b^* \mu$. When as many phases as possible of the mass matrices are absorbed by redefining the physical quark states, there remains one complex phase in the charged current mixing matrix which is the source of CP violation in the present scheme. In order to obtain realistic masses, the farther hierarchy $|b| < |a|$ must be required. Next, in order to obtain a feeling of the predicted values of the mixing angles, we put the undetermined complex phase equal to zero. Then, following Ref. 15), we know that the sinus of the θ_1 , θ_2 and θ_3 mixing angles of the Kobayashi-Maskawa matrix ¹⁶⁾ are :

$$\begin{aligned} \sin \theta_1 &= \frac{(m_d m_c)^{1/2} - (m_u m_s)^{1/2}}{[(m_c + m_u) \cdot (m_s + m_d)]^{1/2}} \\ \sin \theta_2 &= \frac{(m_s/m_b)^{1/2} - (m_c/m_t)^{1/2}}{1 - (m_u m_s / m_c m_d)^{1/2}} \\ \sin \theta_3 &= \frac{(m_s/m_b)^{1/2} - (m_c/m_t)^{1/2}}{1 - (m_c m_d / m_u m_s)^{1/2}} \end{aligned} \quad (10)$$

The ratios of the masses are phenomenologically bounded ¹⁷⁾ to be :

$$\frac{m_d}{m_u} = 1.76 \pm 0.13, \quad \frac{m_s}{m_u} = 34.5 \pm 5.1 \quad (11)$$

with

$$m_c = 1350 \pm 50 \text{ Mev}, \quad m_s = 175 \pm 55 \text{ Mev}, \quad m_b \sim 5.3 \pm 0.1 \text{ Gev}$$

The real Kobayashi-Maskawa matrix obtained is compatible with the phenomenological bounds except for the value of S_1 ¹⁸⁾. It tends to be too small. The values calculated with

$$\frac{m_d}{m_u} = 1.89, \quad \frac{m_s}{m_u} = 29.4 \quad (12)$$

$$m_t = 20 \text{ Gev}, \quad m_c (1 \text{ Gev}) = 1400 \text{ Mev}, \quad m_s (1 \text{ Gev}) = 120 \text{ Mev}, \quad m_b = 5.2 \text{ Gev}$$

are the following

$$\sin \theta_1 = 0.19, \quad \sin \theta_2 = -0.14, \quad \sin \theta_3 = 0.03 \quad (13)$$

Taking into account the undetermined complex phase, the above values will lie in a certain interval ¹⁹⁾, whose detailed determination is out of the scope of this letter. We, however, notice that complex Yukawa couplings are necessary in this model in order to introduce a CP violating phase in the Kobayashi-Maskawa mixing matrix, since the unique phase carried by the VEV of χ_3^1 can always be rotated away.

We have presented here mainly an idea about how to obtain a clean zero order mass matrix as in Eq. (1) due to symmetry restrictions (and not to the minimization of a potential) within the framework of a group $G_V \times SU_H(3) \times U_{PQ}(1)$. Among the many possibilities to complete a realistic mass matrix, we have chosen one, following at every step the simplicity criterion. Another nice feature of this approach is that there are no flavour changing neutral currents, essentially because there is only one doublet of G-W-S, ϕ (see Figs. 1 and 2) responsible for all the masses. Therefore its couplings with fermion are diagonalized together with the mass matrices. The only price to pay is to accept the existence of a heavy hidden sector of fermions. We have spoken about horizontal global symmetries as the natural place for a Higgs in the representation [6] to appear. However, the same basic idea could be applied to horizontal gauge symmetries, if the anomalies are cancelled with ad hoc heavy fermions.

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TABLE : Assignments according to the representations of the group $(SU_L(2) \times U_Y(1))(\text{gauge}) \times (SU_H(3) \times U_{PQ}(1))(\text{global})$. The PQ charges correspond to a total breaking of the $U_{PQ}(1)$ by the strong anomalies, so that no problem with domains in the universe is generated.

Fields	$(SU_L(2) \times SU_H(3))$	$Y = Q - T_3$	PQ
standard fields			
q_L	$(2, \bar{3})$	$1/6$	-1
u_R	$(1, 3)$	$2/3$	-2
d_R	$(1, 3)$	$-1/3$	-2
ℓ_L	$(2, \bar{3})$	$-1/2$	-1
e_R	$(1, 3)$	-1	-2
ϕ	$(2, 1)$	$1/2$	0
$SU_H(3) \times U_{PQ}(1)$ breaking			
χ	$(1, 3)$	0	$-1/2$
χ'	$(1, \bar{3})$	0	-1
σ	$(1, 1)$	0	$3/2$
hidden sector			
$F_{L,R}$	$(1, 1)$	$\begin{cases} -1 \\ 2/3 \\ -1/3 \end{cases}$	$-3/2$
$F'_{L,R}$	$(2, 1)$	$\begin{cases} -1/2 \\ 1/6 \end{cases}$	$-3/2$
$F''_{L,R}$	$(1, \bar{3})$	$\begin{cases} -1 \\ 2/3 \\ -1/3 \end{cases}$	-1

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FIGURE CAPTIONS

Figure 1 : This diagram contributes only to the mass of the third generation, with $\langle \chi \rangle = (0, 0, W)$. It corresponds to an "effective" Higgs $(2, 6)$. The representations quoted refer to the group $SU_L(2) \times SU_H(3)$.

Figure 2 : A diagram which contributes with an antisymmetric mass matrix to the masses of the first two generations, with $\langle \chi' \rangle = (a, 0, b)\omega$. M is the order of the invariant mass terms of the heavy fermions F . The representations quoted refer to the group $SU_L(2) \times SU_H(3)$.

FIG. 1

FIG. 2