Ref.TH.3673-CERN

BEFORE PRIMORDIAL INFLATION

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ABSTRACT

We show that, before the onset of primordial inflation, there is plenty of time for fields with very flat potentials and very weak couplings (such as the local supersymmetry breaking field and the axion field) to roll to the global minima of their potentials. Thus there is no energy stored in these fields today and hence no constraint (such as $f_{\text{axion}} < 10^{12}$ GeV) on the properties of their potentials.

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It is well known that a period of exponential expansion (inflation) at some time in the history of the very early universe can provide solutions to many puzzles of modern cosmology¹⁾. These include the observed homogeneity, isotropy and flatness of the universe - all highly unnatural in the standard hot big bang model - as well as the origin of the density perturbations which led to galaxy formation. The essential ingredient for inflation is the inflaton: a scalar field whose potential produces a supercooled phase transition. In the new inflationary universe scenario^{2),3)}, bubbles of the new phase are formed with the inflaton field still far from the value which minimizes the potential. The associated vacuum energy inside a given bubble causes it to grow exponentially, while quantum fluctuations of the inflaton produce the density perturbations which are the seeds of galaxies. Each bubble becomes an island universe.

Much work during the previous year⁴⁾⁻¹⁰⁾ has established that the inflaton field ϕ cannot be the Higgs field associated with the breaking of $SU(5)$ via a Coleman-Weinberg¹¹⁾ potential. A study of the potentials which produce an acceptable scenario shows that the most natural situation is to have the largest possible distances (in ϕ space) between the metastable and global minima of the potential^{12),13)}. This leads to what we call primordial inflation¹²⁾⁻¹⁷⁾: at the metastable minimum, $\langle \phi \rangle = 0$, while at the global minimum, $\langle \phi \rangle \approx M$, where $M = M_{\text{Planck}}/\sqrt{8\pi} \approx 2.4 \times 10^{18}$ GeV is the scale of gravity. In order to account for at least some of the effects of gravity, we have considered inflationary models^{14),15),17)} with potentials dictated by $N = 1$ supergravity^{18),19)}. Obviously, unless we want to identify the grand unification scale with the Planck scale, we must take ϕ to be a gauge singlet.

In this paper, we show that another important cosmological problem is solved by primordial inflation. Many models of particle physics contain, for one reason or another, fields with very flat potentials and very weak couplings. Examples are the field which breaks $SU(5)$ to $SU(3) \times SU(2) \times U(1)$ in certain locally supersymmetric GUTs²⁰⁾⁻²²⁾, and the "harmless" axion field^{23),24)}. If such fields are not at the global minima of their potentials at the end of the inflationary era, they must spend a very long time rolling towards these minima. Possibly they are still rolling today, and still have stored energy. We must be certain that this energy density does not exceed the observed upper limit. This condition has been used to show, for example, that an axion must have a decay constant f_a less than 10^{12} GeV²⁴⁾.

We will show that constraints of this type do not apply to a universe which has passed through primordial inflation. The reason is that, before very many

bubbles have been nucleated, there is plenty of time for slowly rolling fields to reach their global minima. Thus, almost all bubbles begin with these fields already at their minima, and no further rolling is necessary. This removes the upper bound on f_a , and any possible constraint on the supersymmetry breaking scale.

One initially attractive possibility is to identify one of the slowly rolling fields, such as the one which breaks supersymmetry, with the inflaton²⁵⁾⁻²⁷⁾. Unfortunately, this does not work. The energy stored in such a field is converted to ordinary particles so slowly that the universe does not reheat to a temperature adequate for baryon number generation ($\gtrsim 10^{10}$ GeV)^{26),27)}. The inflaton potential must have a curvature of at least $(10^{10} \text{ GeV})^2$ near its global minimum for sufficient reheating. This is simply not the case for a supersymmetry breaking field, which has a curvature of the order of $(100 \text{ GeV})^2$. We therefore stay with the idea of primordial inflation and an inflaton which is not identified with any other field. This gives a completely acceptable reheating scenario which has been discussed in detail elsewhere¹⁵⁾.

We now turn to the generic problem of slowly rolling fields. Let us begin by recalling some of the features of the primordial inflation model of Ref. 14). The superpotential is taken to be an infinite series in the complex field φ

$$f(\varphi) = \mu^3 \left[\lambda + \sum_{n=0} \frac{\lambda_n}{n+1} \left(\frac{\varphi}{M} \right)^{n+1} \right] \quad (1)$$

In $N = 1$ supergravity, this leads to a scalar potential

$$V(\varphi, \varphi^*) = e^{\frac{\varphi^* \varphi}{M^2}} \left[\left| \frac{\partial f}{\partial \varphi} + \frac{\varphi^* f}{M} \right|^2 - \frac{3}{M^2} |f|^2 \right] \quad (2)$$

where μ is a mass parameter which sets the overall scale, and the λ_n are dimensionless couplings. A detailed study¹⁷⁾ of $V(\varphi, \varphi^*)$ in the complex plane, including finite temperature corrections, showed that an acceptable potential is produced with the choice $\lambda_0 = 0.4$, $\lambda_1 = 0.05$, $\lambda_2 = 0.001$, $\lambda_3 = -0.1$, $\lambda_4 = 0.01$, $\mu/M = 0.01$. We take $\lambda = \lambda_1/2$ exactly, in order to have a local minimum at $\varphi = 0$ *).

*) This should not be regarded as a fine tuning: obviously, since φ is a gauge singlet, it does not matter if the metastable minimum is exactly at $\varphi = 0$ or not. The choice $\lambda = \lambda_1/2$ is made only for convenience and plays no essential role. We do fine tune parameters so that $V = 0$ (no cosmological constant) at the global minimum $\varphi \simeq M$.

Clearly, all of the couplings are ordinary numbers. No special adjustments are needed. The choice given above is made as an illustrative example, and a large range of parameter space yields acceptable potentials. The relevant features of the potential (with this choice of λ_n) were computed in Ref. 17), following the general analysis of Ref. 14). We quote the results, setting $M = 1$ for convenience.

The Hubble parameter H at $\varphi = 0$ is given by

$$H^2 = \frac{1}{3} \mu^6 \left(\lambda_0^2 - \frac{3}{4} \lambda_1^2 \right) \simeq 5 \times 10^{-14} \quad (3)$$

The local maximum to which φ will tunnel via the Hawking-Moss²⁸⁾ mechanism is at $\varphi = \varphi_{\text{MAX}} \simeq 0.01$. The probability per unit volume per unit time to form a bubble is

$$P \simeq \mu^6 \lambda_0^2 \lambda_2^2 e^{-B} \simeq 10^{-31} e^{-B} \quad (4)$$

where B is the Euclidean action for tunnelling from $\varphi = 0$ to $\varphi = \varphi_{\text{MAX}}$,

$$B \simeq 24 \pi^2 \frac{V(\varphi_{\text{MAX}}) - V(0)}{V(0)^2} \simeq 2 \times 10^8 \quad (5)$$

Now let us consider a field z with a long rollover time scale; z might be the field which breaks local supersymmetry or the field associated with the axion. We assume it has the equation of motion:

$$\ddot{z} + 3 H \dot{z} + m^2 (z - v) = 0 \quad (6)$$

where v is the vacuum expectation value of z at the global minimum of its potential, and m is its mass. We assume that, at some very early time when $\varphi = 0$ everywhere, $z = \dot{z} = 0$. The general solution to Eq. (6) with these boundary conditions, for $m \ll H$ (always true for the fields under consideration), is

$$z = v \left[1 - e^{-\left(\frac{m^2}{3H}\right)t} + \left(\frac{m}{3H}\right)^2 e^{-3Ht} \right] \quad (7)$$

Thus the rollover time scale is

$$\tau \approx \frac{3H}{m^2} \quad (8)$$

To get a feel for the numbers, take $m = 100 \text{ GeV}$ (appropriate for the local SUSY breaking field). This gives $\tau \approx 2 \times 10^8 \text{ GeV}^{-1}$, and $H\tau \approx 10^{20}$.

Let us see how likely it is that a bubble has formed by this time. In the Hawking-Moss picture²⁸⁾, an entire horizon volume tunnels simultaneously from $\phi = 0$ to $\phi = \phi_{\text{MAX}}$. Thus the probability that such a tunnelling event has occurred is P from Eq. (4) times the horizon volume H^{-3} times the rollover time τ . We want this probability to be much less than one:

$$P H^{-3} \tau \ll 1, \quad (9)$$

$$\left(\frac{P}{H^4} \right) \left(\frac{H^2}{m^2} \right) \ll 1 \quad (10)$$

Using our values for H and P , we get

$$10^{-4} e^{-B} \left(\frac{H^2}{m^2} \right) \ll 1 \quad (11)$$

For the enormous action of Eq. (5), $B \approx 2 \times 10^8$, this is obviously true for $m \approx 100 \text{ GeV}$ (again, appropriate for the local SUSY breaking field), and even for the very tiny axion mass $m_a \approx m_{\pi} F_{\pi} / F_a$. It is thus overwhelmingly probable that the bubble which encompasses our universe did not form until long after τ . By the time the average bubble was formed, any slowly rolling fields had long since reached the minima of their potentials.

There is one possible complication. The large H induces a temperature for all fields, the Hawking temperature $T_H = H/2\pi \approx 10^{11} \text{ GeV}$. Hence, a slowly rolling field will not necessarily reach the global minimum of its zero temperature potential, but rather the global minimum of its effective potential at the temperature T_H . Then, after the bubble has formed and as ϕ (the inflaton) approaches its minimum, the universe will reheat to a temperature T_R . This reheating will not affect slowly rolling fields, as they are essentially decoupled from all other particles (otherwise they would decay rather than roll slowly). A slowly rolling field will still be at the value corresponding to the minimum of its finite temperature potential at a temperature T_H . The difference between

the minimum values of $V(z, T_H)$ and $V(z, 0)$ is at most of the order of $m^2 T_H^2$, and could be zero¹⁵⁾. This possible energy density is much less than that stored in the reheated particles, which is of the order of T_R^4 (remember that we must have $T_R \geq 10^{10}$ GeV in order to generate the cosmological baryon asymmetry). Thus the universe is radiation dominated. As the universe cools, the energy density stored in the slowly rolling field decreases like T^3 (it behaves like non-relativistic matter). Thus, today, the maximum energy density $\mathcal{E}$ stored in a slowly rolling field is of order

$$\mathcal{E} \simeq m^2 T_H^2 \left(\frac{3^0 K}{T_R} \right)^3 \quad (12)$$

Using $m \simeq 100$ GeV and $T_H \simeq T_R \simeq 10^{11}$ GeV, we get $\mathcal{E} \simeq 10^{-45}$ GeV⁴. This worst case estimate is just above the observed upper limit of about 10^{-46} GeV⁴. Since $\mathcal{E}$ depends strongly on the model-dependent parameters m , T_H and T_R , we do not believe that there is any problem with this possible residual energy density.

For an SU(5) breaking field²⁰⁾⁻²²⁾, we must be certain that T_H is below the critical temperature T_C for the transition to the $SU(3) \times SU(2) \times U(1)$ phase. In local SUSY GUTs, T_C is, in general, no larger than the local SUSY breaking scale $M_{\text{SUSY}} \simeq (M_W M_P)^{1/2} \simeq 10^{10} - 10^{11}$ GeV^{*}). Our T_H is of the same order of magnitude. If, indeed, the SU(5) to $SU(3) \times SU(2) \times U(1)$ transition occurs before primordial inflation^{15), 16)}, any magnetic monopoles will be inflated away along with the particles which are produced during the transition.

As for the axion, the global minimum of its potential does not shift with temperature, provided $T \lesssim F_a$ ²⁴⁾. (For $T > F_a$, the Peccei-Quinn symmetry is restored, and there is no axion.) However, the axion mass is much smaller at high temperature. Roughly²⁴⁾,

$$m_a(T) \simeq \left(\frac{\Lambda^2}{F_a} \right) \left(\frac{\Lambda}{T} \right)^n \quad (13)$$

where Λ is some strong interaction scale (of order 100 MeV) and n is an integer which depends on the number of light-coloured particles (typically n is about 10). Even this ridiculously small mass can be accommodated in Eq. (11) with a B of order 100; our B is 2×10^8 and the axion has settled into its minimum long, long before the average bubble was formed.

^{*}) Models which violate this rule can be constructed; see Ref. 29).

To conclude: very weakly coupled fields with very flat potentials [such as the local SUSY breaking field, the axion field and, possibly, the SU(5) breaking field] may need a long time to roll to the global minima of their potentials. There is plenty of time to complete these rollovers before primordial inflation. Thus, constraints on the parameters of the potentials (such as $F_a \gtrsim 10^{12}$ GeV²⁴) which can be derived for a conventional universe do not apply to one which passes through primordial inflation.

ACKNOWLEDGEMENTS

We thank C. Kounnas, M. Quiros and G.G. Ross for discussions, and K. Olive for removing Fog.

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