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UNIFORM ACCELERATION AND THE QUANTUM FIELD THEORY VACUUM - I

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ABSTRACT

Free massless scalar and Dirac fields are quantized in two-dimensional Minkowski space in a representation in which the energy operator for a uniformly accelerating observer is diagonalized. The usual vacuum appears thermally excited in this representation. The crucial role of PCT symmetry, as regards the general validity of this conclusion, is illustrated.

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1. - INTRODUCTION

The formulation of quantum field theories in the non-inertial co-ordinate systems of flat space-time is of great interest because one encounters gravitational fields and event horizons in a relatively simple setting. Such problems represent a first step towards the construction of quantum field theory on curved space-time.

One of the simplest such co-ordinate systems is that constructed by a uniformly accelerated observer. These co-ordinates, known as Rindler co-ordinates¹⁾⁻³⁾, only cover a segment of Minkowski space-time, known as Rindler space, because of the presence of event-horizons for a uniformly accelerated observer. An observer at rest in Rindler space is equivalent to a uniformly accelerating observer in Minkowski space, and experiences a static gravitational field.

When one discusses the particle interpretation of canonical field theory for a uniformly accelerated observer one must introduce the field "Hamiltonian" in the Rindler co-ordinate system. Field quanta which are eigenstates of this operator are then associated with particles of a definite "energy"⁴⁾. This problem has been studied in non-interacting field theories for a decade, starting with massless and massive^{5),6)} scalar fields. Subsequently, Dirac fields⁷⁾⁻¹⁰⁾, Maxwell fields in a physical gauge¹¹⁾, and the Hara-Schwinger field⁷⁾ have been investigated. Green's function¹²⁾⁻¹⁵⁾, path integral^{16),17)} and other¹⁸⁾ methods have also been used.

The surprising conclusion in all the above cases [but note Ref. 19) to which we will return later] is that the Rindler and Minkowski representations are inequivalent, and, in particular, the (pure) vacuum state of the Minkowski representation is a mixed state of finite temperature of the Rindler representation²⁰⁾. Furthermore, it has been proved that any field theory which satisfies the Wightman axioms has this property^{21),22)}. Unruh²³⁾ has shown that, in the case of a free, massive scalar field, the thermal bath of Rindler quanta in the Minkowski vacuum does excite a model particle detector at rest in Rindler space. This result has since been extended to the Dirac field⁹⁾. The possible experimental detection of such thermal excitations for accelerating thermometers in the Minkowski vacuum has been studied in Ref. 24).

Clearly, there must be some general principle which is responsible for the remarkable uniformity of these results. Euclidean formulations of field theory¹⁷⁾ relate the finite temperature effects to the topology of the space-time. However, no general principle has been found in any of the above-mentioned constructions of canonical free field theory on Minkowski space. On the other hand, the

axiomatic approach of Refs. 21) and 22) has highlighted the important role of the PCT symmetry^{25),26)} of quantum field theory. The elucidation of this general principle by the elementary methods of free field theory is the object of this investigation.

Previous canonical investigations have been obscure in this respect because of their excessive reliance on the Rindler co-ordinate system. Specifically, field theories have been constructed on Rindler space and subsequently related to Minkowski space through a process of continuation. However, we believe that a more fruitful approach is to quantize the field of interest in the whole of Minkowski space in such a way that its restriction to the Rindler segment gives a "sensible" field theory for a uniformly accelerated observer. For us, Rindler co-ordinates merely provide a convenient notation for keeping track of the region of Minkowski space accessible to a uniformly accelerated observer.

This is the approach which we adopt in this paper for the quantization of free massless scalar and Dirac fields in two space-time dimensions. These problems are sufficient for us to identify the general principles which relate the Minkowski and Rindler representations. In a subsequent paper we will study these properties further in four space-time dimensions for a massive scalar field, a Dirac field and the Maxwell field in a covariant gauge.

The remainder of this paper is organized as follows. Section 2 is devoted to a review of the Rindler co-ordinate system constructed by a uniformly accelerated observer. Minkowski space is shown to possess two causally-disconnected Rindler space segments. The operator that generates the evolution in the Rindler time variable is identified as the generator of Lorentz boosts.

In Section 3 the quantization of a free massless Hermitian scalar field in two-dimensional Minkowski space is studied in a representation in which the Lorentz boost is diagonalized. There are an infinite number of ways in which to implement this scheme, but with the aid of PCT symmetry we are able to select out one scheme which corresponds to the "natural" form for a uniformly accelerated observer. This is a representation in which the total boost is equal to the boost measured by a uniformly accelerated observer on one of the Rindler spaces. PCT symmetry then guarantees that this representation is also "natural" on the other (causally disconnected) Rindler space. This representation is shown to be inequivalent with the usual Minkowski one. For instance, the Minkowski vacuum is shown to be a mixed state of finite temperature of the Rindler representation.

In Section 4 we repeat the construction of Section 3 for a massless Dirac field, in order to see how the techniques are modified by the change to Fermi-Dirac statistics, and by the charge of the field.

In Section 5 we summarize and discuss our investigation, and present some conclusions.

Appendix A contains some mathematical details of the functions used in Section 3, and a brief discussion of the relevant aspects of PCT symmetry. Appendix B contains a few mathematical details of the spinors introduced in Section 4.

2. - UNIFORM ACCELERATION AND RINDLER CO-ORDINATES

We consider first the motion of a classical point particle in two-dimensional Minkowski space, $x^\mu = (t, x)$, $\eta^{\mu\nu} = \text{diag}(1, -1)$, for which the two-acceleration, a^μ , satisfies,

$$a_\mu a^\mu = -a^2 \quad (2.1)$$

where a is a constant.

Without loss of generality, we may require that the particle be at rest at $t = 0$, which we also choose to be the origin of the proper time, τ , along the particle's world-line. This world-line is then given by,

$$t = a^{-1} \sinh(a\tau) \quad (2.2a)$$

$$x = a^{-1} \cosh(a\tau) + b \quad (2.2b)$$

where b is an integration constant associated with the freedom of choosing the location of the particle at $t = 0$.

A motion of the form (2.2) is termed hyperbolic²⁷⁾, in contrast with the "parabolic" motion of a uniformly accelerated particle in Newtonian mechanics. It is apparent that the motion (2.2) amounts to a Lorentz boost of magnitude a of the event $(0, b + a^{-1})$, about the event $(0, b)$. This motion is generated by,

$$B_b = a(K - bP^0) \quad (2.3)$$

where K is the generator of Lorentz boosts about $(0, 0)$ and P^0 is the generator of translations in Minkowski time. These operators satisfy the Poincaré algebra,

$$[K, P^0] = -iP^1 \quad (2.4a)$$

$$[K, P^1] = -iP^0 \quad (2.4b)$$

where P^1 is the generator of translations in x .

In passing, it is interesting to note that the inertial limit, $a \rightarrow 0$, of (2.2) corresponds to a world-line at spatial infinity, unless we make the choice,

$$b = -a^{-1} + c \quad (2.5)$$

where c is a constant. In this case we may also verify that,

$$\lim_{a \rightarrow 0} B_b = P^0 \quad (2.6)$$

in accordance with our intuition.

However, we are here interested in setting up a co-ordinate system for a single accelerated observer. For this purpose we wish to use the world-lines (2.2) with different values of a as surfaces of constant "position" for our accelerating observer. Clearly, the choice (2.5) is not allowed in this case, and indeed the maximum simplification occurs when

$$b = 0 \quad (2.7)$$

giving,

$$t = a^{-1} \sinh(a\tau) \quad (2.8a)$$

$$x = a^{-1} \cosh(a\tau) \quad (2.8b)$$

and,

$$B = B_{b=0} = aK \quad (2.9)$$

We see that the non-accelerating world-lines are now banished to spatial infinity, and that the generator of τ translations is a pure Lorentz boost about the origin. This is a positive boost for accelerations along the positive x direction, $a > 0$, but a negative one for accelerations along the negative x direction, $a < 0$.

The world-lines (2.8) with $a > 0$ fill up a wedge-shaped region, R , on the right-hand side of space-time, where,

$$R = \{(t, x) : x > |t|\} \quad (2.10)$$

All of these world-lines are asymptotic to the one for which $a \rightarrow \infty$, given by the union of the two half light-cones, R_+ , R_- , through the origin, where,

$$R_+ = \{(t, x) : t = x \text{ AND } x > 0\} \quad (2.11a)$$

and,

$$R_- = \{(t, x) : t = -x \text{ AND } x > 0\} \quad (2.11b)$$

so that infinite proper acceleration corresponds to a motion with the speed of light.

It is well known²⁷⁾ that R_+ and R_- constitute event horizons for an observer on any one of the world-lines (2.8) in R ($a > 0$). The region R therefore constitutes the whole of space-time for such an observer, and he may construct a co-ordinate system to cover it.

The adjacent hyperbolic world-lines in R describe a Born rigid motion²⁸⁾, and so may be used to define a position co-ordinate of an event in R for our observer. This is done by giving the value of a^{-1} for the particular hyperbolic world-line which passes through the event in question, as its position.

A time may also be assigned to any event in R once the clocks on adjacent hyperbolic world-lines have had their rates adjusted to be Einstein synchronized. This is necessary because of the different gravitational potentials experienced by the clocks on adjacent world-lines. It amounts to a simple re-scaling of the proper time.

In this way our uniformly accelerated observer constructs the Rindler co-ordinate system¹⁾, $x^\alpha = (T, X)$, $X > 0$, to cover R , with

$$\left. \begin{aligned} t &= X \sinh T \\ x &= X \cosh T \end{aligned} \right\} (t, x) \in R \quad \begin{aligned} (2.12a) \\ (2.12b) \end{aligned}$$

We note that Rindler time, T , and Minkowski time, t , advance in the same sense on R , and that the world-lines R_- , R_+ correspond to the infinite past and future, respectively, of the Rindler time variable, T . This is in accordance with their roles as event horizons for our accelerating observer.

The metric in the Rindler co-ordinate system is,

$$g^{\alpha\beta} = \text{diag}(X^{-2}, 1) \quad (2.13)$$

and a space-time with this metric is known as Rindler space. The co-ordinate singularity of (2.12) and (2.13) encountered at $x^\mu = (0, 0)$ is a feature of non-inertial reference frames³⁰⁾.

Translations in Rindler time, T , clearly correspond to a boost of the x axis about the origin, and are therefore generated by K . When acting on scalar functions of space and time the restriction of this operator to R has the form

$$K_R = i \frac{\partial}{\partial T} \quad (2.14)$$

which has the familiar Hamiltonian form. Eigenvalues of B are simply related to those of K_R by the re-scaling required to recover proper time, t , from Rindler time, T . One may well ask, therefore, if there are any advantages to be gained from introducing the Rindler co-ordinate system? For us, the main purpose of Rindler co-ordinates will be to keep track of the segment of Minkowski space which is accessible to our uniformly accelerated observer. We will not use the metric (2.13).

Thus far we have neglected the world-lines (2.8) with $a < 0$. These fill up a wedge shaped region, L , on the left-hand side of Minkowski space, where

$$L = \{(t, x) : x < -|t|\} \quad (2.15)$$

This is obtained from R by space reflection, $x \rightarrow -x$, and is causally disconnected from it. All the properties which we have discussed for a uniformly accelerated observer on R , therefore have analogues on L . In particular, Rindler co-ordinates, $x^\alpha = (T, X)$, $X < 0$, may be introduced to cover L by

$$\left. \begin{aligned} t &= -X \sinh T \\ x &= X \cosh T \end{aligned} \right\} (t, x) \in L \quad (2.16)$$

where we have chosen T to evolve in the same sense as t . Evolution in T is generated by the negative boost operator, $(-K)$, whose restriction to L has the form,

$$-K_L = i \frac{\partial}{\partial T} \quad (2.1)$$

when acting on scalar functions of space and time. However, since R and L are not causally connected the timelike operators K_L and K_R may be diagonal simultaneously.

Finally, we note that Minkowski space is completed by taking the union of L and R with the regions,

$$F = \{(t, x) : t > |x|\} \quad (2.18)$$

and,

$$P = \{(t, x) : t < -|x|\} \quad (2.18)$$

and that K is spacelike on F and P .

3. - FREE SCALAR FIELD THEORY ON THE RINDLER SPACE R

In this section we will be concerned with the quantum theory of a Hermitian scalar field, ϕ , on two-dimensional Minkowski space, which satisfies the wave equation,

$$\square \phi(t, x) = 0 \quad (3.)$$

subject to the quantization condition,

$$[\pi_\phi(t, x), \phi(t, y)] = -i \delta(x - y) \quad (3.)$$

where the momentum canonically conjugate to ϕ is,

$$\pi_\phi(t, x) = \frac{\partial}{\partial t} \phi(t, x) \quad (3.)$$

It can easily be shown⁵⁾ that in terms of the Rindler co-ordinate system for R , the canonical momentum is just (3.3) transcribed into the new co-ordinate system, while the equal t commutation relation, (3.2), is transcribed into an equal T one, which coincides with (3.2) at $T = 0$, and is equivalent with it on a surface of constant $T \neq 0$, since such a surface is spacelike⁵⁾. Therefore, if we quantize ϕ in Minkowski space it is automatically quantized on R too. However, before studying the quantization of $\phi|_R$ we will first review the salient features of the usual realization of (3.2) in Minkowski space, which we will call the Minkowski representation.

In any representation the field ϕ is to be expanded over some complete, orthonormal set of c number solutions of (3.1). When space-time is the whole of Minkowski space, completeness and orthogonality are defined with respect to the conserved, Hermitian, but not positive definite, scalar product,

$$\langle \phi | \psi \rangle_M = \int_{-\infty}^{\infty} dx \phi^*(t, x) i \overleftrightarrow{\partial}_t \psi(t, x) \quad (3.4)$$

where

$$\overleftrightarrow{\partial}_t = \overrightarrow{\partial}_t - \overleftarrow{\partial}_t \quad (3.5)$$

The usual choice for such a set of functions is the set of eigenfunctions of the energy operator, $i(\partial/\partial t)$. These functions are labelled by the x momentum eigenvalue, k , and the sign of the energy. For positive energy we have,

$$\psi_{k,+}(t, x) = (4\pi|k|)^{-1/2} e^{-i|k|t + ikx}, \quad k \in]-\infty, \infty[\quad (3.6a)$$

and for negative energy,

$$\psi_{k,-}(t, x) = \psi_{-k,+}^*(t, x) \quad (3.6b)$$

These eigenfunctions satisfy the orthogonality conditions

$$\langle \psi_{k,+} | \psi_{k',+} \rangle_M = \delta(k - k') \quad (3.7a)$$

$$\langle \psi_{k,-} | \psi_{k',-} \rangle_M = -\delta(k - k') \quad (3.7b)$$

$$\langle \psi_{k,+} | \psi_{k',-} \rangle_M = 0 \quad (3.7c)$$

and the completeness property,

$$\int_{-\infty}^{\infty} dk \left[\psi_{k,+}(t,x) (-i\vec{\partial}_t) \psi_{k,+}^*(t,y) + \psi_{k,+}^*(t,x) (i\vec{\partial}_t) \psi_{k,+}(t,y) \right] = \delta(x-y) \quad (3.7d)$$

so that

$$\phi(t,x) = \int_{-\infty}^{\infty} dk \left[b(k) \psi_{k,+}(t,x) + b^\dagger(k) \psi_{k,+}^*(t,x) \right] \quad (3.8)$$

satisfies the quantization condition, (3.2), if the normal mode amplitudes obey the commutation relation,

$$[b(k), b^\dagger(k')] = \delta(k-k') \quad (3.9)$$

with all of their other commutators vanishing.

This operator algebra, (3.9), may be realized by the introduction of the Minkowski vacuum, $|0\rangle_M$, by

$$b(k) |0\rangle_M = 0, \quad \forall k \quad (3.10)$$

This state is the ground state of the field energy operator. Excitations of it generated by the $b^\dagger(k)$ carry definite energy and momentum, and are identified as "particles" with those properties.

Although (3.8)-(3.10) are perfectly good representations of ϕ throughout Minkowski space, including R , it is apparent that the operators $b^\dagger$ do not create "energy" eigenstates for our uniformly accelerated observer. For him the natural set of functions to expand the field ϕ over will be the eigenfunctions of the Lorentz boost operator, $\{\phi_j\}$. These satisfy,

$$K \phi_j(t,x) = j \phi_j(t,x) \quad (3.11a)$$

where j is a real number, since K is Hermitian, and,

$$-\infty < j < \infty \quad (3.11b)$$

forming a continuous eigenvalue spectrum.

The effect of a Lorentz boost on the arguments of the ϕ_j is a phase change. Specifically, on R and L ,

$$\phi_j(x \sinh \alpha, x \cosh \alpha) = e^{-ij\alpha} \phi_j(0, x) \quad (3.12)$$

and this finite transformation is more useful than (3.11a) in the construction of the ϕ_j as a superposition of the energy momentum eigenfunctions, (3.6).

We note that the sign of the energy is an invariant of the restricted Poincaré group, and that the sign of the x momentum is Lorentz boost invariant for a massless field, so that we have two types of positive energy solutions of (3.11a) and (3.12), namely

$$\phi_{j,1}(t,x) = (2\pi)^{-1/2} \int_0^\infty dk k^{-1/2} \psi_{k,+}(t,x) \quad (3.13a)$$

and,

$$\phi_{j,2}(t,x) = (2\pi)^{-1/2} \int_{-\infty}^0 dk |k|^{1/2} \psi_{k,+}(t,x) \quad (3.13b)$$

which have positive and negative momentum respectively.

Negative energy solutions of (3.11a) and (3.12) also exist for the same eigenvalue j . There are again two types, namely,

$$\phi_{j,3}(t,x) = \phi_{-j,1}^*(t,x) \quad (3.14a)$$

and

$$\phi_{j,4}(t,x) = \phi_{-j,2}^*(t,x) \quad (3.14b)$$

which are formed from negative and positive momentum components respectively.

The form of these functions in terms of Rindler co-ordinates on R and L is given in the Appendix. They satisfy the completeness relation,

$$\sum_{j=-\infty}^{\infty} \int d_j \left[\phi_{j,3}(t,x) (-i\vec{\partial}_t) \phi_{j,3}^*(t,y) + \phi_{j,3}^*(t,x) (i\vec{\partial}_t) \phi_{j,3}(t,y) \right] = \delta(x-y) \quad (3.15)$$

and the orthogonality conditions,

$$\langle \phi_{j,\lambda} | \phi_{j',\lambda'} \rangle_M = m_{\lambda\lambda'} \delta(j-j') \quad (3.16)$$

where the metric matrix is,

$$m_{\lambda\lambda'} = \text{diag}(1, 1, -1, -1) \quad (3.17)$$

so that there is no correlation between the sign of the eigenvalue, j , and the norm of the corresponding eigenfunction, ϕ_j . There are therefore an infinite number of ways in which to expand the quantum field, ϕ , over boost eigenfunctions, and hence an infinity of different choices of basis states in the one-particle sector of Fock space.

One such expansion is,

$$\phi(t,x) = \sum_{\lambda=1}^2 \int_{-\infty}^{\infty} dj [c(j,\lambda) \phi_{j,\lambda}(t,x) + c^\dagger(j,\lambda) \phi_{j,\lambda}^*(t,x)] \quad (3.18)$$

which satisfies the quantization condition, (3.2), if the normal mode amplitudes, $c, c^\dagger$ have the commutation relation,

$$[c(j,\lambda), c^\dagger(j',\lambda')] = \delta_{\lambda\lambda'} \delta(j-j'), \quad \lambda, \lambda' = 1, 2 \quad (3.19)$$

with all of their other commutators vanishing.

A representation of this type was first studied by Boulware¹⁹⁾. It is equivalent to the representation (3.8) and (3.9) since,

$$c(j,\lambda) |0\rangle_M = 0, \quad \forall j, \lambda = 1, 2 \quad (3.20)$$

Because the field boost operator is,

$$K = -\frac{1}{2} \int_{-\infty}^{\infty} dx \alpha [(\partial_t \phi)^2 + (\partial_x \phi)^2] \quad (3.21a)$$

$$= \sum_{\lambda=1}^2 \int_{-\infty}^{\infty} dj \left(\frac{1}{2} \right) [c^\dagger(j,\lambda) c(j,\lambda) + c(j,\lambda) c^\dagger(j,\lambda)] \quad (3.21b)$$

we indeed find that the operators $c^\dagger(j,\lambda)$ create quanta which carry an amount of boost j . However, j can be positive or negative, so that K has no ground state. On the other hand, it is our prejudice that the "energy" operator for our uniformly accelerated observer should be positive definite.

The way out of this apparent impasse is to note that K is not the boost which the field in R carries, which is actually given by

$$K_R = -\frac{1}{2} \int_0^{\infty} dx \alpha [(\partial_t \phi)^2 + (\partial_x \phi)^2] \Big|_{t=0} \quad (3.22)$$

whereas some of the boost carried by the quantum created by $c^\dagger(j,\lambda)$ resides on L , which is inaccessible to an accelerated observer on R . In order to evaluate (3.22) we must study the orthogonality properties of the $\phi_{j,\lambda}$ functions in R . That is, we must determine their behaviour with respect to the conserved scalar product on R , given by,

$$\langle \phi | \psi \rangle_R = \int_0^{\infty} dx \phi^*(t,x) i \overleftrightarrow{\partial}_t \psi(t,x) \Big|_{t=0} \quad (3.23a)$$

Similarly, a conserved scalar product on L is,

$$\langle \phi | \psi \rangle_L = \int_{-\infty}^0 dx \phi^*(t,x) i \overleftrightarrow{\partial}_t \psi(t,x) \Big|_{t=0} \quad (3.23b)$$

so that,

$$\langle \phi | \psi \rangle_L + \langle \phi | \psi \rangle_R = \langle \phi | \psi \rangle_M \quad (3.23c)$$

The orthogonality properties of the $\phi_{j,\lambda}$ with respect to (3.23a) and (3.23b) are listed in the Appendix. We note that the $\phi_{j,\lambda}$ are not orthogonal with respect to (3.23a), so that (3.22) is,

$$K_R = \int_{-\infty}^{\infty} dj \frac{|j|}{4 \sinh(\pi j)} \left[e^{\pi j} \{ c^\dagger(j,\lambda) c(j,\lambda) + c(j,\lambda) c^\dagger(j,\lambda) \} + \{ c^\dagger(j,\lambda) c(-j,\lambda) + c(j,\lambda) c(-j,\lambda) \} \right] \quad (3.24a)$$

while the analogous operator on L is,

$$K_L = - \int_{-\infty}^{\infty} dj \frac{|j|}{|4 \sinh(\pi j)|} \left[e^{-\pi j} \{ c^{\dagger}(j, \lambda) c(j, \lambda) + c(j, \lambda) c^{\dagger}(j, \lambda) \} \right. \\ \left. + \{ c^{\dagger}(j, \lambda) c^{\dagger}(-j, \lambda) + c(j, \lambda) c(-j, \lambda) \} \right] \quad (3.24b)$$

giving,

$$K = K_L + K_R \quad (3.25a)$$

and,

$$[K_L, K_R] = 0 \quad (3.25b)$$

as expected.

However, we now find that $c^{\dagger}(j, \lambda)$ does not create an eigenstate of K_R , since

$$[K_R, c^{\dagger}(j, \lambda)] = \frac{|j|}{|2 \sinh(\pi j)|} \left[e^{\pi j} c^{\dagger}(j, \lambda) + c(-j, \lambda) \right] \quad (3.26)$$

Furthermore, $|0\rangle_M$ is not an eigenstate of K_R , since,

$$\langle 0 | (K_R)^2 | 0 \rangle_M - \langle 0 | K_R | 0 \rangle_M^2 \neq 0 \quad (3.27)$$

It is clear that K_R must be diagonalized, together with K . We call the representation in which this is true the Rindler representation.

One way in which to achieve this goal would be to diagonalize the field boost operator, (3.24a), directly. However, it will be more illuminating first to find the boost eigenfunctions that diagonalize the differential operator,

$$K_R = \begin{cases} i \partial_{\partial T} & \text{in } R \\ 0 & \text{in } L \end{cases} \quad (3.28)$$

These are boost eigenfunctions for which the total boost resides on R . Such functions must vanish on L , by virtue of (3.28). It is apparent from the formulae given in the Appendix that it is possible to choose linear combinations of the $\phi_{j, \lambda}$ functions which have this property. This is because these functions are not linearly independent on L , (or R). However, there is a general principle, which will hold in any free-field theory, which is responsible for this. It follows from the relation²⁵⁾ between reversal in space and time and complex Lorentz transformations which is at the root of PCT symmetry^{26), 25)}.

The point is that (3.12) will remain valid for complex α provided ϕ can be continued analytically to the relevant values of t and x . Such a continuation can be defined by (3.13) and (3.6a), and we see that for positive energy solutions and positive x , $i\alpha$ can be taken positive in (3.12). Then, with $\alpha = -i\pi$,

$$\phi_{j, \lambda}(0, x) = e^{-\pi j} \phi_{j, \lambda}(0, x), \quad \lambda = 1, 2, \quad x > 0 \quad (3.29a)$$

and invoking (3.12) for arbitrary real α ,

$$\phi_{j, \lambda}(-t, x) = e^{-\pi j} \phi_{j, \lambda}(t, x), \quad \lambda = 1, 2, \quad (t, x) \in R \quad (3.29b)$$

or since,

$$\phi_{j, \lambda}(-t, -x) = \phi_{-j, \lambda}^*(t, x) \quad (3.30)$$

we have

$$\phi_{-j, \lambda}^*(t, x) = e^{-\pi j} \phi_{j, \lambda}(t, x), \quad \lambda = 1, 2, \quad (t, x) \in R \quad (3.31)$$

This relation [see (3.14)] characterizes the non-independence of the $\phi_{j, \lambda}$ on R , and is a consequence of the general principle of PCT invariance of field theory (see Appendix A). A relation of this form will also hold for boost eigenfunctions constructed for other free fields, independently of their mass or spin. On L the situation is reversed,

$$\phi_{j, \lambda}(t, x) = e^{-\pi j} \phi_{-j, \lambda}^*(t, x), \quad \lambda = 1, 2, \quad (t, x) \in L \quad (3.32)$$

since K generates a motion in the sense of decreasing Minkowski time here.

The relations (3.31) and (3.32) may be verified from the formulae given in the Appendix.

We now construct the following eigenfunctions of the boost operator, K , of eigenvalue j :

$$\phi_{j,\lambda,R}(t,x) = i/2 \sinh(\pi j) \left\{ e^{\pi j/2} \phi_{j,\lambda}(t,x) - e^{-\pi j/2} \phi_{-j,\lambda}^*(t,x) \right\}, \quad (3.33a)$$

$$\lambda = 1, 2$$

which are dominantly positive energy for $j > 0$, and

$$\phi_{j,\lambda,L}(t,x) = -i/2 \sinh(\pi j) \left\{ e^{-\pi j/2} \phi_{j,\lambda}(t,x) - e^{\pi j/2} \phi_{-j,\lambda}^*(t,x) \right\}, \quad (3.33b)$$

$$\lambda = 1, 2$$

which are dominantly negative energy for $j > 0$. By virtue of (3.31b) and (3.32) these have the properties that,

$$\phi_{j,\lambda,L}(t,x) = 0, \quad (t,x) \in R \quad (3.34a)$$

and,

$$\phi_{j,\lambda,R}(t,x) = 0, \quad (t,x) \in L \quad (3.34b)$$

while

$$\phi_{j,\lambda,L}(-t,-x) = \phi_{j,\lambda,R}(t,x) \quad (3.34c)$$

so that the $\phi_{j,\lambda,L}$ are irrelevant for field theory on R . The form of these functions is given in the Appendix.

The functions (3.33) satisfy the orthogonality properties

$$\langle \phi_{j,\lambda,R} | \phi_{j',\lambda',L} \rangle_M = 0 \quad (3.35a)$$

$$\langle \phi_{j,\lambda,R} | \phi_{j',\lambda,R} \rangle_M = \langle \phi_{j,\lambda,R} | \phi_{j',\lambda,R} \rangle_R = \delta_{\lambda\lambda'} \delta(j-j') \operatorname{sgn}(j) \quad (3.35b)$$

$$\langle \phi_{j,\lambda,L} | \phi_{j',\lambda',L} \rangle_M = \langle \phi_{j,\lambda,L} | \phi_{j',\lambda',L} \rangle_L = \delta_{\lambda\lambda'} \delta(j-j') \operatorname{sgn}(-j) \quad (3.35c)$$

and the completeness relation,

$$\sum_{\lambda=1}^2 \int_0^\infty dj \left[\phi_{j,\lambda,R}(t,x) (-i\vec{\partial}_t) \phi_{j,\lambda,R}^*(t,y) + \phi_{j,\lambda,R}^*(t,x) (-i\vec{\partial}_t) \phi_{j,\lambda,R}(t,y) \right. \\ \left. + \phi_{j,\lambda,L}^*(t,x) (-i\vec{\partial}_t) \phi_{j,\lambda,L}(t,y) + \phi_{j,\lambda,L}(t,x) (-i\vec{\partial}_t) \phi_{j,\lambda,L}^*(t,y) \right] = \delta(x-y) \quad (3.35d)$$

We may now write the quantum field operator, ϕ , as

$$\phi(t,x) = \phi_L(t,x) + \phi_R(t,x) \quad (3.36a)$$

where

$$\phi_L(t,x) = \sum_{\lambda=1}^2 \int_0^\infty dj \left[a(j,\lambda,L) \phi_{j,\lambda,L}^*(t,x) + a^\dagger(j,\lambda,L) \phi_{j,\lambda,L}(t,x) \right] \quad (3.36b)$$

and

$$\phi_R(t,x) = \sum_{\lambda=1}^2 \int_0^\infty dj \left[a(j,\lambda,R) \phi_{j,\lambda,R}(t,x) + a^\dagger(j,\lambda,R) \phi_{j,\lambda,R}^*(t,x) \right] \quad (3.36c)$$

with

$$[a(j,\lambda,L), a^\dagger(j',\lambda',L)] = \delta_{\lambda\lambda'} \delta(j-j') \quad (3.37a)$$

$$[a(j,\lambda,R), a^\dagger(j',\lambda',R)] = \delta_{\lambda\lambda'} \delta(j-j') \quad (3.37b)$$

and all other commutators of the normal mode amplitudes vanishing. We note that ϕ_L and ϕ_R are not independent fields, but are related by PCT.

The algebra (3.37) may be realized by the introduction of two states, $|0\rangle_L$ and $|0\rangle_R$ with

$$a(j,\lambda,R)|0\rangle_R = 0, \quad \forall j > 0, \quad \lambda = 1, 2 \quad (3.38a)$$

and

$$a(j, \lambda, L) |0\rangle_L = 0, \quad \forall j > 0, \lambda = 1, 2 \quad (3.38b)$$

which we may call the left and right Rindler vacua, respectively.

A Fock space, F_R , may now be constructed from the states resulting from the action of the $a^\dagger(j, \lambda, R)$ on $|0\rangle_R$, and likewise, a Fock space F_L from the action of the $a^\dagger(j, \lambda, L)$ on $|0\rangle_L$. States of the Minkowski representation are elements of $F_R \otimes F_L$, whereas field theory on R is concerned with ϕ_R and F_R only. The boost operator may now be written

$$K = K_L + K_R \quad (3.39a)$$

where

$$K_R = \frac{1}{2} \sum_{\lambda=1}^2 \int_0^\infty dj j [a^\dagger(j, \lambda, R) a(j, \lambda, R) + a(j, \lambda, R) a^\dagger(j, \lambda, R)] \quad (3.39b)$$

and

$$K_L = -\frac{1}{2} \sum_{\lambda=1}^2 \int_0^\infty dj j [a^\dagger(j, \lambda, L) a(j, \lambda, L) + a(j, \lambda, L) a^\dagger(j, \lambda, L)] \quad (3.39c)$$

so that, K_R is positive definite, and $|0\rangle_R$ is the ground state of K_R , as desired. Furthermore, we note that $|0\rangle_L$ is the ground state of $(-K_L)$, which is the generator of the motion on L . Therefore, we have simultaneously achieved a representation which is appropriate for any one of the uniformly accelerated observers on L . This is a consequence of PCT symmetry, since

$$\phi_R(t, x) \xrightarrow{PCT} \phi_L(-t, -x), \quad (t, x) \in R \quad (3.40a)$$

and

$$|0\rangle_R \xrightarrow{PCT} |0\rangle_L \quad (3.40b)$$

Our next concern is the relation between the representation, (3.36) and (3.37), and the Minkowski representation, (3.18) and (3.19). We have,

$$a^\dagger(j, \lambda, R) = -\langle \phi(t, x) | \phi_{j, \lambda, R}^*(t, x) \rangle_M, \quad j > 0 \quad (3.41a)$$

$$= i \left\{ (1 - e^{-2\pi j})^{-1/2} c(j, \lambda) + (e^{2\pi j} - 1)^{-1/2} c(-j, \lambda) \right\}, \quad j > 0 \quad (3.41b)$$

for instance. The two representations are clearly inequivalent, and the number of Rindler quanta in the Minkowski vacuum is given by,

$$\langle 0 | a^\dagger(j, \lambda, R) a(j, \lambda, R) | 0 \rangle_M = \delta_{\lambda\lambda} \delta(j-j) (e^{2\pi j} - 1)^{-1} \quad (3.42)$$

which shows the familiar Rindler-Fulling temperature of $T_R = (2\pi)^{-1}$, first found by Fulling⁵⁾. When allowance is made for the relation between proper time along a given uniformly accelerated world-line and Rindler time, this turns into a temperature of $a/2\pi$, where a is the acceleration parameter.

Another important point concerns the form of $|0\rangle_M$ in terms of elements of $F_R \otimes F_L$. We note that $|0\rangle_M$ is defined by (3.20) and

$$c(j, \lambda) = \langle \phi | \phi_{j, \lambda} \rangle_M \quad (3.43a)$$

$$= \begin{cases} i \left[(1 - e^{-2\pi j})^{-1/2} a(j, \lambda, R) - (e^{2\pi j} - 1)^{-1/2} a^\dagger(j, \lambda, L) \right], & j > 0 \\ i \left[(1 - e^{2\pi j})^{-1/2} a(-j, \lambda, L) - (e^{-2\pi j} - 1)^{-1/2} a^\dagger(-j, \lambda, R) \right], & j < 0 \end{cases} \quad (3.43b) \quad (3.43c)$$

so that,

$$c(j, \lambda) = \begin{cases} i U a(j, \lambda, R) U^{-1}, & j > 0 \\ i U a(-j, \lambda, L) U^{-1}, & j < 0 \end{cases} \quad (3.44a)$$

$$(3.44b)$$

where

$$U = \exp \left\{ - (2\pi)^{-1/2} \sum_{\lambda=1}^2 \int_0^\infty dj j \ln \tanh \left(\frac{\pi j}{2} \right) \left[a^\dagger(j, \lambda, R) a^\dagger(j, \lambda, L) - a(j, \lambda, R) a(j, \lambda, L) \right] \right\} \quad (3.45)$$

giving

$$|\phi\rangle_M = U|\phi\rangle_L |\phi\rangle_R \quad (3.46a)$$

$$= Z^{-1/2} \prod_{j=0}^{\infty} \prod_{\lambda=1}^2 e^{-n\pi j} |n, j, \lambda\rangle_L |n, j, \lambda\rangle_R \quad (3.46b)$$

where

$$Z = \prod_{j=0}^{\infty} \prod_{\lambda=1}^2 e^{-2n\pi j} = \prod_{j=0}^{\infty} \prod_{\lambda=1}^2 (1 - e^{-2n\pi j})^{-1} \quad (3.47a)$$

$$|n, j, \lambda\rangle_R = (n!)^{-1/2} [a^\dagger(j, \lambda, R)]^n |\phi\rangle_R \quad (3.47b)$$

and similarly for $|n, j, \lambda\rangle_L$. We note that the total boost of each $|\lambda\rangle_L, |\lambda\rangle_R$ component of (3.46b) is zero.

If we now consider an observable, A_R , which is restricted to R , its expectation value in the Minkowski vacuum is,

$$\langle \phi | A_R | \phi \rangle_M = Z^{-1} \prod_{j=0}^{\infty} \prod_{\lambda=1}^2 e^{-2n\pi j} \langle n, j, \lambda | A_R | n, j, \lambda \rangle_R \quad (3.48)$$

which is precisely the expectation value which it would have in a thermally mixed state of temperature, $T_R = (2\pi)^{-1}$. We see that $Z^{-1} e^{-2n\pi R}$ plays the role of the density matrix. The representation of the density matrix as a vector in some larger Hilbert space is precisely the way in which quantum statistical mechanics has been formulated by Takahashi and Umezawa³²⁾. This connection with thermofield dynamics was first pointed out by Israel²⁰⁾. Equivalently, (3.46b) is a thermally mixed state as far as expectation values of observables restricted to L are concerned. The density matrix in this case is $Z^{-1} e^{2nK_L}$.

Operators like K_L and K_R , (3.39), are conventionally "normal ordered" by the subtraction of a (divergent) c number, so that

$$\langle \phi | :K_R: | \phi \rangle_M = 0 \quad (3.49)$$

when the normal ordering is with respect to the Minkowski vacuum. However, if such a definition of operators is adopted, we find

$$:K_R: |\phi\rangle_R = -2 \int_0^\infty dj j (e^{2\pi j} - 1)^{-1} \delta(0) |\phi\rangle_R \quad (3.50a)$$

and

$$:K_L: |\phi\rangle_L = 2 \int_0^\infty dj j (e^{2\pi j} - 1)^{-1} \delta(0) |\phi\rangle_L \quad (3.50b)$$

which is, of course, consistent with the thermal properties of the Minkowski vacuum in the Rindler representation, but demonstrates the rather singular nature of $|\phi\rangle_L$ and $|\phi\rangle_R$, when operators are defined in this way.

4. - MASSLESS DIRAC FIELD ON R

In this section we will be concerned with the quantum theory of a massless Dirac field, ψ , on two-dimensional Minkowski space. This field satisfies the Dirac equation,

$$\not{\partial} \psi = 0 \quad (4.1)$$

and is subject to the quantization condition,

$$\{\psi_a^\dagger(t, x), \psi_b(t, y)\} = \delta_{ab} \delta(x - y) \quad (4.2)$$

where a, b are spinor indices. (Spinor indices will be suppressed wherever they constitute an unnecessary complication.) This problem is of interest since we will see how the arguments of Section 3 become modified by the change to Fermi statistics, and by the use of a charged field.

We choose a representation of the Dirac γ matrices in which,

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (4.3a)$$

$$\gamma_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (4.3b)$$

so that

$$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu} \quad (4.3c)$$

We further define

$$\gamma_5 \equiv \gamma_0 \gamma_1 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (4.3d)$$

and

$$\sigma_{\mu\nu} \equiv i/2 [\gamma_\mu, \gamma_\nu]$$

so that

$$\sigma_{01} = i\gamma_5$$

and

$$\{\gamma_\mu, \gamma_5\} = 0$$

We now repeat the construction of Section 3 for the spinor field, ψ . So, we must expand ψ over some complete, orthonormal set of c number solutions of (4.1). In Minkowski space, completeness and orthogonality are defined with respect to the conserved, Hermitian, positive definite scalar product,

$$\langle \psi | \phi \rangle_M = \int_{-\infty}^{\infty} dx \psi_a^\dagger(t, x) \phi_a(t, x)$$

where $\dagger$ denotes complex conjugation and transposition.

We will first construct the usual (Minkowski) representation in which we choose the set of energy eigenspinors as our complete set. As before, these are labelled by the x momentum eigenvalue, k , and the sign of the energy ($\pm$). However, we now have an additional label, namely the γ_5 eigenvalue, which may be diagonalized simultaneously by virtue of (4.1) and (4.3g). This matrix has two eigenspinors, χ_λ , $\lambda = 1, 2$, with,

$$\gamma_5 \chi_1 = -\chi_1$$

$$\gamma_5 \chi_2 = \chi_2$$

which in our representation have the form,

$$\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Thus, for positive energy, $|k|$, we have the spinors,

$$\psi_{k, \lambda, +}(t, x) = \chi_\lambda \phi_{k, \lambda, +}(t, x) \quad (\text{no sum on } \lambda) \quad (4.7)$$

where the functions, $\phi_{k, \lambda, +}$, have the form

$$\phi_{k, +}(t, x) = (2\pi)^{-1/2} e^{-ik|t| + ikx}, \quad k > 0 \quad (4.8a)$$

and

$$\phi_{k, +}(t, x) = (2\pi)^{-1/2} e^{-ik|t| + ikx}, \quad k < 0 \quad (4.8b)$$

Similarly, for negative energy, $-|k|$, we have

$$\psi_{k, \lambda, -}(t, x) = \chi_\lambda \phi_{k, \lambda, -}(t, x) \quad (4.9)$$

with

$$\phi_{k, -, -}(t, x) = (2\pi)^{-1/2} e^{ik|t| + ikx}, \quad k < 0 \quad (4.10a)$$

$$\phi_{k, -, -}(t, x) = (2\pi)^{-1/2} e^{ik|t| + ikx}, \quad k > 0 \quad (4.10b)$$

and we note that,

$$\phi_{k, \lambda, -}(t, x) = \phi_{-k, \lambda, +}^*(t, x), \quad \lambda = 1, 2 \quad (4.11)$$

The spinors (4.7) and (4.9) obey the orthogonality relations,

$$\langle \psi_{k, \lambda, \pm} | \psi_{k', \lambda', \pm} \rangle = \delta_{\lambda\lambda'} \delta(k-k') \quad (4.12a)$$

$$\langle \psi_{k,\lambda,+} | \psi_{k',\lambda',-} \rangle = 0 \quad (4.12b)$$

and the completeness relation,

$$\sum_{\lambda=1}^2 \int_{-\infty}^{\infty} dk \left[\psi_{k,\lambda,+}^\dagger(t,x) \psi_{k,\lambda,+}(t,y) + \psi_{k,\lambda,+}^{*\dagger}(t,x) \psi_{k,\lambda,+}^*(t,y) \right] = \delta(x-y) \quad (4.13)$$

where "*" denotes complex conjugation.

If we now write the Dirac field operator as,

$$\psi(t,x) = \sum_{\lambda=1}^2 \int_{-\infty}^{\infty} dk \left[b(k,\lambda) \psi_{k,\lambda,+}(t,x) + d^\dagger(k,\lambda) \psi_{k,\lambda,+}^*(t,x) \right] \quad (4.14)$$

the quantization condition, (4.2), is satisfied if the normal mode amplitudes obey the anticommutation relations,

$$\{ b^\dagger(k,\lambda), b(k',\lambda') \} = \delta_{\lambda\lambda'} \delta(k-k') \quad (4.15a)$$

$$\{ d^\dagger(k,\lambda), d(k',\lambda') \} = \delta_{\lambda\lambda'} \delta(k-k') \quad (4.15b)$$

with all of their other anticommutators vanishing.

The algebra (4.15) is realized with the introduction of the Minkowski vacuum state, $|0\rangle_M$, via,

$$b(k,\lambda) |0\rangle_M = d(k,\lambda) |0\rangle_M = 0, \quad \forall k,\lambda \quad (4.16)$$

In order to find a representation in which the "energy" operator for a uniformly accelerated observer is diagonalized we now seek eigenspinors of the Lorentz boost operator, K . The sign of the energy is conserved under a boost, as is the γ_5 eigenvalue, λ . We, therefore, seek spinors $\xi_{j,\lambda,\pm}(t,x)$ with

$$K \xi_{j,\lambda,\pm}(t,x) = j \xi_{j,\lambda,\pm}(t,x) \quad (4.17a)$$

or, under a finite boost,

$$e^{-i\frac{\alpha}{2} \sigma_0} \xi_{j,\lambda,\pm}(t',x') = e^{-ij\alpha} \xi_{j,\lambda,\pm}(t,x) \quad (4.17b)$$

where

$$t' \pm x' = e^{\pm\alpha} (t \pm x) \quad (4.17c)$$

and

$$K = i(x \frac{\partial}{\partial t} + t \frac{\partial}{\partial x}) + \frac{1}{2} \sigma_0 \quad (4.17d)$$

Solutions of (4.17b) can be constructed as superpositions of the $\psi_{k,\lambda,\pm}$. Specifically,

$$\xi_{j,1,+}(t,x) = (2\pi)^{-1/2} \int_0^\infty dk k^{-ij-1/2} \psi_{k,1,+}(t,x) \quad (4.18a)$$

$$\xi_{j,2,+}(t,x) = (2\pi)^{-1/2} \int_{-\infty}^0 dk (-k)^{ij-1/2} \psi_{k,2,+}(t,x) \quad (4.18b)$$

while for negative energies,

$$\xi_{j,1,-}(t,x) = (2\pi)^{-1/2} \int_{-\infty}^0 dk (-k)^{-ij-1/2} \psi_{k,1,-}(t,x) \quad (4.18c)$$

$$\xi_{j,2,-}(t,x) = (2\pi)^{-1/2} \int_0^\infty dk k^{ij-1/2} \psi_{k,2,-}(t,x) \quad (4.18d)$$

and we have the relations,

$$\xi_{j,\lambda,-}(t,x) = \xi_{-j,\lambda,+}^*(t,x) \quad (4.19)$$

The $\xi_{j,\lambda}$ spinors are given in terms of elementary functions in Appendix B. We note that positive boost spinors can be constructed from both positive and negative energy spinors. This again leads to an infinite variety of possible representations for which the boost is diagonalized.

The spinors $\xi_{j,\lambda,\pm}$ satisfy the orthogonality relations

$$\langle \xi_{j,\lambda,\pm} | \xi_{j',\lambda',\pm} \rangle = \delta_{\lambda\lambda'} \delta(j-j') \quad (4.20a)$$

$$\langle \xi_{j,\lambda,+} | \xi_{j',\lambda',-} \rangle = 0 \quad (4.20b)$$

and the completeness relation,

$$\sum_{\lambda=1}^{\infty} \int_{-\infty}^{\infty} dj \left[\xi_{j,\lambda,+}^{\dagger}(t,x) \xi_{j,\lambda,+}(t,y) + \xi_{j,\lambda,+}^{\dagger}(t,x) \xi_{j,\lambda,+}^*(t,y) \right] = \delta(x-y) \quad (4.21)$$

so that

$$\psi(t,x) = \sum_{\lambda=1}^{\infty} \int_{-\infty}^{\infty} dj \left[a(j,\lambda) \xi_{j,\lambda,+}(t,x) + c^{\dagger}(j,\lambda) \xi_{j,\lambda,+}^*(t,x) \right] \quad (4.22)$$

satisfies the quantization condition (4.2), if

$$\{a(j,\lambda), a^{\dagger}(j',\lambda')\} = \{c(j,\lambda), c^{\dagger}(j',\lambda')\} = \delta_{\lambda\lambda'} \delta(j-j') \quad (4.23)$$

with all other anticommutators of the normal mode amplitudes vanishing. Hence

$$a(j,\lambda) |0\rangle_{\mathcal{M}} = c(j,\lambda) |0\rangle_{\mathcal{M}} = 0 \quad (4.24)$$

and so, (4.22) and (4.23) may be termed "Boulware's representation" for the Dirac field, ψ . As such, it suffers from the same drawbacks, from the point of view of a uniformly accelerating observer, as it did for the scalar field of Section 3. We therefore now move on to the diagonalization of K_L and K_R for our Dirac field, by again making use of PCT symmetry.

We have,

$$\xi_{j,\lambda,+}(-t,-x) = \xi_{j,\lambda,+}^*(t,x) \quad (4.25)$$

and applying a complex boost of $\alpha = -i\pi$ in (4.17b), we find,

$$\xi_{j,1,+}(-t,-x) = -ie^{-i\pi} \xi_{j,1,+}(t,x) \quad (4.26a)$$

$$\xi_{j,2,+}(-t,-x) = ie^{-i\pi} \xi_{j,2,+}(t,x) \quad (4.26b)$$

Equations (4.26) are the analogues of (3.29b) for the scalar case. The additional factor of "i" in (4.26) relative to (3.29b) arises from the fact that a spinor field itself changes under a Lorentz boost, whereas a scalar field does not.

When combined with the identity (4.25), Eqs. (4.26) yield,

$$\xi_{j,1,+}^*(t,x) = -ie^{-i\pi} \xi_{j,1,+}(t,x) \quad (4.27a)$$

$$\xi_{j,2,+}^*(t,x) = ie^{-i\pi} \xi_{j,2,+}(t,x) \quad (4.27b)$$

which characterize the non-independence of the $\xi_{j,\lambda,\pm}$ on R , by virtue of (4.19). Likewise, on L we have

$$\xi_{j,1,+}(t,x) = -ie^{-i\pi} \xi_{j,1,+}^*(t,x) \quad (4.28a)$$

$$\xi_{j,2,+}(t,x) = ie^{-i\pi} \xi_{j,2,+}^*(t,x) \quad (4.28b)$$

The identities (4.27) and (4.28) are analogous to (3.31) and (3.32) in the scalar case, and may be explicitly verified with the expressions given in Appendix B.

We next construct eigenspinors of K_R :

$$\xi_{j,1,R}(t,x) = [2 \cosh(\pi j)]^{-1/2} \left\{ e^{\pi j/2} \xi_{j,1,+}(t,x) + ie^{-\pi j/2} \xi_{j,1,+}^*(t,x) \right\} \quad (4.29a)$$

$$\xi_{j,2,R}(t,x) = [2 \cosh(\pi j)]^{-1/2} \left\{ e^{\pi j/2} \xi_{j,2,+}(t,x) - ie^{-\pi j/2} \xi_{j,2,+}^*(t,x) \right\} \quad (4.29b)$$

which satisfy

$$\xi_{j,\lambda,R}(t,x) = 0, \quad (t,x) \in L \quad (4.29c)$$

The functional form of these spinors is given in Appendix B. Likewise, eigen-spinors of K_L are:

$$\xi_{j,1/2}(t,x) = [2 \cosh(\pi j)]^{1/2} \left\{ e^{-\pi j/2} \xi_{j,1/2}^*(t,x) - i e^{\pi j/2} \xi_{j,1/2}^*(t,x) \right\} \quad (4.30a)$$

$$\xi_{j,2/2}(t,x) = [2 \cosh(\pi j)]^{1/2} \left\{ e^{-\pi j/2} \xi_{j,2/2}^*(t,x) + i e^{\pi j/2} \xi_{j,2/2}^*(t,x) \right\} \quad (4.30b)$$

which satisfy,

$$\xi_{j,\lambda/2}(t,x) = 0, \quad (t,x) \in R \quad (4.30c)$$

These spinors satisfy the orthogonality properties,

$$\langle \xi_{j,\lambda/2} | \xi_{j',\lambda',R} \rangle_M = 0 \quad (4.31a)$$

$$\langle \xi_{j,\lambda/2} | \xi_{j,\lambda',R} \rangle_M = \delta_{\lambda\lambda'} \delta(j-j') \quad (4.31b)$$

$$\langle \xi_{j,\lambda/2} | \xi_{j',\lambda',L} \rangle_M = \delta_{\lambda\lambda'} \delta(j-j') \quad (4.31c)$$

and the completeness relation,

$$\sum_{\lambda=1}^2 \int_0^\infty dj \left[\xi_{j,\lambda,R}(t,y) \xi_{j,\lambda,R}^*(t,y) + \xi_{j,\lambda,R}^\dagger(t,x) \xi_{j,\lambda,R}^*(t,y) \right] + \xi_{j,\lambda,L}^\dagger(t,x) \xi_{j,\lambda,L}(t,y) + \xi_{j,\lambda,L}^*(t,y) \xi_{j,\lambda,L}^\dagger(t,x) = \delta(x-y) \quad (4.32)$$

The Dirac field operator, ψ , may now be written

$$\psi(t,x) = \psi_L(t,x) + \psi_R(t,x) \quad (4.33a)$$

where

$$\psi_R(t,x) = \sum_{\lambda=1}^2 \int_0^\infty dj [a(j,\lambda,R) \xi_{j,\lambda,R}(t,x) + c^\dagger(j,\lambda,R) \xi_{j,\lambda,R}^*(t,x)] \quad (4.33b)$$

and

$$\psi_L(t,x) = \sum_{\lambda=1}^2 \int_0^\infty dj [a(j,\lambda,L) \xi_{j,\lambda,L}^*(t,x) + c^\dagger(j,\lambda,L) \xi_{j,\lambda,L}(t,x)] \quad (4.33c)$$

with

$$\{a(j,\lambda,R), a^\dagger(j',\lambda',R)\} = \delta_{\lambda\lambda'} \delta(j-j') \quad (4.34a)$$

$$\{c(j,\lambda,R), c^\dagger(j',\lambda',R)\} = \delta_{\lambda\lambda'} \delta(j-j') \quad (4.34b)$$

$$\{a(j,\lambda,L), a^\dagger(j',\lambda',L)\} = \delta_{\lambda\lambda'} \delta(j-j') \quad (4.34c)$$

$$\{c(j,\lambda,L), c^\dagger(j',\lambda',L)\} = \delta_{\lambda\lambda'} \delta(j-j') \quad (4.34d)$$

and all other anticommutators of the normal mode amplitudes vanishing. The fields ψ_L and ψ_R are related by PCT. Just as in the scalar case, the algebra (4.34) may be realized with the introduction of two Rindler vacua, $|0\rangle_L$ and $|0\rangle_R$, where

$$a(j,\lambda,R)|0\rangle_R = c(j,\lambda,R)|0\rangle_R = 0, \quad \forall j > 0, \lambda = 1,2 \quad (4.35a)$$

$$a(j,\lambda,L)|0\rangle_L = c(j,\lambda,L)|0\rangle_L = 0, \quad \forall j > 0, \lambda = 1,2 \quad (4.35b)$$

The relation between the Minkowski, (4.22) and (4.23), and Rindler (4.33) and (4.34), representations is given by

$$c^\dagger(j,\lambda,R) = \langle \xi_{j,\lambda,R} | \psi \rangle, \quad j > 0 \quad (4.36a)$$

so that

$$c^\dagger(j,1,R) = (1 + e^{-2\pi j})^{1/2} c^\dagger(j,1) + i(e^{2\pi j} + 1)^{-1/2} a(-j,1), \quad j > 0 \quad (4.36b)$$

$$c^\dagger(j,2,R) = (1 + e^{-2\pi j})^{1/2} c^\dagger(j,2) - i(e^{2\pi j} + 1)^{-1/2} a(-j,2), \quad j > 0 \quad (4.36c)$$

Likewise,

$$a(j,1,L) = (e^{2\pi j}_{+1})^{\frac{1}{2}} c^\dagger(j,1) - i(1+e^{-2\pi j})^{\frac{1}{2}} a(-j,1) \quad , j > 0 \quad (4.36d)$$

$$a(j,2,L) = (e^{2\pi j}_{+1})^{\frac{1}{2}} c^\dagger(j,2) + i(1+e^{-2\pi j})^{\frac{1}{2}} a(-j,2) \quad , j > 0 \quad (4.36e)$$

The number of Rindler antiparticles in the Minkowski vacuum is given by

$$\langle 0 | c^\dagger(j,\lambda,R) c(j,\lambda,R) | 0 \rangle_M = \delta_{\lambda\lambda'} \delta(j-j') (e^{2\pi j}_{+1})^{-1} \quad (4.37)$$

We see that the Minkowski vacuum contains a Fermi-Dirac distribution of Rindler antiparticles at the Fulling-Rindler temperature of $T_R = (2\pi)^{-1}$, as found in other investigations⁷⁾⁻⁹⁾.

It is of interest to note that we found a Fermi-Dirac distribution for the Dirac problem, rather than a Bose-Einstein one, as a result of the positive definite nature of the Dirac scalar product, (4.4). This led to the appearance of the cosh functions in (4.30) and hence a Fermi-Dirac distribution. In contrast, the scalar product for the scalar field, (3.4), is not positive-definite, and the boost eigenfunctions (3.33) contained the sinh functions instead.

The inverse form of the identities (4.36) is easily found. We have,

$$c^\dagger(j,\lambda) = \langle \sum_{j,\lambda,+}^* | \psi \rangle_M \quad (4.38a)$$

$$= \begin{cases} (1+e^{-2\pi j})^{\frac{1}{2}} c^\dagger(j,\lambda,R) + (e^{2\pi j}_{+1})^{\frac{1}{2}} a(j,\lambda,L) & , j > 0 \\ \pm i(e^{-2\pi j}_{+1})^{\frac{1}{2}} a(-j,\lambda,R) \mp i(1+e^{2\pi j})^{\frac{1}{2}} c(-j,\lambda,L) & , j < 0, \lambda=1,2 \end{cases} \quad (4.38b)$$

$$a(j,\lambda) = \langle \sum_{j,\lambda,+} | \psi \rangle_M \quad (4.38c)$$

$$= \begin{cases} (1+e^{-2\pi j})^{\frac{1}{2}} a(j,\lambda,R) + (e^{2\pi j}_{+1})^{\frac{1}{2}} c^\dagger(j,\lambda,L) & , j > 0 \\ \mp i(1+e^{2\pi j})^{\frac{1}{2}} c^\dagger(-j,\lambda,R) \pm i(e^{2\pi j}_{+1})^{\frac{1}{2}} a(-j,\lambda,L) & , j < 0, \lambda=1,2 \end{cases} \quad (4.38d)$$

The field boost operators, K_L and K_R are given by

$$K_R = \int_0^\infty dx \left[\frac{i}{2} x \psi^\dagger \vec{\sigma}_0 \psi + \psi^\dagger \frac{1}{2} \vec{\sigma}_0 \psi \right] \Big|_{t=0} \quad (4.39a)$$

$$K_L = \int_{-\infty}^0 dx \left[\frac{i}{2} x \psi^\dagger \vec{\sigma}_0 \psi + \psi^\dagger \frac{1}{2} \vec{\sigma}_0 \psi \right] \Big|_{t=0} \quad (4.39b)$$

so that

$$K_R = \frac{1}{2} \sum_{\lambda=1}^2 \int_0^\infty dx j \left[a^\dagger(j,\lambda,R) a(j,\lambda,R) - c(j,\lambda,R) c^\dagger(j,\lambda,R) \right] \quad (4.40a)$$

$$K_L = -\frac{1}{2} \sum_{\lambda=1}^2 \int_{-\infty}^0 dx j \left[a^\dagger(j,\lambda,L) a(j,\lambda,L) - c(j,\lambda,L) c^\dagger(j,\lambda,L) \right] \quad (4.40b)$$

and hence, $|0\rangle_R$ is the ground state of K_R , while $|0\rangle_L$ is the ground state of $(-K_L)$, which generates the hyperbolic motion on L . The representation of (4.33) and (4.34) is therefore of the desired form.

5. - DISCUSSION, SUMMARY AND CONCLUSIONS

In this paper we have studied the quantum theory of free massless Hermitian scalar and Dirac fields in one space dimension in a representation where we diagonalize the "energy" operator for a uniformly accelerating observer. After discussing the sector of space-time accessible to such an observer we found that his "energy" operator corresponded to the portion of the Lorentz boost operator which originated on the sector in question. The resulting representation for each of the quantum fields was found to be inequivalent with the usual one in Minkowski space, and the Minkowski vacuum was shown to be a finite temperature state of the Rindler representation.

These thermal properties have their origin in the fact that a pure state on a whole space is a mixed state, in general, on a subspace. The fact that the mixture is thermal is a consequence of the PCT symmetry of field theory, and of the simple behaviour of boost eigenfunctions under this symmetry, which may be represented by a complex Lorentz boost of $i\pi$. So that, although we are dealing with particular examples of free-field theories we see that PCT will play exactly the same role in diagonalizing the boost operator in other free-field theories. Our investigation therefore illustrates the general principle underlying the remarkable uniformity of the relation between Rindler and Minkowski representations of free-field theories. This has been particularly straightforward because we have insisted on quantizing the field in the whole of Minkowski space, and not merely on the Rindler segment, as has been done in the past.

Sciama et al.³²⁾ have pointed out that the Rindler vacuum, $|0\rangle_R$, is subject to less stringent symmetry requirements than the Minkowski vacuum, and have suggested that this may underly the thermal effects. Indeed, $|0\rangle_R$ is not translationally invariant, but it is perhaps more important to note that it is not PCT invariant either.

We have studied massless fields, but there would have been no additional complication with including a mass, except that the direction of the one-momentum would no longer have been a boost invariant. A similar complication would also have arisen if we had had more than one space-dimensions. In such a case, with a massive field, the spectrum of the "energy" of the accelerated observer would not exhibit an "energy" gap¹⁴⁾. This is perhaps not surprising since such an observer experiences a gravitational potential. Furthermore, the operator P^2 is a Casimir invariant of the Poincaré group, so that the accelerated observer would be able to detect the mass of his quanta.

In a recent paper 't Hooft³³⁾ has questioned the magnitude of the Hawking temperature. We see no such ambiguity in our investigation of the Rindler temperature, and we note that neither does the Unruh gedanken experiment²³⁾.

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APPENDIX A

The boost eigenfunctions (3.13) and (3.14) have the following forms on R ($X > 0$):

$$\phi_{j,1}(X \sinh T, X \cosh T) = e^{-ijT} X^{ij} \Gamma(-ij) e^{\pi j/2} (8\pi^2)^{-1/2} \quad (A.1a)$$

$$\phi_{j,2}(X \sinh T, X \cosh T) = -e^{-ijT} X^{-ij} \Gamma(ij) e^{\pi j/2} (8\pi^2)^{-1/2} \quad (A.1b)$$

$$\phi_{j,3}(X \sinh T, X \cosh T) = e^{-ijT} X^{ij} \Gamma(-ij) e^{-\pi j/2} (8\pi^2)^{-1/2} \quad (A.1c)$$

$$\phi_{j,4}(X \sinh T, X \cosh T) = -e^{-ijT} X^{-ij} \Gamma(ij) e^{-\pi j/2} (8\pi^2)^{-1/2} \quad (A.1d)$$

while on L , ($X < 0$) we have

$$\phi_{j,1}(-X \sinh T, X \cosh T) = e^{ijT} |X|^{ij} \Gamma(-ij) e^{-\pi j/2} (8\pi^2)^{-1/2} \quad (A.2a)$$

$$\phi_{j,2}(-X \sinh T, X \cosh T) = -e^{ijT} |X|^{ij} \Gamma(ij) e^{-\pi j/2} (8\pi^2)^{-1/2} \quad (A.2b)$$

$$\phi_{j,3}(-X \sinh T, X \cosh T) = e^{ijT} |X|^{ij} \Gamma(-ij) e^{\pi j/2} (8\pi^2)^{-1/2} \quad (A.2c)$$

$$\phi_{j,4}(-X \sinh T, X \cosh T) = -e^{ijT} |X|^{ij} \Gamma(ij) e^{\pi j/2} (8\pi^2)^{-1/2} \quad (A.2d)$$

The cut in the logarithmic functions has been chosen to lie along the negative real axis.

We list below the properties of the functions (A.1) and (A.2) under the conserved scalar products on R and L , given by (3.23a) and (3.23b):

$$\langle \phi_{j,\lambda} | \phi_{j',x} \rangle_R = \delta(j-j') \frac{\text{sgn}(j)}{|2\sinh(\pi j)|} M_{xx}(j) \quad (\text{A.3a})$$

where the matrix M is given by,

$$M_{xx}(j) = \begin{pmatrix} e^{\pi j} & 0 & 1 & 0 \\ 0 & e^{\pi j} & 0 & 1 \\ 1 & 0 & e^{-\pi j} & 0 \\ 0 & 1 & 0 & e^{-\pi j} \end{pmatrix} \quad (\text{A.3b})$$

and

$$\langle \phi_{j,\lambda} | \phi_{j',x} \rangle_L = \delta(j-j') \frac{\text{sgn}(-j)}{|2\sinh(\pi j)|} N_{xx}(j) \quad (\text{A.4a})$$

with

$$N_{xx}(j) = \begin{pmatrix} e^{-\pi j} & 0 & 1 & 0 \\ 0 & e^{-\pi j} & 0 & 1 \\ 1 & 0 & e^{\pi j} & 0 \\ 0 & 1 & 0 & e^{\pi j} \end{pmatrix} \quad (\text{A.4b})$$

Here we have used the identities

$$|\Gamma(ij)\Gamma(-ij)|^2 = \frac{\pi}{j \sinh(\pi j)} \quad (\text{A.5})$$

In terms of Minkowski co-ordinates, (t,x) , these boost eigenfunctions have the following forms:

$$\phi_{j,1}(t,x) = (8\pi^2)^{-1/2} \Gamma(-ij) |t-x|^{ij} \begin{cases} e^{-\pi j/2} & , (t-x) > 0 \\ e^{\pi j/2} & , (t-x) < 0 \end{cases} \quad (\text{A.6a})$$

$$\phi_{j,2}(t,x) = -(8\pi^2)^{-1/2} \Gamma(ij) |t+x|^{-ij} \begin{cases} e^{\pi j/2} & , (t+x) > 0 \\ e^{-\pi j/2} & , (t+x) < 0 \end{cases} \quad (\text{A.6b})$$

$$\phi_{j,3}(t,x) = (8\pi^2)^{-1/2} \Gamma(-ij) |t-x|^{ij} \begin{cases} e^{\pi j/2} & , (t-x) > 0 \\ e^{-\pi j/2} & , (t-x) < 0 \end{cases} \quad (\text{A.6c})$$

$$\phi_{j,4}(t,x) = -(8\pi^2)^{-1/2} \Gamma(ij) |t+x|^{-ij} \begin{cases} e^{-\pi j/2} & , (t+x) > 0 \\ e^{\pi j/2} & , (t+x) < 0 \end{cases} \quad (\text{A.6d})$$

We see that these functions are boost eigenfunctions of eigenvalue j throughout Minkowski space. The functions $\phi_{j,\lambda,R}$ of (3.33a) have the following forms on R ($X > 0$):

$$\phi_{j,1,R}(X \sinh T, X \cosh T) = i e^{-ijT} X^{ij} \Gamma(-ij) \text{sgn}(j) |2 \sinh(\pi j)|^{1/2} (8\pi^2)^{-1/2} \quad (\text{A.7a})$$

$$\phi_{j,2,R}(X \sinh T, X \cosh T) = -i e^{-ijT} X^{-ij} \Gamma(ij) \text{sgn}(j) |2 \sinh(\pi j)|^{1/2} (8\pi^2)^{-1/2} \quad (\text{A.7b})$$

and in terms of Minkowski co-ordinates,

$$\phi_{j,1R}(t,x) = \begin{cases} i(8\pi^2)^{-1/2} \Gamma(-ij) |t-x|^{ij} \operatorname{sgn}(j) |2\sinh(\pi j)|^{1/2}, & (t-x) < 0 \\ 0, & (t-x) > 0 \end{cases} \quad (A.7c)$$

$$\phi_{j,2R}(t,x) = \begin{cases} -i(8\pi^2)^{-1/2} \Gamma(ij) |t+x|^{-ij} \operatorname{sgn}(j) |2\sinh(\pi j)|^{1/2}, & (t+x) > 0 \\ 0, & (t+x) < 0 \end{cases} \quad (A.7d)$$

in accordance with (3.34b).

Likewise, the functions $\phi_{j,\lambda,L}$ of (3.33b) have the following forms on L , ($X < 0$):

$$\phi_{j,1L}(-X\sinh T, X\cosh T) = ie^{ijT} |X|^{ij} \Gamma(-ij) \operatorname{sgn}(j) |2\sinh(\pi j)|^{1/2} (8\pi^2)^{-1/2} \quad (A.8a)$$

$$\phi_{j,2L}(-X\sinh T, X\cosh T) = -ie^{ijT} |X|^{-ij} \Gamma(ij) \operatorname{sgn}(j) |2\sinh(\pi j)|^{1/2} (8\pi^2)^{-1/2} \quad (A.8b)$$

and in terms of Minkowski co-ordinates,

$$\phi_{j,1L}(t,x) = \begin{cases} i(8\pi^2)^{-1/2} \Gamma(-ij) |t-x|^{ij} \operatorname{sgn}(j) |2\sinh(\pi j)|^{1/2}, & (t-x) > 0 \\ 0, & (t-x) < 0 \end{cases} \quad (A.8c)$$

$$\phi_{j,2L}(t,x) = \begin{cases} -i(8\pi^2)^{-1/2} \Gamma(ij) |t+x|^{-ij} \operatorname{sgn}(j) |2\sinh(\pi j)|^{1/2}, & (t+x) < 0 \\ 0, & (t+x) > 0 \end{cases} \quad (A.8d)$$

We note that the functions (3.33) vanish on a half of Minkowski space. This is a peculiarity of solutions of the wave equation in two space-time dimensions, which are functions of $(x \pm t)$. In the massive case, the analogous boost eigenfunctions would only vanish on one quarter of Minkowski space (L or R).

The identity (3.31) may also be found by studying the behaviour of the field operator, ϕ , of (3.18) under the action of the anti-unitary PCT operator θ . We have,

$$\theta \phi(t,x) \theta^{-1} = \phi^\dagger(-t,-x) \quad (A.9a)$$

so that,

$$\theta c(j,\lambda) \theta^{-1} = c(-j,\lambda), \quad \lambda = 1, 2 \quad (A.9b)$$

On the other hand,

$$\phi(-t,-x) |\phi\rangle_M = e^{-\pi K} \phi(t,x) |\phi\rangle_M, \quad (t,x) \in R \quad (A.10a)$$

so that,

$$\phi(t,x) |\phi\rangle_M = \theta e^{-\pi K} \phi^\dagger(t,x) |\phi\rangle_M, \quad (t,x) \in R \quad (A.10b)$$

This last equation remains true when $\phi(t,x)$ is replaced by any operator with arguments totally within R . Sewell's general proof of the Fulling-Unruh effect follows from this identity. However, for us (A.10b) leads directly to the identity (3.31), via,

$$\langle \phi | c(j,\lambda) [\phi(t,x) - \theta e^{-\pi K} \phi^\dagger(t,x)] |\phi\rangle_M = 0, \quad (t,x) \in R \quad (A.11)$$

APPENDIX B

We list below some properties of the spinors defined in Section 4. On R , ($X > 0$), we have

$$\sum_{j,1,+} (X \sinh T, X \cosh T) = (2\pi)^{-1} \chi_1 e^{-ijT+T/2} X^{ij-1/2} \Gamma(-ij+1/2) e^{i\pi/4} \quad (B.1a)$$

$$\sum_{j,2,+} (X \sinh T, X \cosh T) = (2\pi)^{-1} \chi_2 e^{-ijT-T/2} X^{-ij-1/2} \Gamma(ij+1/2) e^{-i\pi/4} \quad (B.1b)$$

$$\sum_{j,1,-} (X \sinh T, X \cosh T) = (2\pi)^{-1} \chi_1 e^{-ijT+T/2} X^{ij-1/2} \Gamma(-ij+1/2) e^{-i\pi/4} \quad (B.1c)$$

$$\sum_{j,2,-} (X \sinh T, X \cosh T) = (2\pi)^{-1} \chi_2 e^{-ijT-T/2} X^{-ij-1/2} \Gamma(ij+1/2) e^{i\pi/4} \quad (B.1d)$$

and we see directly that

$$\sum_{j,1,-} (t,x) = -i e^{-\pi j} \sum_{j,1,+} (t,x) \quad (B.2a)$$

$$\sum_{j,2,-} (t,x) = i e^{\pi j} \sum_{j,2,+} (t,x) \quad (B.2b)$$

in accordance with (4.19) and (4.27). Hence, we have on R ,

$$\sum_{j,1,R} (X \sinh T, X \cosh T) = [2 \cosh(\pi j)]^{1/2} (2\pi)^{-1} \chi_1 e^{-ijT+T/2} X^{ij-1/2} \Gamma(-ij+1/2) e^{i\pi/4} \quad (B.3a)$$

and

$$\sum_{j,2,R} (X \sinh T, X \cosh T) = [2 \cosh(\pi j)]^{1/2} (2\pi)^{-1} \chi_2 e^{-ijT-T/2} X^{-ij-1/2} \Gamma(ij+1/2) e^{-i\pi/4} \quad (B.3b)$$

A conserved scalar product on R for c number solutions of the Dirac equation is

$$\langle \psi | \phi \rangle_R = \int_0^\infty \frac{dx}{x} \psi^\dagger x_0 e^{i\phi} \delta_\mu \phi \quad (B.4)$$

where

$$e^\mu(0) = X(\cosh T, \sinh T) \quad (B.5)$$

is the normal to a surface of constant T . We have,

$$\langle \sum_{j,\lambda,+} | \sum_{j',\lambda',+} \rangle_R = \delta_{\lambda\lambda'} \delta(j-j') (1 + e^{-2\pi j})^{-1} \quad (B.6a)$$

$$\langle \sum_{j,\lambda,-} | \sum_{j',\lambda',-} \rangle_R = \delta_{\lambda\lambda'} \delta(j-j') (e^{2\pi j} + 1)^{-1} \quad (B.6b)$$

$$| \langle \sum_{j,\lambda,+} | \sum_{j',\lambda',-} \rangle_R | = \delta_{\lambda\lambda'} \delta(j-j') (e^{\pi j} + e^{-\pi j})^{-1} \quad (B.6c)$$

and

$$\langle \sum_{j,\lambda,R} | \sum_{j',\lambda',R} \rangle_R = \delta_{\lambda\lambda'} \delta(j-j') \quad (B.7)$$

Similarly, on L , $X < 0$, we have

$$\sum_{j,1,+} (-X \sinh T, X \cosh T) = (2\pi)^{-1} \chi_1 e^{ijT-T/2} |X|^{ij-1/2} \Gamma(-ij+1/2) e^{-i\pi/4} \quad (B.8a)$$

$$\sum_{j,2,+} (-X \sinh T, X \cosh T) = (2\pi)^{-1} \chi_2 e^{ijT+T/2} |X|^{-ij-1/2} \Gamma(ij+1/2) e^{i\pi/4} \quad (B.8b)$$

$$\sum_{j,1,-} (-X \sinh T, X \cosh T) = (2\pi)^{-1} \chi_1 e^{ijT-T/2} |X|^{ij-1/2} \Gamma(-ij+1/2) e^{i\pi/4} \quad (B.8c)$$

$$\sum_{j,2,-} (-X \sinh T, X \cosh T) = (2\pi)^{-1} \chi_2 e^{ijT+T/2} |X|^{-ij-1/2} \Gamma(ij+1/2) e^{-i\pi/4} \quad (B.8d)$$

In terms of functions of Minkowski co-ordinates these boost eigenspinors are:

$$\sum_{j_1, j_2} \chi_1 \chi_2 |t-x|^{j_1-j_2} \Gamma(-ij+\frac{1}{2}) \begin{cases} e^{\pi j/2} e^{i\pi/4} & , (t-x) < 0 \\ e^{-\pi j/2} e^{-i\pi/4} & , (t-x) > 0 \end{cases} \quad (B.9a)$$

$$\sum_{j_1, j_2} \chi_1 \chi_2 |t+x|^{j_1-j_2} \Gamma(ij+\frac{1}{2}) \begin{cases} e^{\pi j/2} e^{-i\pi/4} & , (t+x) > 0 \\ e^{-\pi j/2} e^{i\pi/4} & , (t+x) < 0 \end{cases} \quad (B.9b)$$

$$\sum_{j_1, j_2} \chi_1 \chi_2 |t-x|^{j_1-j_2} \Gamma(-ij+\frac{1}{2}) \begin{cases} e^{-\pi j/2} e^{-i\pi/4} & , (t-x) < 0 \\ e^{\pi j/2} e^{i\pi/4} & , (t-x) > 0 \end{cases} \quad (B.9c)$$

$$\sum_{j_1, j_2} \chi_1 \chi_2 |t+x|^{j_1-j_2} \Gamma(ij+\frac{1}{2}) \begin{cases} e^{-\pi j/2} e^{i\pi/4} & , (t+x) > 0 \\ e^{\pi j/2} e^{-i\pi/4} & , (t+x) < 0 \end{cases} \quad (B.9d)$$

and hence the spinors $\xi_{j,\lambda,L}$ and $\xi_{j,\lambda,R}$ vanish on one half of Minkowski space.

The identities (4.27) may also be found by studying the behaviour of the field operator, ψ , of (4.22) under the action of the anti-unitary PCT operator, θ . We have,

$$\theta \psi(t,x) \theta^{-1} = i \gamma_5 \psi^\dagger(-t,-x) \quad (B.10a)$$

so that

$$\theta a(j,\lambda) \theta^{-1} = \mp i c(-j,\lambda) \quad , \lambda=1,2 \quad (B.10b)$$

$$\theta c^\dagger(j,\lambda) \theta^{-1} = \mp i a^\dagger(-j,\lambda) \quad , \lambda=1,2 \quad (B.10c)$$

On the other hand,

$$e^{-\pi K} \psi^\dagger(t,x) |\hat{0}\rangle_M = i \gamma_5 \psi^\dagger(-t,-x) |\hat{0}\rangle_M \quad , (t,x) \in R \quad (B.11)$$

from which

$$\psi(t,x) |\hat{0}\rangle_M = -\theta e^{-\pi K} \psi^\dagger(t,x) |\hat{0}\rangle_M \quad , (t,x) \in R \quad (B.12)$$

and so identities (4.27) follow from,

$$\langle 0 | c(j,\lambda) [\psi(t,x) + \theta e^{-\pi K} \psi^\dagger(t,x)] |\hat{0}\rangle_M = 0 \quad , (t,x) \in R \quad (B.13)$$

We note that a double application of θ to ψ leads to a sign change, which is apparent in (B.12).

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