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ON THE PERTURBATIVE CONTRIBUTIONS TO THE
STRING TENSION ESTIMATES

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ABSTRACT

We analyze the contamination of perturbative contributions to string tension estimates based on the analysis of Wilson loops. We define suitable quantities to identify the non-perturbative value of the string tension, which shows a sizeable non-scaling behaviour between $\beta = 5.4$ and $\beta = 6$.

A standard estimate of the string tension in numerical simulations of lattice QCD is based on the following quantity¹⁾:

$$\chi(I, J) = -\ln \left\{ \frac{W(I, J) W(I-1, J-1)}{W(I-1, J) W(I, J-1)} \right\} \quad (1)$$

Indeed, if the average value of the Wilson loop $W(I, J)$ calculated on a rectangle of dimensions I, J behaves like:

$$W(I, J) = \exp - \{ C_0 + C_1 (I+J) + C_2 \cdot I \cdot J \} \quad (2)$$

the quantity $\chi(I, J)$ in Eq. (1) is equal to C_2 which is identified with the string tension σ , regulating the linear increase of the static $q\bar{q}$ potential. However, recent results on the value of $\chi(I, J)$ at fixed value of the bare coupling constant g show a dependence upon the values of I and J . This is in contradiction with the parametrization in Eq. (2). These results have been obtained with the Wilson action at different values of $\beta \equiv 6/g^2$: the most precise ones²⁾ refer to $\beta = 5.4$ and $\beta = 5.7$, and give $\Lambda_{\text{lattice}}/\sqrt{\sigma} \geq (6.1 \pm 0.1) \cdot 10^{-3}$ and $\Lambda/\sqrt{\sigma} \geq (6.9 \pm 0.5) \cdot 10^{-3}$ at $\beta = 5.4$ and 5.7 , respectively.

A measurement of the string tension based on the correlation of Polyakov loops with a length of 10 lattice spacings gives the value $\Lambda_{\text{lattice}}/\sqrt{\sigma} \sim (12 \pm 1) \cdot 10^{-3}$ at $\beta = 6$ ³⁾. There is then a general trend to find values for σ smaller than the usually accepted ones⁴⁾ ($\Lambda/\sqrt{\sigma} \approx 6 \cdot 10^{-3}$). This may invalidate the assumption on the behaviour of the Wilson loop as given in Eq. (2). In fact, if the value of the lattice spacing in physical units found at $\beta = 6$ in Ref. 3), which is also consistent with an independent determination based on the hadron spectrum⁵⁾, is extrapolated back to $\beta = 5.7$, assuming the validity of the one-loop scaling law, one obtains that a size of 3 to 4 lattice spacings corresponds to about 0.4 ± 0.5 Fermi. This makes it plausible that contributions which are non-leading in the $q\bar{q}$ potential at large distance may be essential in determining the value of χ at such distances.

In this letter, we simply take into account those contributions which are perturbative and responsible for Coulomb-like forces. The perturbative behaviour of the Wilson loop for different loop sizes has been exactly calculated in first order, and good estimates for the second order corrections are available⁶⁾. In Table 1 we report, for sake of completeness, the coefficients of the $g^2 [a(I)]$ and $g^4 [b(I)]$ expansion of $\chi(I, J)$. One notices that the second order term

is, in general, comparable to the first order for $\beta \sim 5.4, 5.7$: a straightforward use of those formulae is then inadequate in this region of the bare coupling. Our strategy consists of two steps: first, we rewrite the perturbative expansion of $X(I, I)$ in a way which aims to resum the biggest terms beyond $O(g^4)$. This is done by assuming for $X(I, I)$ the form

$$X(I, I)_{\text{perturbative}} = a(I) g^2(\eta I) + \tilde{b}(I) g^4(\eta I) \quad (3)$$

where $g^2(\eta I)$ is the running coupling constant at the scale ηI . The coefficients $a(I)$ follow a Coulomb-like behaviour, as one can see from the independence from I of the quantity $A(I) \equiv a(I) \cdot I(I-1)$, reported in Table 2. Given in the same table are the coefficients $\tilde{b}(I)$. Their values are much smaller than those of $b(I)$ in Table 1, once the scale η is normalized to cancel $b(2)$; i.e., to set $\tilde{b}(2) = 0$. If we had chosen to set to zero a different $\tilde{b}(I)$, the results would not have changed significantly.

The validity of the formula in Eq. (3) is confirmed by the numerical results in the weak coupling regime: in the figure we compare the results taken from Ref. 4) to the fit of Eq. (3) (dashed line) and to the naïve perturbative expansion (full line).

The second step is to form quantities which are not affected by perturbative contributions up to order $(g(\eta I))^6$: if we believe that the expansion in Eq. (3) converges well, those quantities should be more suitable for an investigation of the non-perturbative behaviour of the potential. We propose to use:

$$\begin{aligned} K(I, J) = \frac{1}{a(I) - a(J)} \{ & a(I) X(J, J) - a(J) X(I, I) - \\ & - a(I) a(J) [g^2(\eta J) - g^2(\eta I)] - a(I) g^4(\eta J) \tilde{b}(J) + \\ & + a(J) g^4(\eta I) \tilde{b}(I) \} \end{aligned} \quad (4)$$

The above quantity is free of perturbative corrections up to $O(g^6(\eta I))$. By using the data in Ref. 2) we obtain the results in Table 3. The major part of the R dependence of $X(R, R)$ ($= +\partial V/\partial R$ for R large enough) is given by the perturbative contribution.

The non-perturbative behaviour of $X(R)$ is not known. A fit with a constant string tension σ shows a sizeable scaling violation between $\beta = 5.4$ and $\beta = 5.7$ *). Notice that other non-perturbative contributions like $V(R) \sim (c/R)^7$

*) The values of σ that we extract are close to the extrapolated values of Ref. 2).

simulating a Coulomb-like potential, would be negligible in the combination entering $K(I,J)$: in fact, for large I , $a(I) \sim 1/I^2$ and $K(I,J)$ becomes independent of Coulomb-like terms.

We have been considering the data in Ref. 4): the values at $\beta = 5.7$ are compatible with the ones of Ref. 2); for those at $\beta = 6$ we are unable to extract from the figures numbers which allow a precise determination of $K(I,J)$. As a general feature, the different values of $K(I,J)$ $J \geq I \geq 2$, $4 \geq J \geq 2$ are quite similar among themselves and indicate an average value:

$$K(I,J) \sim \text{const} \sim .045 \quad (5)$$

The identification of $K(I,J)$ with the string tension brings up the problem of the non-scaling behaviour between $\beta = 5.4$ and $\beta = 5.7$ [and between 5.7 and 6 if we believe our rough analysis of the data of Ref. 4)]^{*}). The presence of higher order terms in $1/\beta$ may correct the non-scaling behaviour: however, the magnitude of the correction required would be very large and incompatible with a perturbative expansion in $1/\beta$.

The other extreme possibility is to ascribe the discrepancy to higher dimension operators; the scaling formula for σ in lattice spacing units ($\tilde{\sigma}$) then reads:

$$\tilde{\sigma} = \frac{\sigma}{\Lambda^2} f(\beta) \left[1 + c \frac{\sigma}{\Lambda^2} f(\beta) \right] \quad (6)$$

where

$$f(\beta) = \left[\frac{8\pi^2}{33} \beta \right]^{\frac{102}{121}} \exp \left[- \frac{8\pi^2}{33} \beta \right]$$

If one fits c and σ/Λ^2 to agree with the values of $\tilde{\sigma}$ at $\beta = 5.4$ ($= 0.50$) and at $\beta = 5.7$ ($= 0.173$) one finds $c \sim 11$ and $\Lambda/\sqrt{\sigma} \sim 11 \cdot 10^{-3}$. The same parameters would lead, at $\beta = 6$, to a value of $\tilde{\sigma} = 0.67$ which is not so far from the one in Eq. (5). Given that the correction to the naïve scaling law resulting from the term proportional to c in Eq. (6) is very large, the above estimate should be only taken at a qualitative level: if higher dimension operators are important, the hypothesis that only those of $O(a^4)$ are relevant at $\beta = 5.4$ may be too crude. If the scaling violations are due to effects which are spurions of the lattice regularization, a different pattern of scaling might occur for different physical quantities. In fact, while the critical

^{*}) The use of $g(\eta I)$ instead of g in the verification of the scaling law would not improve the agreement.

temperature T_c shows a similar non-scaling behaviour between $\beta = 5.55$ and $\beta = 5.7$ ⁸⁾, the hadron masses seem to show a better scaling behaviour between $\beta = 5.7$ and $\beta = 6$ ⁹⁾. A measurement of the same quantities with lattice actions improved à la Symanzik¹⁰⁾ would help in understanding the origin of the scaling violations.

We summarize our conclusions: the quoted values of the string tension measured with Wilson loops of moderate size are heavily affected by perturbative tails. When these are removed, the values for the derivative of the potential coming from plaquettes of different sizes agree much better among themselves and are compatible with the presence of non-perturbative contributions slowly varying with the distance, including the case of a fixed string tension.

Under this interpretation, the non-scaling behaviour between $\beta = 5.4$ and $\beta = 6$ can be phenomenologically accounted for by $O(a^2)$ corrections and, in turn, the new resulting string tension in physical units becomes close to the one measured at $\beta = 6$ with the correlation of long Polyakov loops.

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NOTE ADDED IN PROOF

After the completion of this paper, we received a paper by H.D. Politzer [Caltech preprint 68-1035 (1983)] where the importance of perturbative contaminations in string tension estimates is also stressed.

Table 1

	I = 1	I = 2	I = 3	I = 4
a(I)	0.3333	0.0965	0.0345	0.0162
b(I)	0.0963	0.0472	0.0194	0.0098

The coefficients $a(I)$ and $b(I)$ of the perturbative expansion of $\chi(I,I) = a(I)g^2 + b(I)g^4$.

Table 2

	I = 1	I = 2	I = 3	I = 4
A(I)		0.193	0.207	0.194
$\tilde{b}(I)$	-0.0346	0	0.00058	0.00031

The coefficients $\tilde{b}(I)$ for improved perturbative expansion of Eq. (3): $\chi(I,I) = a(I)g^2(\eta I) + \tilde{b}(I)g^4(\eta I)$ and the values of $A(I) \equiv a(I) \cdot I(I-1)$.

Table 3

$\beta = 5.4$		
K(1,2)	0.466 ± 0.003	6.74 ± 0.02
K(1,3)	0.50 ± 0.02	6.53 ± 0.15
K(2,3)	0.51 ± 0.03	6.5 ± 0.2

$\beta = 5.7$		
K(1,2)	0.186 ± 0.003	7.63 ± 0.06
K(1,3)	0.170 ± 0.007	7.98 ± 0.16
K(1,4)	0.18 ± 0.05	7.8 ± 1.0
K(2,3)	0.164 ± 0.009	8.12 ± 0.22
K(2,4)	0.175 ± 0.050	7.9 ± 1.0
K(3,4)	0.18 ± 0.07	7.7 ± 1.6

The first column contains the results for $K(I,J)$ at different β and I, J . The second column contains the corresponding values of $\Lambda/\sqrt{\sigma} \cdot 10^{-3}$ where $\sigma^2 \equiv K(I,J)$.

REFERENCES

- 1) For a review, see, for example:
M. Creutz, L. Jacobs and C. Rebbi, "Monte Carlo computations in lattice gauge theories", Brookhaven National Lab. preprint BNL 32438 (1983).
- 2) F. Gutbrod, P. Hasenfratz, Z. Kunszt and I. Montvay, CERN preprint TH.3591 (1983).
- 3) G. Parisi, R. Petronzio and F. Rapuano, CERN preprint TH.3596 (1983).
- 4) R. Ardill, M. Creutz and K.J.M. Moriarty, Phys. Rev. D27 (1983) 1956.
- 5) H. Lipps, G. Martinelli, R. Petronzio and F. Rapuano, CERN preprint TH.3548 (1983), to be published in Phys. Lett. B.
- 6) T. Hattori and H. Kawai, Phys. Lett. 105B (1981) 43;
J.P. Kowall and H. Neuberger, Nucl. Phys. B189 (1981) 535;
R. Kirschner, J. Kripfganz, J. Ranft and A. Schiller, Nucl. Phys. B210 (FS6) (1982) 567.
- 7) M. Luscher, K. Symanzik and P. Weisz, Nucl. Phys. B173 (1980) 365.
- 8) T. Celik, J. Engels and H. Satz, Bielefeld preprint, BI-TP 83/07 (1983).
- 9) See the talk by
E. Wallace at the Johns Hopkins Meeting, Bad Honnef, Germany (June 1983).
- 10) K. Symanzik, in Mathematical Problems in Theoretical Physics, eds. R. Schrader et al., (Springer, Berlin, 1982);
P. Weisz, Nucl. Phys. B212 (1983) 1;
G. Curci, P. Menotti and G.P. Paffuti, Univ. of Pisa preprint IFUP-TH 83/6 (1983).

FIGURE CAPTION

Numerical results for the $X(I,I)$ are compared to the naïve perturbative expansion up to $O(g^4)$ and to the resummed formula of Eq. (3).