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LOCALLY SUPERSYMMETRIC $SO(10)$
WITH NATURAL DOUBLET/TRIPLET SPLITTING

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A B S T R A C T

We propose a mechanism for solving the second hierarchy problem in locally supersymmetric $SO(10)$. It involves the Higgs multiplets 21_0 , 12_6 and $\overline{12}_6$. A high mass is given to the colour triplets of 10 through the couplings $\langle 21_0 \rangle \cdot 12_6 \cdot 10$ and $\langle 21_0 \rangle \cdot \overline{12}_6 \cdot 10$, while the couplings of the weak doublets in 10 and $\overline{12}_6$ are forbidden. The requirements of this mechanism are related to the question of fermion masses

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One of the problems in supersymmetric grand unified theories (SUSY GUT's) is to create a large mass hierarchy between the weak Higgs doublets and their colour triplet partners. This so-called second hierarchy problem can be solved either through fine tunings¹⁾, the "missing partner" mechanism²⁾ or the light "sliding" singlet³⁾. Although the sliding singlet is a reasonable substitute for unnatural adjustments, any light singlet connecting the weak doublets to some heavy superfields will spoil the hierarchy at one loop level when supersymmetry is made local⁴⁾. Instead of a light singlet one could think of using a heavy one, but producing a high mass either via self-couplings, couplings to other fields or explicit mass terms conflicts with the requirement that the singlet F-term must vanish in the vacuum. In view of these problems, the only reliable method available at present in local SUSY GUT's seems to be the missing partner mechanism.

In the missing partner mechanism one needs "incomplete multiplets" which contain colour triplets but no weak doublets. When the scalars that break the GUT symmetry couple these representations to the ordinary doublet/triplet multiplets, the colour triplets will acquire a high mass while the weak doublets remain massless. This mechanism is found to work in connection with SU(5) SUSY GUT's²⁾.

In SO(10), which is our concern in this letter, the situation is different since there are no incomplete multiplets. We will show in the following that a natural solution to the second hierarchy problem also exists here. It is based on the occurrence of "missing couplings" in the sense that the Yukawa couplings of the Higgs colour triplets are present, while those of the weak doublets are absent. We will also discuss the consequences of this mechanism for neutrino and other fermion masses.

In SU(5) the simplest incomplete representations are 5_0 and $\bar{5}_0$. The couplings of these to the doublet/triplet multiplets 5 and $\bar{5}$ require that the larger representation 75 is used to break SU(5), instead of the standard 24 ²⁾. The couplings

$$\lambda \langle 75 \rangle \cdot 5_0 \cdot \bar{5} + \lambda' \langle 75 \rangle \cdot \bar{5}_0 \cdot 5 \quad (1)$$

mix the colour triplets, i.e., $(3, 1, -1/3)$ and $(\bar{3}, 1, 1/3)$ of $SU(3)_C \times SU(2)_L \times U(1)_Y$, from $(\bar{50})$ and $(\bar{5})$ with each other giving them mass of the order of the unification mass. The doublets $(1, 2, \pm 1/2)$ of 5 and $\bar{5}$ have no partners in 50 or $\bar{50}$ and thus remain massless. An alternative to (1) is possible in a locally supersymmetric model with $\langle 75 \rangle$ replaced by $m_p (\langle 24 \rangle / m_p)^{n/5}$, since nonrenormalizable couplings should be allowed in supergravity.

Let us now turn to $SO(10)$. It is straightforward to see that the simple generalization to $SO(10)$ of the missing partner mechanism in $SU(5)$ does not work. The ordinary weak doublet and colour triplet Higgses are contained in a 10 of $SO(10)$. The minimal representation that contains the 75 of $SU(5)$ is 210 , the fourth order anti-symmetric tensor representation. The minimal accessory $SO(10)$ multiplets containing the $SU(5)$ 50 and $\bar{50}$ are 126 and $\bar{126}$. Then the terms

$$\lambda \langle 210 \rangle_{75} \cdot \bar{126} \cdot 10 + \lambda' \langle 210 \rangle_{75} \cdot 126 \cdot 10 \quad (2)$$

contain the couplings (1) which in the $SU(5)$ model provided the doublet/triplet splitting. However, the couplings (2) do not protect the weak doublets in 10 from acquiring a high mass. In fact, from the $SU(5)$ decomposition of 126 ,

$$126 = 1 + \bar{5} + 10 + \bar{15} + 45 + \bar{50}, \quad (3)$$

it can be seen that in addition to the desired $\langle 75 \rangle \cdot 50 \cdot \bar{5}$ and $\langle 75 \rangle \cdot \bar{50} \cdot 5$ terms, Eq.(2) contains also $\langle 75 \rangle \cdot \bar{45} \cdot 5$ and $\langle 75 \rangle \cdot 45 \cdot \bar{5}$. The latter terms are disastrous, because 45 and $\bar{45}$, in contrast to 50 and $\bar{50}$, are not "incomplete" but include both colour triplets and weak doublets. Consequently the hierarchy cannot be generated in this manner.

It would be, however, incorrect to assume on the basis of this negative conclusion that there is no natural group theoretical way in the framework of $SO(10)$ to keep the weak doublets light, even with the

$210 \cdot \overline{126} \cdot 10$ and $210 \cdot 126 \cdot 10$ couplings. It is still possible that if the 210 acquires a vev in some other direction than the 75 of $SU(5)$, group theory could prevent the doublets of $(\overline{126})$ from coupling to the doublets of 10. In order to see this, let us consider the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ subgroup of $SO(10)$. The 10 representation decomposes under this group as

$$10 = (6, 1, 1) + (1, 2, 2) , \quad (4)$$

where $(6, 1, 1)$ contains the colour triplets and $(1, 2, 2)$ the weak doublets. The decompositions of 126 and 210 are

$$126 = (6, 1, 1) + (15, 2, 2) + (10, 3, 1) + (\overline{10}, 1, 3) , \quad (5)$$

$$210 = (1, 1, 1) + (15, 1, 1) + (15, 1, 3) + (15, 3, 1) \quad (6) \\ + (10, 2, 2) + (\overline{10}, 2, 2) + (6, 2, 2) .$$

Thus if we let the singlet component of the 210 acquire a large vev $\langle 210_{(1,1,1)} \rangle = O(M_X)$, then the coupling

$$\langle 210_{(1,1,1)} \rangle \cdot 126 \cdot 10 \quad (7.a)$$

alone, or otherwise the coupling

$$\langle 210_{(1,1,1)} \rangle \cdot \overline{126} \cdot 10 , \quad (7.b)$$

will provide the desired hierarchy. We note that the $SU(2)_L$ doublets in $(\overline{126})$ with the same quantum numbers as the weak doublets belong to

$(15,2,2)$ and that their couplings are therefore absent in Eqs.(7). We also note that, as opposed to $SU(5)$ where two accessory multiplets $(50$ and $\overline{50})$ are needed, here only one 126 or $\overline{126}$ is required with no need for both. However, in order to provide these fields with a large mass, both 126 and $\overline{126}$ should be used. Such mass is obtained from the term

$$M \, 126 \cdot \overline{126} \quad , \quad (8)$$

where $M \geq O(M_X)$.

A possible alternative to the above method uses the vevs of $(15,1,1)$ and $(15,1,3)$ in 210 . These contain each a singlet under the standard model and break $SO(10)$ into⁶⁾ $SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R$ and to $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_R$ respectively. The possible couplings are, in terms of the Pati-Salam representations,

$$\begin{aligned} & \langle 15,1,1 \rangle_{210} \cdot [(6,1,1)_{126} \cdot (6,1,1)_{10} + (15,2,2)_{126} \cdot (1,2,2)_{10}] \\ & \langle 15,1,3 \rangle_{210} \cdot [(\overline{10},1,3)_{126} \cdot (6,1,1)_{10} + (15,2,2)_{126} \cdot (1,2,2)_{10}] \end{aligned} \quad (9)$$

with analogous terms for $\overline{126}$. It is obvious that the doublet/triplet splitting can only occur if

$$\langle 15,1,1 \rangle_{210} = - \langle 15,1,3 \rangle_{210} \quad , \quad (10)$$

which is actually required by supersymmetry. Indeed, the only contributions to the $(15,2,2)_{126}$ F-term come from (9) and give

$$\begin{aligned} F_{(15,2,2)_{126}} &= [(15,1,1)_{210} + (15,1,3)_{210}] \cdot (1,2,2)_{10} \\ &= F_{(15,2,2)_{126}} \quad . \end{aligned} \quad (11)$$

(The corresponding contributions from (8) vanish because we will assume 126 and $\overline{126}$ not to acquire vevs.) The vanishing of (11) in the vacuum implies relation (10).

A possible difficulty with this alternative method is associated with giving high masses to the 210. A large collection of light fields from 210 is not admissible since it conflicts with cosmology and perturbation theory requirements⁵⁾. In our first case this mass can be generated by including a 45 representation and letting it acquire a vev $O(M_X)$ in its (15,1,1) component. Then the coupling

$$\langle 45_{(15,1,1)} \rangle \cdot 210 \cdot 210 \quad (12)$$

gives mass to all the 210, except a singlet (1,1,1). (This singlet will not lead to dangerous radiative corrections since the coupling 210.10.10 is not allowed.) In our second case the masses for the 210 cannot be generated in the same simple manner. The reason is that the couplings $45_{(1,1,3)} \cdot 210_{(15,1,1)} \cdot 210_{(15,1,3)}$ and $45_{(15,1,1)} \cdot 210_{(15,1,1)} \cdot 210_{(15,1,1)}$ allowed by (12) would generate too large terms for the superpotential in the vacuum that cannot be cancelled in a natural way.

The breaking of $SO(10)$ is still incomplete since neither the 210 nor the 45 decrease the rank of the group. In order to break the remaining $U(1)$ symmetry⁶⁾ we will use a 16 and $\overline{16}$ with a vev for their " ν_R " components. We note that 126 and $\overline{126}$ cannot be used for this purpose since these representations are not allowed to acquire a large vev, because the couplings $\overline{126} \cdot 210 \cdot 10$ would give a high mass to the weak doublets. One consequence of this is that the standard $\overline{126} \cdot 16 \cdot 16$ is not available to generate a large Majorana mass to the right-handed neutrino ν_R . Instead one should use the nonrenormalizable quartic coupling⁷⁾

$$\lambda_1 \left(\langle \overline{16} \rangle^2 / m_P \right) \cdot 16 \cdot 16 \quad (13)$$

As far as fermion masses are concerned, one can use either the standard

$$\lambda_2 \langle 10 \rangle_{(1,2,2)} \cdot 16 \cdot 16 \quad (14)$$

or a quartic coupling like

$$\lambda_3 \left(\langle 210 \rangle_{(1,1,1)} \cdot \langle 10 \rangle_{(1,2,2)} / m_P \right) \cdot 16 \cdot 16, \quad (15)$$

where $\langle 10 \rangle_{(1,2,2)} = O(m_W)$ and $\langle 210 \rangle_{(1,1,1)} = O(M_X)$. The latter possibility has the advantage that the coupling constant λ_3 need not be highly suppressed^{*)}. In fact, taking $\lambda_3 = O(10^{-2})$ the masses from (15) are typically of the order of 1 MeV. The same good feature is also present in the coupling

$$\lambda_4 \left(\langle 45 \rangle_{(15,1,1)} \cdot \langle 10 \rangle_{(1,2,2)} / m_P \right) \cdot 16 \cdot 16. \quad (16)$$

A difference between (15) and (16) is that the former gives the flavour diagonal masses in a similar manner to $\overline{126}$ in the standard $SO(10)$ model, while the latter acts like a 10.

We summarize:

- i) In the locally supersymmetric $SO(10)$ the weak-doublet/colour-triplet mass splitting can be generated with representations 210, 126 and $\overline{126}$ through the couplings (7). The vev of 210 should be a singlet under $SU(4)_C \times SU(2)_L \times SU(2)_R$. The 126 and $\overline{126}$ are not allowed to acquire non-zero vevs.
- ii) The 210 is made heavy through coupling to 45 [cf. Eq.(12)]. The accessory Higgses 126 and $\overline{126}$ will acquire mass from the term $126 \cdot \overline{126}$.

^{*)} The analogous situation in $SU(5)$ was first remarked in Ref. 9).

iii) The fermion masses can be generated by non-renormalizable terms as given in Eqs.(13), (15) and (16).

iv) All above features are contained in the superpotential

$$\lambda \, 210 \cdot 126 \cdot 10 + M \, 126 \cdot \overline{126} + \lambda'_{45} \cdot 210 \cdot 210 \\ + (\lambda_1/m_p) \, \overline{16} \cdot \overline{16} \cdot 16 \cdot 16 + (\lambda_3/m_p) \, 210 \cdot 10 \cdot 16 \cdot 16 \, .$$

As a final remark we note that all the above analysis is still valid with the 210 replaced by the self-couplings of 45. In the mass term (15) this would mean a further suppression, therefore suggesting a possible origin for the mass differences between fermion generations.

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