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LIGHT TRIPLETS IN SUPERGRAVITY GUTS

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A B S T R A C T

Effects of a light weak triplet in the effective potential of supergravity GUTs are investigated. The light triplet can trigger the breaking of $SU(2) \times U(1)$, without violating the constraint put by the ρ parameter on its vacuum expectation value. Contrary to the light singlet case, the effective superpotential cannot be homogeneous, but it must contain, along with trilinear couplings, explicit mass terms.

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The coupling of simple (N=1) supergravity¹⁾ to unified models of strong and electroweak interactions has several interesting consequences. The most remarkable one is perhaps that spontaneous breaking of supergravity at a high mass scale may simultaneously lead to the breaking of $SU(2) \times U(1)$ weak symmetry at low mass scale set up by the gravitino mass m_g , thus generating the so far poorly understood gauge hierarchy²⁾. The weak scale need not be inserted by hand but has its origin deeper in the theory.

There has been two main strategies to realize this program and to produce the desired breakdown of $SU(2) \times U(1)$. One possibility is the breaking of $SU(2) \times U(1)$ by radiative corrections^{3),4)}. This may happen if there is a heavy top quark (or some other heavy fermion) with a large Yukawa coupling. Renormalization group calculations show that top quark loops can drive the Higgs mass-squared operators negative at the gravitino mass scale, thus inducing the breaking of the weak symmetry. However, for this mechanism to work one may need a very heavy top quark^{3),4)}, $m_t \gtrsim 60$ GeV.

The second method for breaking $SU(2) \times U(1)$ is to add an extra Higgs superfield to the low energy theory. The simplest possibility is to use a light singlet, Y . It has been shown⁵⁾ that the low energy effective superpotential

$$f_{\text{eff}} = \lambda H \bar{H} Y + \sigma Y^3 \quad (1)$$

leads to spontaneous breaking of $SU(2) \times U(1)$. Here H and $\bar{H}$ are the usual weak doublets. Note that f_{eff} satisfies the naturalness condition⁵⁾ by having no dimensionful parameters. The light singlet superfields are, however, potentially dangerous, as has been pointed out recently⁶⁾, since they tend to destabilize the supergravity-induced hierarchy. Large mass splittings within heavy supermultiplets would leak through them to the doublet sector, hence spoiling the hierarchy.

In the present note we will study the possibility of using a light weak triplet as a substitute of the dangerous singlet in the low energy superpotential. This possibility has been recently discussed also by Alvarez-Gaumé, Polchinski and Wise⁴⁾. A triplet is the only non-singlet that can couple to the Higgs doublets in renormalizable terms. Although the triplet, too, could generate loop diagrams that may destabilize the hierarchy, it is quite evident that in this case a model builder can more easily avoid the dangerous couplings.

Light triplets, in fact, frequently appear in supergravity GUTs, and in some cases their presence might even be desirable⁷⁾. For these theories the examination of the effects of light triplets is of utmost importance, since one should worry about violating the $\Delta I = \frac{1}{2}$ rule of the electroweak breaking experimentally measured by the parameter $\rho = M_W^2/M_Z^2 \cos^2 \theta_W$.

In the following we will not specify the underlying GUT but assume only that the GUT sector is such that two light doublets and a triplet result. All other light multiplets, if appearing, are supposed to have negligible small couplings and will be disregarded.

It turns out that for a natural range in the parameters of the Higgs potential it is possible to break $SU(2) \times U(1)$ without violating the constraint on ρ . However, we will emphasize that to achieve this, the effective low-energy superpotential cannot be scaleless but must contain explicit mass terms $O(m_g)$ for the doublets and/or the triplet.

We will adopt the standard assumption that the breakdown of the local supersymmetry occurs in the "hidden" gauge singlet sector which is separated from the "ordinary" sector (GUT sector), the superpotential having the form^{8),5)}

$$F(z^a, \tilde{z}) = f_o(z^a) + f_h(\tilde{z}) . \quad (2)$$

The hidden fields $\tilde{z}$ develop vevs of the order of the Planck mass M_p , while $f_h = O(m_g M_p^2)$. Some of the ordinary fields z^a can also be heavy, of the order of the GUT mass M_x .

The effective low energy scalar potential V_{eff} is obtained by "integrating out" the heavy fields by expanding the complete potential in parameters m_g/M_p and m_g/M_x ⁹⁾. In the general case the complete scalar potential is given by

$$V = e^d \left(\sum_{NM} (g^{-1})^N_M F_N F_M^* - 3|F|^2 \right) + \text{gauge terms} , \quad (3)$$

where d is the Kähler potential (in units $8\pi G$), F is given in (2), F_N is defined as

$$F_N = \frac{\partial F}{\partial Z^N} + \frac{\partial d}{\partial Z^N} F \quad (4)$$

and

$$g_N^M = \frac{\partial^2 d}{\partial Z^N \partial Z^{M*}} (8\pi G)^{-1} \quad (5)$$

is the Kähler metric. The sum runs over all chiral scalars Z^N . In what follows we will make use of the general result of Hall, Lykken and Weinberg⁹⁾. They have recently derived the following expression for the effective low-energy scalar potential

$$\begin{aligned} V_{\text{eff}} = \sum_a \left| \frac{\partial f_{\text{eff}}}{\partial z^a} \right|^2 + |m_1|^2 |z^a|^2 + 2\text{Re}\{(4m_2 - m_3^*) f^{(1)} + \\ + 2m_2^* f^{(2)} + m_3^* f^{(3)}\} + \text{gauge terms} \\ + O(m_2/M_P, m_3/M_X) \end{aligned} \quad (6)$$

where the sum now runs over light scalars. The effective superpotential of the light fields is $f_{\text{eff}} = f^{(1)} + f^{(2)} + f^{(3)}$, where $f^{(1)}$ is linear, $f^{(2)}$ bilinear and $f^{(3)}$ trilinear in light fields. The mass parameters m_1 , m_2 and m_3 are all of the order of the

gravitino mass. Their precise values depend both on the hidden sector, the GUT symmetry and details of the Kähler potential.

Eq.(6) is valid for a more general class of Kähler potentials than just the usual flat one. The flat limit corresponds to the case $m_1 = m_2$. The gauge terms are modified from their flat metric form, but for us it suffices to note that they vanish when the usual D-terms, $D = z^\dagger \bar{z}$, vanish.

We will now apply the general formula (6) to the case when the light chiral scalars are the doublets $H(\underline{1}, \underline{2}, -1)$ and $\bar{H}(\underline{1}, \underline{2}, 1)$ and the triplet $\Delta(\underline{1}, \underline{3}, 0)$. When only renormalizable terms are considered, the most general effective superpotential for the light sector is given by

$$f_{\text{eff}} = \lambda H^T \epsilon \vec{\Delta} \cdot \vec{\epsilon} \bar{H} + \frac{1}{2} \mu_1 \text{Tr} \Delta^2 + \mu_2 H^T \epsilon \bar{H}, \quad (7)$$

where $\epsilon = i\tau_2$. The natural scale for μ_1 and μ_2 is $O(m_g)$. There should also be Yukawa couplings of quark and lepton superfields, but we will neglect these small terms. The effective potential V_{eff} can be derived from Eq.(6) by noting that $f^{(1)} = 0$, $f^{(2)} = \frac{1}{2} \mu_1 \text{Tr} \Delta^2 + \mu_2 H^T \epsilon \bar{H}$ and $f^{(3)} = \lambda H^T \epsilon \vec{\Delta} \cdot \vec{\epsilon} \bar{H}$.

What interests us is to know whether the resulting potential V_{eff} has a minimum in the direction of $SU(2) \times U(1)$ breaking. The appropriate stationary conditions are given as

$$\left\langle \frac{\partial V_{\text{eff}}}{\partial H^0} \right\rangle = 0 = (2m_2^* \mu_2 + |m_1|^2 - m_3^* \lambda \delta - \lambda \mu_1^* \delta^*) v + (|\lambda v|^2 + |\lambda \delta|^2 + |\mu_2|^2 - \mu_2 \lambda^* \delta^* - \mu_2^* \lambda \delta) v^*, \quad (8)$$

$$\left\langle \frac{\partial V_{\text{eff}}}{\partial \Delta^0} \right\rangle = 0 = 2m_2^* \mu_1 \delta + (|\mu_1|^2 + 2|\lambda v|^2 + |m_1|^2) \delta^* - \\ - m_3^* \lambda |v|^2 - 2\lambda |v|^2 \mu_2^* - \mu_1 \lambda^* (v^*)^2$$

where $\delta = \langle \Delta \rangle$ and we have assumed, to minimize the gauge terms, that $v = \langle H \rangle = |\langle \bar{H} \rangle|^*$). The Eq.(8) should be satisfied for a finite and natural range of the parameters.

We will rewrite Eq.(8) in a more compact form by defining $a = \delta/v$, $x_1 = \mu_1/m_g$, $x_2 = -\mu_2/m_g$, $x_3 = m_1/m_g$, $x_4 = (\mu_1 + m_3 + 2\mu_2)/m_g$ and $y = -\lambda v/m_g$. We also restrict ourselves to the real manifold of the parameters. We then have

$$x_4 y = -a(x_1^2 + 2x_1 + x_3^2 + 2y^2), \\ (1+a^2)y^2 = 2x_2 - (x_2^2 + x_3^2) - ax_4 y. \quad (9)$$

The natural scale for y and the various z -parameters is $O(1)$, since all the mass parameters are $O(m_g) = O(m_w)$.

*) These conditions are not affected if one includes superfields transforming as $(\underline{8}, \underline{1}, 0)$ or $(\underline{1}, \underline{1}, \pm 2)$ to the superpotential. The case when there is also a singlet Y in (7) would be obtained by making substitutions $\mu_1 \rightarrow \mu_1 + \lambda_1 \langle Y \rangle$, $\mu_2 \rightarrow \mu_2 + \lambda_2 \langle Y \rangle$.

The experimental limit on the ρ -parameter puts a quite stringent-looking constraint on the ratio $a = \delta/v^{10}$:

$$\rho = 1 + 2a^2 \leq 1.03 . \quad (10)$$

However, it should be noted that for the vacuum expectation values this implies only $\delta \leq 0.125 v$.

We have found out by numerical computations that for a finite, natural range in the parameters of V_{eff} there are solutions that simultaneously satisfy the constraint (10) and lead to a minimum for the potential. To illustrate this we show in Fig. 1 a projection in parameter space for a set of such solutions. We have assumed $|\mu_1| = |\mu_2|$ for simplicity and taken μ_2 negative. All mass parameters fall in their natural range $0(m_g)$. Other projections were found to have a similar behaviour. When $m_3 \approx 0.5m_2$ the allowed region in Fig. 1 corresponds roughly to doublet vevs of the order of $(0.2-1.0)m_g/\lambda$, while V_{eff} at the minimum varies roughly in the range $-(0.1-0.5)m_g^4/\lambda^2$.

It is now easy to see that a purely trilinear superpotential $f_{\text{eff}} = \lambda H^T \epsilon \vec{\Delta} \cdot \vec{\tau} \bar{H}$ would not work, although one would hope the initial low energy theory to be scale invariant⁵⁾. If we set $x_1 = x_2 = 0$ in (9) it follows that $a=1$, i.e., the doublet and the triplet vevs are equal, $v = \delta$, contradicting with (10). This can be understood as a consequence of the fact that there is no parameter in the purely trilinear superpotential that could set a different scale for v and δ . In the singlet case [cf. Eq.(1)] this does not cause any problem since there is no constraint

for the vev of the singlet. It should be pointed out also that if only the trilinear term is present, f_{eff} obeys the global symmetry $H \rightarrow e^{i\theta} H$, $\bar{H} \rightarrow e^{i\theta} \bar{H}$, $\Delta \rightarrow e^{-2i\theta} \Delta$. This symmetry would become spontaneously broken leading to a harmful Peccei-Quinn axion. The mass terms correct the situation by breaking that symmetry explicitly. In the case of light singlet no mass terms are needed for this purpose, since there the cubic singlet term γ^3 explicitly breaks the corresponding global symmetry.

The presence of the mass terms in the low energy effective potential (7) does not, however, necessarily mean that small mass parameters, aside from m_g , had to put into the theory by hand. The terms bilinear in light fields may originate in trilinear terms of the full GUT superpotential. The breaking of supersymmetry causes shifts of the order of m_g in the vevs of the heavy fields thus creating the small mass terms of the effective low-energy theory⁹⁾. We should mention that the need for explicit mass terms for the triplets in the low-energy superpotential has also been emphasized by Alvarez-Gaumé et al.⁴⁾. However, in contrast to us they suppose that the mass parameter in the triplet term must be much larger than m_g . They realize that this is contradictory since the triplet would then decouple from the low-energy theory. Also the origin of the new mass scale would be hard to explain. We have shown that all the mass parameters can be of the order of m_g .

Returning back to Eq.(9), one observes that there is no solution with $a=0$, $y \neq 0$, i.e., $\delta=0$, $v \neq 0$. This is probably a general feature: whenever one has a triplet in the low energy

theory, it will develop a vev, which might be dangerous when the parameter ρ is concerned. We have shown that in our case there is a natural range of parameters for which this vev is safe. We expect this to be true in more general cases, too. After all, the limit coming from ρ is not very severe on the actual vevs, as we pointed out earlier.

In conclusion, we have investigated the effects of a light $Y=0$ weak triplet Higgs in the effective low-energy superpotential of simple supergravity GUT's. We found out that i) the effective superpotential f_{eff} in Eq.(7) leads to a scalar potential that has a minimum in the direction of the correct $SU(2) \times U(1)$ breaking; ii) for a finite and natural range of parameters the vev of the triplet corresponding to this minimum obeys the experimental constraint set by the ρ parameter; iii) the conclusion ii) is only true if f_{eff} breaks homogeneity by having both trilinear and bilinear terms.

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REFERENCES

- 1) E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, Nucl.Phys. B147 (1979) 105;

E. Cremmer, S. Ferrara, L. Girardello and A. van Proeyen, Phys.Lett. 116B (1982) 29 and Nucl.Phys. B212 (1983) 413.
- 2) H.P. Nilles, Nucl.Phys. B217 (1983) 366.
- 3) K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Prog.Th.Phys. 68 (1982) 927;
L.E. Ibáñez, Phys. Lett. 118B (1982) 73;
L.E. Ibáñez and C. Lopez, Madrid preprint FTUAM/83-2(1983).
- 4) L. Alvarez-Gaumé, J. Polchinski and M.B. Wise, Nucl.Phys. B221 (1983) 495.
- 5) H.P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. 120B (1983) 346.
- 6) H.P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. 124B (1983) 337;
A.B. Lahanas, Phys.Lett. 124B (1983) 341.
- 7) L. Ibáñez, Madrid University preprint FTUAM/83-10 (1983).

- 8) R. Barbieri, S. Ferrara and C.A. Savoy, Phys. Lett. 119B (1982) 343.
- 9) L. Hall, J. Lykken and S. Weinberg, Phys.Rev. D 27 (1983) 2359.
- 10) I. Liede and M. Roos, Nucl.Phys. B167 (1980) 397.

FIGURE CAPTION

A projection of the region of the parameter space allowed by the stationary conditions [of Eq.(9)] and the ρ -parameter constraint [cf.Eq.(10)]. The allowed region in the different cases $m_3=m_2$, $3m_2/4$ and $m_2/2$ is the shaded area above the appropriate curves.