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SU(2) GAUGE - HIGGS THEORY WITH
RADIAL DEGREE OF FREEDOM

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A B S T R A C T

We determine the phase-diagram for the SU(2) gauge-Higgs theory where the Higgs fields are in the fundamental representation and the radial degree of freedom is not frozen. For certain limiting values of the couplings we discuss the structure by analytic methods, elsewhere by Monte Carlo simulations on 4^4 and 8^4 lattices.

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1 INTRODUCTION

Monte Carlo simulations of coupled gauge-Higgs systems^[1] have confirmed the theoretical expectations^[2] and have provided a quantitative description of the phase structure. Most of these discussions have been restricted to Higgs fields of a fixed length. Near a second order phase transition, where one eventually might approach a continuum theory, the length of the Higgs field should become irrelevant. Still the phase diagram might be different if one allows for this radial degree of freedom. This has been demonstrated only recently in the investigation of an Abelian Z_2 - gauge-Higgs system^[3]. The specific shape of the critical line is important for assuming the continuum limit in practice as we know from gauge-fermion calculations.

Here we present results of an investigation of the $SU(2)$ gauge theory with a Higgs field in the fundamental representation of $SU(2)$ and arbitrary length. To control the radial modes we employ quartic Higgs self-coupling, chosen so as to allow the obvious connection to the results for fixed length^[4] and to the system without quartic term. The second case leads to a phase diagram that closely resembles the gauge-fermion situation. Several aspects of dynamical fermion fields (the "non-quenched" case) might be studied in this bosonic theory.

First we introduce the model and our notation in sec. 2, then in sec. 3 some limiting situations are discussed. The results of the MC-simulations are presented in sec. 4 and in the last section we discuss them and conclude.

2 THE MODEL

The gauge group is $SU(2)$. However, for technical reasons we work with the discrete 120-elements group $\tilde{Y}$ in the numerical evaluations. It is well known by now that this approximation is reliable below $\beta \approx 6$, where there is a freezing transition typical for discrete groups. We introduce the Higgs field ϕ_x as complex 2-vectors, unitary representations of $SU(2)$, attached to each point of the 4-dimensional lattice Λ . The link variables $U_{x,\mu}$ are 2×2 complex $SU(2)$ matrices given at the links $x \rightarrow x+\mu$ where μ denotes any one of the four unit vectors. We introduce the action

$$S = S_g(\{U\}) + \lambda \sum_{x \in \Lambda} (\Phi_x^\dagger \Phi_x - 1)^2 + \sum_{x, y \in \Lambda} \Phi_x^\dagger Q_{xy} \Phi_y, \quad (2.1)$$

where S_g denotes the usual Wilson-type [5] gauge field action

$$S_g(\{U\}) = -1/g^2 \sum_{p \in \Lambda} \text{Tr} (U_p + U_p^\dagger) \quad (2.2)$$

with $4/g^2 \equiv \beta$, U_p the ordered product of link variables around the plaquette p , and

$$Q_{xy} = \delta_{xy} - \kappa [\delta_{x, y-\mu} U_{x, \mu} + \delta_{x, y+\mu} U_{x-\mu, \mu}^\dagger] . \quad (2.3)$$

We have chosen this specific form to make obvious the resemblance with the fermionic action.

The relation of the lattice parameters λ , κ and the dimensionless field Φ to the quantities of the (naive) continuum limit of the Higgs part of the action, i.e.

$$S_c = \int dx [(D_\mu \phi^c)^2 + m_c^2 (\phi^c)^2 + \lambda_c (\phi^c)^4] , \quad (2.4)$$

is given by

$$\begin{aligned} \phi^c &= \Phi \sqrt{\kappa/a} , \\ \lambda_c &= \lambda/\kappa^2 , \\ m_c^2 &= (1-2\lambda-8\kappa)/\kappa a^2 . \end{aligned} \quad (2.5)$$

Expectation values are given by the functional integral over configuration space with the measure

$$d\mu = \exp(-S) \{ d\Phi^\dagger d\Phi dU \} / \int \exp(-S) \{ d\Phi^\dagger d\Phi dU \} , \quad (2.6)$$

where $\{ d\Phi^\dagger d\Phi dU \}$ denotes the product of corresponding Haar measures for all lattice variables. Monte Carlo simulation replaces this integral by a sum over configurations sampled according to the probability distribution given by this measure.

We are going to represent Φ_x by the pair of variables (ρ_x, σ_x) where $\rho_x \in \mathbb{R}^+$, real and non-negative, and σ_x a 2×2 matrix of $SU(2)$ (in the actual calculations: $\tilde{Y}$). The Haar measure then becomes

$$d\Phi^\dagger d\Phi \rightarrow \rho^3 d\rho d\sigma , \quad (2.7)$$

and $d\sigma$ is the $SU(2)$ Haar measure like for dU reducing to a constant for the discrete subgroup $\tilde{Y}$.

In this notation we find

$$S = S_g + \lambda \sum_{x \in \Lambda} (\rho_x^2 - 1)^2 + \sum_{x \in \Lambda} \rho_x^2 - \kappa \sum_{x, \mu \in \Lambda} \rho_x \rho_{x+\mu} \text{Tr}(\sigma_x U_{x, \mu} \sigma_{x+\mu}) . \quad (2.8)$$

Finally we observe that by a variable change

$$\Phi_{(xyzt)} \rightarrow (-1)^{x+y+z+t} \Phi_{(xyzt)} , \quad (2.9)$$

(-1 denotes the negative unit element of the center Z_2 of $SU(2)$) one finds a symmetry $\kappa \rightarrow -\kappa$ for all β, λ . This symmetry holds for all $SU(2n)$ gauge-Higgs systems.

For $\kappa = 0$ there is also a symmetry $\beta \rightarrow -\beta$ which can be demonstrated by applying the transformation

$$\begin{aligned}
 U_{xyzt,x} &\rightarrow U_{xyzt,x} \\
 U_{xyzt,y} &\rightarrow (-1)^x U_{xyzt,y} \\
 U_{xyzt,z} &\rightarrow (-1)^{x+y} U_{xyzt,z} \\
 U_{xyzt,t} &\rightarrow (-1)^{x+y+z} U_{xyzt,t} \quad ,
 \end{aligned} \tag{2.10}$$

as has been shown in the context of a fixed length Z_2 gauge-Higgs system by Bhanot and Creutz^[1]. This transformation defines a one-to-one correspondence between configurations with $\langle U_p \rangle$ and $-\langle U_p \rangle$.

3 BOUNDARIES

There is a certain understanding of the expected behaviour of the system based on the extreme cases for $\lambda \rightarrow \infty$ and $\lambda = 0$.

3.1 Limit of fixed length Higgs fields

For $\lambda \rightarrow \infty$ the radial mode freezes to the value $\rho = 1$ such that the action effectively becomes

$$S = S_g - \kappa \sum_{x, \mu \in \Lambda} \text{Tr}(\sigma_x U_{x, \mu} \sigma_{x+\mu}) \quad . \tag{3.1}$$

The corresponding phase structure has been anticipated^[2] and determined in Monte Carlo calculations^[4].

For $\beta \rightarrow \infty$ the system becomes a $SU(2)$ spin system in four dimensions with a symmetry breaking 2nd order phase transition at $\kappa_c(\beta=\infty) \approx 0.355 \pm 0.010$ ^[4]. The Higgs phase (large β , sufficiently large $\kappa > \kappa_c$) is, however, analytically connected with the so-called confinement phase^[6] (for $SU(2)$: arbitrary β , $\kappa < \kappa_c$). This is due to the complete breaking of the symmetry by the Higgs fields in the fundamental representation. For $\beta = 0$ one may solve the model analytically by implementing a gauge where all $\sigma = 1$. One is left with the one-link integral which may be expressed through modified Bessel functions, i.e. analytic and smooth.

The critical line $\kappa = \kappa_c$ in the (κ, β) plane starts at $\kappa_c(\beta=\infty)$ and continues, bending slightly to higher κ values for decreasing β , until it disappears near $\kappa \approx 0.70$ and $\beta \approx 1.2$ ^[4].

3.2 No Φ^4 coupling

The case $\lambda = 0$ leaves us with the pure gauge-boson theory without bosonic quartic self-interactions; the action

$$S = S_g + \sum_{x,y \in \Lambda} \Phi_x^\dagger Q_{xy} \Phi_y \quad (3.2)$$

is Gaussian in the Φ fields and may be integrated over the Φ variables explicitly, i.e.

$$\begin{aligned} \int \{ d\Phi^\dagger d\Phi dU \} \exp[-S_g - \sum \Phi_x^\dagger Q_{xy} \Phi_y] \\ = \int \{ dU \} \exp[-S_g] [\det Q(\kappa, U)]^{-1} . \end{aligned} \quad (3.3)$$

3.2.1 Free bosons ($\beta \rightarrow \infty$)

For $\beta \rightarrow \infty$ we are left with the theory of two free scalar particles on the lattice. Fourier transformation^[7] leads to the solution for

$$\det Q^{-1} = \exp\{ - \text{Tr}(I) \sum_{p \in \Lambda_p} \log(1 - 2\kappa \sum_{\mu} \cos p_{\mu}) \} \quad (3.4)$$

where p assumes values of the corresponding momentum space lattice Λ_p , e.g. $p_x = 2\pi n_x / N_x$ for $n_x = 0, 1, 2, \dots, N_x - 1$ for periodic boundary conditions in direction x (N_x sites in x -direction) or, for antiperiodic boundary conditions: $n_x - 1/2 = 0, 1, 2, \dots, N_x - 1$.

Similarly one obtains the propagator

$$\begin{aligned} \langle \Phi_x^\dagger \Phi_y \rangle &= (Q^{-1})_{xy} \\ &= \text{Tr}(I) N_s^{-1} \sum_{p \in \Lambda_p} \exp(ip(x-y)) / (1 - 2\kappa \sum_{\mu} \cos p_{\mu}) , \end{aligned} \quad (3.5)$$

where N_s denotes the number of sites.

For periodic boundary conditions the propagator has a pole singularity at $\kappa = 1/8$, contributing $\text{Tr}(1)/N_s(1-\kappa/\kappa_c)$, for boundary conditions antiperiodic in the μ direction the closest pole is at $\kappa = 1/(6+2\cos(\pi/N_\mu))$. In the thermodynamic limit the singularity at $\kappa_c=1/8$ is of the weak type contributing $(\kappa-1/8)\log(\kappa-1/8)$ and there exists a finite value of $\langle \Phi^2 \rangle$ at $\kappa=\kappa_c=1/8$. For $\kappa > 1/8$ the action is unbounded from below and the theory not well defined. Thus, like for the fermions, $\kappa_c(\beta=\infty) = 1/8$ is an upper bound for the free theory. For small masses a.m one has $\kappa \approx 1/(8+a^2m^2)$ where a is the lattice spacing.

On an infinite lattice we obtain the series expansion

$$\begin{aligned} \langle \Phi^\dagger \Phi \rangle = & 2 + 16 \kappa^2 + 336 \kappa^4 + 10240 \kappa^6 \\ & + 380240 \kappa^8 + 15878016 \kappa^{10} + O(\kappa^{12}) \end{aligned} \quad (3.6)$$

3.2.2 No gauge field action ($\beta = 0$)

For $\beta = 0$ we perform the variable transformation $U_{x,\mu} \rightarrow \sigma_x U_{x,\mu} \sigma_{x+\mu}^\dagger$ (the so-called U-gauge) such that the action becomes

$$S = \sum_{x \in \Lambda} \rho_x^2 - \kappa \sum_{x, \mu \in \Lambda} \rho_x \rho_{x+\mu} \text{Tr} U_{x,\mu} \quad (3.7)$$

This action describes a specific spin system with random interaction, similar to spin-glass systems^[8] with spins $\rho_x \in \mathbb{R}^\dagger$, measure $\exp(-\rho^2)\rho^3 d\rho$, and random coupling $\kappa_{x,x+\mu} = \kappa \text{Tr} U_{x,\mu} \in [-2\kappa, 2\kappa]$ distributed according to $SU(2)$.

One may obtain a power series in κ for e.g. $\langle \Phi^2 \rangle$ or, in a certain approximation, even a closed expression. We introduce first

$$\begin{aligned}
 Z(\alpha, \kappa) &= \int \{ d\Phi^\dagger d\Phi dU \} \exp[-\alpha \sum_x \Phi_x^\dagger Q_{xy}(\kappa) \Phi_y] \\
 &= \alpha^{-2N} Z(1, \kappa)
 \end{aligned} \tag{3.8}$$

and remark that

$$\begin{aligned}
 -1/N_s \partial \log Z(\alpha, \kappa) / \partial \alpha \Big|_{\alpha=1} &= \langle \Phi^\dagger \Phi \rangle - 8 \kappa \langle \Phi^\dagger U \Phi \rangle \\
 &= 2,
 \end{aligned} \tag{3.9}$$

where $\langle \Phi^\dagger U \Phi \rangle$ denotes the nearest neighbour coupling term, and thus obtain the useful identity

$$\begin{aligned}
 \langle \Phi^\dagger \Phi \rangle &= 2 + 8 \kappa \langle \Phi^\dagger U \Phi \rangle \\
 &= 2 + \kappa/N_s \partial \log Z(1, \kappa) / \partial \kappa.
 \end{aligned} \tag{3.10}$$

Now to determine $Z(\kappa) \equiv Z(1, \kappa)$ one may integrate over the gauge fields first, resulting in

$$\begin{aligned}
 Z(\kappa) &= \int \{ \rho^3 d\rho \} \prod_{x, \mu \in \Lambda} [I_1(2\kappa \rho_x \rho_{x+\mu}) / \kappa \rho_x \rho_{x+\mu}] \\
 &= \int \{ \sigma^3 d\sigma \} \prod_{x, \mu \in \Lambda} [\sum_n k^{2n} (\sigma_x \sigma_{x+\mu})^{2n} / n!(n+1)!]
 \end{aligned} \tag{3.11}$$

(I_1 denotes the modified Bessel function), and collect order by order in κ the non-vanishing contributions.

On the other hand, one might as well first perform the Gaussian integral over the boson variables, giving

$$Z(\kappa) = \int \{dU\} \det Q^{-1} \\ = \int \{dU\} \exp[-\text{Tr} \log Q] \quad (3.12)$$

Since Q is of the form $(1 - \kappa M)$ this leads to the so-called "hopping expansion"[7].

$$Z(\kappa) = \int \{dU\} \exp\left[\sum_{n>0} \kappa^{2n} \text{Tr}(M^{2n}) / 2n\right], \quad (3.13)$$

where $\text{Tr} M^{2n}$ may be represented graphically by sums over all contributing closed paths of length $2n$. This gives rise to the trace over the ordered product of U fields along the loops. For $\beta = 0$ only a subset of loops survives the U integration, essentially those graphs, where along each link one finds an even number of bosons running along, including spike-like objects. Most of the paths may be constructed from contributions like $\rightleftharpoons$ where two bosons run anti-parallel, but there are some paths or parts of paths with bits of type $\Longrightarrow$. The latter type is due to products of paths of same shape, e.g. a plaquette, which is encircled by a boson twice; these paths are of higher order in κ and presumably not important (cf. the discussion in ref. 9).

If one neglects these latter paths one may sum the resulting series explicitly. This has been done for Wilson-type fermions for arbitrary values of the parameter r (from the factor $(r + \gamma_\mu)$ in the fermion action)[10]. For bosons the factor $(r^2 - 1) \cdot \kappa^2$ has to be substituted by κ^2 , and one has to take into account a trivial multiplicative factor of 4. Thus, essentially, the bosonic $\langle \Phi^\dagger \Phi \rangle$ at κ is $\langle \Psi^\dagger \Psi \rangle$ for naive Wilson fermions ($r=0$) at ik . One obtains

$$\langle \Phi^\dagger \Phi \rangle = 14 / [3 + 4\sqrt{1 - 28\kappa^2}] \quad (+ \text{neglected paths}) \quad (3.14)$$

The term due to the neglected paths starts with order κ^8 and is a small modification. The singularity is of square-root type at $\kappa_c^2 = 1/28$ or $\kappa_c \approx 0.1890$, where $\langle \Phi^2 \rangle$ assumes the value 4.6667.

Thus we expect that for this case $\lambda = 0$ the phase diagram looks very much like that for the fermions, with a phase boundary at values $\kappa = \kappa_c(\beta)$ running from ≈ 0.1890 at $\beta = 0$ to 0.125 as $\beta \rightarrow \infty$.

3.3 Limits in κ

For $\kappa = 0$ the Higgs fields decouple (the mass becomes infinite) and we are left with the pure gauge field theory.

For $\lambda \neq 0$ and $\kappa \rightarrow \infty$ all spins tend to be parallel in group space which is possible since the Higgs fields are in the fundamental representation. Therefore we expect $\langle p_x p_{x+\mu} \rangle \sim \langle p^2 \rangle$ to become frozen in the minimum of the effective potential

$$\lambda (p^2 - 1)^2 - 8 \kappa p^2,$$

i.e.

$$\text{i.e. } \langle p^2 \rangle \underset{\text{large } \kappa}{\sim} 4\kappa/\lambda \text{ for } \lambda \neq 0. \quad (3.15)$$

4 RESULTS FROM MONTE-CARLO SIMULATION

We work with lattices of size 4^4 and 8^4 , using the lattice of smaller size to obtain the gross structure of the phase diagram and the larger lattice for more precise measurements at selected values of (β, κ) .

In the calculations we did not fix the gauge (like e.g. transforming away the Higgs SU(2) degree of freedom). The only exception was at $\beta=\infty$ where we took the trivial U-field configuration, all links equal to the unit element.

The results presented in the figures are for periodic boundary conditions for the Higgs fields. We work with antiperiodic boundary conditions as well, but it turned out that there the results for 4^4 differ much more from 8^4 than for periodic boundary conditions; also the periodic boundary conditions enhance the sensitivity to singular behaviour, i.e. the hysteresis effect. This is quite useful for the determination of the phase transition lines. For 8^4 there is little difference between the values obtained with periodic or anti-periodic

boundary conditions though. In both cases there are "fake poles" in $\langle \Phi^2 \rangle$ at or closely above κ_c for $\lambda = 0$ (cf. the discussion in sec.(3.2)) although it should be well defined at κ_c in the thermodynamic limit. These poles are closer for periodic boundary conditions which is a helpful feature in determining the phase boundary in this case.

We employ the Metropolis method with 6 hits per icosahedral variable (U and σ) and 3 hits for the p variables. We do not choose the new p values out of a set distributed according to the p -measure; the "effective" potential of the p variables has always a single minimum^[11]. Thus we just allow for random changes whose maximal size is optimized to get an acceptance rate of 0.5 on the average. (Note that the eventual symmetry breaking occurs in the $SU(2)$ variable σ and not in the variable p .) The link and site variables are upgraded in randomly chosen order. The CPU time requirements on a CDC 7600 are 2 seconds for one update of all variables on an 8^4 lattice. The program is an extension of the pure gauge program discussed in ref.12.

For hysteresis runs we usually perform 20 MC-iterations at a point before moving to the next and use the last 10 configurations to obtain an expectation value. For precise measurements at selected points we do several hundred iterations at a given value of (β, κ) before we start measuring over another hundred consequent 8^4 configurations.

Fig. 1a shows the thermal cycles of $\langle \Phi^2 \rangle$ for $\lambda = 0.5$ at different values of β . For large κ one finds the expected behaviour (cf. sec. 3.3). We found it useful to plot also the ratio of the averages $\langle \Phi^\dagger U \Phi \rangle / \langle \Phi^2 \rangle$ (see fig. 1b); for $\lambda \rightarrow \infty$ this is of course just the link-term expectation value. One finds that above $\beta \approx 0.8$ a hysteresis develops indicating a phase transition line. The hysteresis vanishes quickly if one increases the number of iterations at each value of κ .

Fig. 2a shows the result of longer runs on an 8^4 lattice, for the coupling values $\lambda = 0.5$, $\beta = 2.25$ and different values of κ . Near the critical point $\kappa_c \approx 0.27$ we notice a significant slowing down of the relaxation time. Whereas for the values $\kappa \leq 0.26$ and $0.28 \leq \kappa$ around 40 iterations per point are sufficient to obtain equilibrium values we need more than 150 for values close to 0.27. The full circles in the figure are mean values over the last 10 configurations. Fig. 2b shows the relaxation behaviour over 200 iterations at $\kappa = 0.27$, starting from a cold ($\kappa=0.28$) and hot ($\kappa=0.26$) configuration respectively. For the first 100 iteration the behaviour resembles that of a first order transition,

but then the configuration mean values approach each other more or less continuously and stay together further on. From this and a similar result at $\beta = 2.5$ we are led to the conclusion that the transition very likely is second order all along the critical line - as it is for the fixed length case^[4].

We checked that for large λ (e.g. $\lambda \geq 5$) the results approach those of ref. 4 for frozen radial degree of freedom (for comparison notice that $\kappa = \gamma/2$ where γ is the coupling introduced in ref. 4).

In Fig. 3 we show the phase-diagram for different values of λ . With decreasing λ the phase-transition line straightens and starts to extend to smaller values of β . We expect the accuracy of the κ_c values in the figure to be of the order of $\sim 5\%$.

Finally for $\lambda = 0$ the critical line $\kappa_c(\beta)$ provides a boundary to the physical region. MC-runs for $\lambda = 0$ and $\kappa \geq \kappa_c(\beta)$ diverge in the length of the Higgs field indicating that the effective potential is unbounded from below. From this we conclude that the phase diagram for $\lambda = 0$ is very much like that for fermion-gauge systems^[9,13]. Thus this system should provide a suitable (and cheaper) laboratory for the study of methods to deal with fermions; related work is in progress.

Fig. 4 exhibits values of $\langle \Phi^2 \rangle$ for $\lambda = 0$, $\beta = 0$ versus κ compared to the analytic strong coupling expansion (cf. sec. (3.2)). From the numerical results we obtain $0.185 < \kappa_c < 0.190$. The correction due to the finite size pole $2/N_s(1-\kappa/\kappa_c)$ (cf. our discussion in sec. (3.2)) is negligible at the values of κ where we determined $\langle \Phi^2 \rangle$, only at the last point $\kappa=0.185$ we note a slight discrepancy due to the influence of the finite size contribution.

We also investigated the situation for $\lambda = 0$ and negative β . At $\beta = -2.25$ we obtain $\kappa_c \approx 0.22$; at $\beta \rightarrow -\infty$ we apply the transformation eq.(2.10) to the trivial link configuration (all links unity); the expectation value of $\text{Tr } U_p$ then is -2 . We find a structure like for $\beta = \infty$ but now κ_c is at ≈ 0.25 . We conclude that the phase boundary for $\lambda = 0$ runs through from $\beta = -\infty$ to $+\infty$.

5 DISCUSSION AND CONCLUSION

We have obtained the phase diagram for a non-abelian $SU(2)$ gauge-Higgs system where the Higgs fields exploit full radial degree of freedom.

For $\infty \geq \lambda > 0$ the critical line changes smoothly, the end point for the spin-system between 0.355 and 0.125 and the other end point moving towards smaller β and κ values for decreasing λ . However, the theory obtained for $\lambda = 0$ is no continuous limit of $\lambda \rightarrow 0$ but differs drastically from the situation $\lambda > 0$. There one has an analytic connection between the so-called Higgs phase (large β , $\kappa > \kappa_c(\beta)$) and the confinement phase ($\kappa < \kappa_c$). For $\lambda = 0$ only the confinement phase survives whereas the other phase is unphysical. The effective ϕ^2 -term in the Lagrangian is negative and no ϕ^4 -term stabilizes the theory. It turns out that this holds for all values of β . Still this theory is asymptotically free^[14] and one will obtain its continuum limit for $g \rightarrow 0$, $\kappa \rightarrow 1/8$.

In fig. 5 we exhibit the critical structure of the theory for $\beta = \infty$ and 1. The first situation is that of the usual Φ^4 theory where κ is playing the role of $1/m^2$. For $\beta = \infty$ and $\lambda = 0$ both regions, $\kappa > \kappa_c$ (the phase of spontaneous breaking) and $\kappa < \kappa_c$ (the disordered phase) exist; for $\lambda = 0$ there is just the disordered phase left.

Switching on gauge interactions there is no order-parameter, the Higgs phase is connected to the confinement phase at smaller values of β . Otherwise the change is not dramatic, and, up to numerical differences, the structure at $\lambda=0$ remains the same as for $\beta = \infty$.

Like for the fixed-length case^[4] we found evidence that the critical line extending into the (β, κ) region is second order everywhere. Based on work by Coleman and Weinberg^[15] it has been conjectured that the phase transition might become of first order for $\beta < \infty$. The arguments are based on a perturbative approach for the continuum theory. It is not clear to us that this has to hold for the lattice theory. We did not check for a discontinuity in the mass of the vector boson as one crosses the critical line - for this one would need substantially more computer-power - but from the behaviour of the free energy alone we see no indication of a first order transition.

Munehisa et al.^[3] have investigated the Z_2 gauge-Higgs model with variable length and also found a second order transition line starting at $\beta = \infty$, continuing towards smaller β until it joins the 1st order transition line originating from the pure gauge theory, continuing further 1st order. These authors suggest that the radial degree of freedom might be responsible for this type of transition.

In our case, i.e. a continuous, non-abelian gauge group, the results of the MC-simulation lead us to the conclusion that the phase transition has to do more with the gauge group degree of freedom of the Higgs fields than the radial one. Fluctuation and relaxation behaviour of the link-term is much conspicuous than that of the purely radial $\langle\Phi^2\rangle$ -term. In the ratio $\langle\Phi^\dagger U\Phi\rangle/\langle\Phi^2\rangle$ where one expects a damping of the fluctuation due to $\langle\Phi^2\rangle$ one notices less fluctuation around the current mean value but still the same long time relaxation behaviour to the final value, whereas $\langle\Phi^2\rangle$ approaches its equilibrium value much faster.

Now if the whole critical line actually is second order, what would be the continuum theories that one ends up with when approaching the line? Brézin and Drouffe^[16] investigate the 2nd order end point of the Z_2 gauge-Higgs system by mean field techniques and conclude that it leads to a Φ^4 theory. One might argue that this is also the case all along the critical line for our system. Approaching the line then one always would end up with a Φ^4 theory. The only way to obtain the continuum limit of an interacting gauge-Higgs theory would be (like in the gauge-fermion case) to fix some physical (mass) parameter and move towards $g \rightarrow 0$, $\kappa \rightarrow \kappa_c(\beta=\infty)$ in this way.

As is well known^[15] for $\lambda > 0$ the discussed gauge-Higgs model is not asymptotically free (unless one adds sufficiently many fermions). There is even evidence that the limit of the regularized Φ^4 theory is a non-interacting free theory^[17]. One option therefore is that taking the continuum limit on the critical line leads to trivial theories everywhere.

Some clarification could be obtained by investigating the renormalization flow by measuring expectation values of more complex operators, like e.g. the propagator of bound states of Higgs bosons. (Note that there can be no free Higgs boson due to the gauge group confinement.) This, however, would require substantially more computer time compared to the present work.

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FIGURE CAPTIONS

- Figure 1: (a) Thermal cycles of $\langle \Phi^2 \rangle$ increasing and decreasing κ for $\lambda = 0.5$ and different values of β . The origins of the different curves are shifted with respect to each other for clearer presentation. The hystereses for $\beta = 0$ and ∞ have been obtained on 8^4 lattices, the other ones on 4^4 lattices. (b) Like in (a) for the quantity $\langle \Phi^* U \Phi \rangle / \langle \Phi^2 \rangle$.
- Figure 2: (a) The results of long runs at $\lambda=0.5$, $\beta=2.25$ and different values of κ . (b) Relaxation behaviour for $\langle \Phi^2 \rangle$ and $\langle \Phi^* U \Phi \rangle$ at $\lambda=0.5$, $\beta=2.25$ and $\kappa=0.27$. The start configurations are from thermal cycles, i.e. at $\kappa=0.26$ (hot) and 0.28 (cold). All results are obtained on an 8^4 lattice.
- Figure 3: The phase diagram in the (β, κ) plane for different values of λ ($0.$, 0.1 , 0.5 , $1.$, $5.$, ∞).
- Figure 4: $\langle \Phi^2 \rangle$ as obtained for MC-runs at $\lambda=0$, $\beta=0$, and different values of κ on an 8^4 lattice. The full circles denote the MC-results and the curve gives the analytic hopping parameter prediction. The difference between the MC-value and the value from curve due to the analytic expression at 0.185 is due to the implicit finite-size pole contribution discussed in sec. 3.2.
- Figure 5: The critical line of the theory for $\beta=\infty$ (the Φ^4 -theory with two scalars) and for $\beta=1$. The theory is undefined in the shaded region, for $\lambda=0$ this is the case for $\kappa > \kappa_c$.

Fig. 1b

Fig. 1a

Fig. 2a

Iterations Fig. 2b

Fig. 3

κ Fig. 4

Fig. 5