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Energy loss distributions for single particles and several particles
in a thin Silicon absorber

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Abstract: The energy-loss distributions for protons, with kinetic energies between 220 and 850 MeV, and for several relativistic particles produced at incoming beam momentum of 115 GeV/c, have been measured in an ion-implanted silicon detector of 300 μ m thickness. The data are well described by an improved energy-loss distribution which takes into account the long-distance collisions disregarded in both the Landau and Vavilov distributions.

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1. Introduction

The energy-loss spectrum of a charged particle traversing a thin silicon detector is given, to a first approximation, by the Landau[1] and Vavilov[2] theories, which are based on Rutherford scattering on free electrons.

In very thin ($<300\mu\text{m}$) silicon detectors, significant deviations from the Landau-Vavilov theory are expected. These deviations have already been observed [3] for single relativistic pions and protons in the momentum range 0.7 to 115GeV/c. As consequence, the energy-loss spectrum of n relativistic particles traversing a very thin silicon device is expected to deviate from that predicted by the Landau and Vavilov theories.

The purpose of the present investigation is to measure the energy-loss spectra of protons with kinetic energy lower than 850MeV (where a strong β dependence is expected) and the energy-loss spectra of several relativistic particles.

2. Energy-straggling

The statistical nature of the ionization process on the passage of a fast charged particle through matter results in large fluctuations of the energy-loss in absorbers which are thin compared with particle range.

The calculation of these fluctuations was performed first by Landau [1] and subsequently Vavilov [2] provided a more accurate solution by introducing the kinematical limit on the maximum transferable energy in a single collision. The Vavilov distribution approaches the Landau spectrum for :

$$k=\xi/E_m \rightarrow 0$$

where

$$\xi=0.1535(z^2 Z \rho x)/(\beta^2 A) \text{ MeV} \quad (1)$$

z and βc are the charge and the velocity of the incoming particle, Z , A and $\rho(\text{g/cm}^3)$ are the atomic number, atomic weight and density of the material, x is the crossed thickness of the substance in cm and $E_m = 2m_e c^2 \beta^2 \gamma^2 [1 + 2m_e \gamma^2 / m + (m_e / m)^2]^{-1}$ is the maximum transferable

energy in a single collision between the incoming particle of mass m and the atomic electron of mass m_e . The above condition is already satisfied for $k \approx 0.06-0.08$ (see [3], [4]).

The improved energy-loss distribution is given by [3],[5],[6],[7],[8]:

$$f(\epsilon, x) = 1/(\sigma\sqrt{2\pi}) \int_{-\infty}^{+\infty} f_{l,v}(\epsilon', x) \exp(-(\epsilon - \epsilon')^2/2\sigma^2) d\epsilon' \quad (2)$$

where ϵ is the actual energy-loss when a thickness x of material is traversed, $f_{l,v}(\epsilon, x)$ are either the Landau or the Vavilov distribution.

The standard deviation σ of the convolving gaussian function can be computed using the expression of Shulek et al.:

$$\sigma^2 = (8/3) \xi \sum_i I_i (Z_i/Z) \ln(2m_e c^2 \beta^2 / I_i) = (8/3) \xi F(\beta) \quad (3)$$

where $F(\beta) = 10^{-3} (2.319 + 0.670 \ln \beta)$ MeV in Si, I_i is the ionization potential of the i th shell [9], Z_i is the number of electrons in the i th shell of the stopping material and the summation is carried out over those shells for which $I_i < 2m_e c^2 \beta^2$. The $F(\beta)$ value given below has been computed assuming that all the shells satisfy this limit.

For $k < 0.06$, the energy-loss distribution sensed by a Si-detector is simply given by :

$$f_{ex}(\epsilon, x) = 1/(\sigma_t \sqrt{2\pi}) \int_{-\infty}^{+\infty} f_l(\epsilon', x) \exp(-(\epsilon - \epsilon')^2/2\sigma_t^2) d\epsilon' \quad (4)$$

where $\sigma_t^2 = \sigma^2 + \sigma_{\text{noise}}^2$, and σ_{noise} is the standard deviation of the gaussian noise distribution, introduced by the detector and its coupled electronics.

The Landau distribution is given by :

$$f_l(\epsilon, x) = (1/\xi) \phi(\lambda)$$

where the function $\phi(\lambda)$ is a universal function of the dimensionless variable λ :

$$\lambda = (1/\xi) (\epsilon - \langle \epsilon \rangle) - \beta^2 - \ln(\xi/E_m) - 1 + C = (1/\xi) [\epsilon - (\epsilon_{mp} - \xi \lambda_0)] \quad (5)$$

where C is the Euler constant 0.577215, λ_0 is the value for which ϕ is a maximum, $\langle \epsilon \rangle$ is the mean energy-loss, ϵ_{mp} is the most probable energy-loss for the Landau distribution. The function $\phi(\lambda)$ has been tabulated by Boersch-Supan [10] and more recently by Kolbig and Schorr[11]. The full-width-at-half-maximum of the Landau distribution is $\approx 4\xi$.¹

The mean energy-loss, $\langle \epsilon \rangle$, can be computed from the Bethe-Bloch formula :

$$\langle \epsilon \rangle = \xi [\ln(2m_e \beta^2 \gamma^2 E_m / I^2) - 2\beta^2 - \delta - U] \quad (6)$$

where I is the mean excitation potential of the atoms of the substance ($I=172$ eV for Si), U is a term due to the non participation of the inner shells for very low velocities of the particle, δ is the correction due to the density effect. U is negligible for $\beta > 0.3$ [6], [13], [14]. The density effect has been calculated by Sternheimer [9], [15], [16].

Assuming $\lambda_0 = -0.225$, from eq (5) and (6) we get :

$$\begin{aligned} \epsilon_{mp} &= \xi (0.198 + \beta^2 + \ln(\xi/E_m) + \langle \epsilon \rangle / \xi) = \\ &\xi (0.891 + \ln[m_e c^2 (10^6 \text{ eV}) / I^2] - \beta^2 + 2\ln(\beta\gamma) + \ln(\xi) - \delta) \quad (7) \end{aligned}$$

where ξ is expressed in MeV, and $\beta\gamma = P/mc$ (P is the momentum of the incoming particle).

For relativistic heavy particles with $\beta\gamma > 30$ (namely protons with momentum > 28 GeV/c and pions with momentum > 4 GeV/c) and for Si absorber, eq (7) may simply be written as :

$$\epsilon_{mp} = \xi (20.936 + \ln(\xi)) \quad (8)$$

The energy-loss distribution, $\Lambda(\epsilon, n)$, of n parallel relativistic particles sensed by a Si-detector of thickness x , is given by :

¹ From [10] we have : FWHM = 3.98 ξ . However in a successive computation it was found [12]: FWHM = 4.02 ξ and $\lambda_0 = -0.229$ and in [11] $\lambda_0 = -0.2227829812564$.

$$\Lambda(\epsilon, n) = f_{ex}(\epsilon, nx) \quad (9)$$

namely by the convolution of a Gaussian of variance $\sigma_t^2 = \sigma_{noise}^2 + \sigma_n^2$ (where $\sigma_n = \sigma\sqrt{n}$), with a Landau distribution where $\xi_n = n\xi$ and $\epsilon_{n,mp} = n\xi(20.936 + \ln(n\xi)) = n(\epsilon_{mp} + \xi \ln(n))$. The parameters σ , ξ , and ϵ_{mp} are those computed for a single relativistic particle traversing a thickness x .

3. Energy-loss measurements for slow protons

3.1. Experimental method

In the present investigation, energy-loss has been measured for protons in the kinetic energy range between 220 and 850 MeV (see Table I), at Saturne II in Saclay. The momentum spread, $\Delta P/P$, was never more than 0.1%. A well-defined momentum is important, particularly at low values, since any momentum spread broadens the energy-loss distribution and thereby masks the effect due to long-distance collisions.

An Enertec ion-implanted passivated-silicon junction detector with an active area of 1.0 cm² and a thickness of 300 ± 5 μ m was used. Total depletion of the detector was achieved at reverse bias of about 100 V. The initial leakage current was 16 nA. The signal processing from this detector and the experimental arrangement have already been described in a previous paper [3].

The detector, together with its associated electronics, had a gaussian noise distribution with a measured standard deviation, σ_{noise} , whose typical value was 5.0 ± 0.4 keV. The exact value was determined separately before each data taking at the different energies.

A Ru β^- source has been used for calibration purposes. It had been determined that the value of ϵ_{mp} for relativistic electrons is 86.8 ± 2.8 keV on the same scale that yields the canonical value of 84.0 ± 2.8 keV for relativistic pions, once only electrons with kinetic energy greater than 1 MeV are selected [3].

3.2. Data Analysis

The experimental energy-loss distributions for each kinetic energy were fitted to the Landau probability density convolved with a Gaussian, eq (4), with free parameters σ , ξ and ϵ_{mp} . The details of the fitting procedure and examples of energy-loss spectra have been given elsewhere[3].

Some deviations from the model are observed, as expected, in the tail, due to the neglect of escaping δ rays and the effect of the maximum energy transfer to electrons, not taken into account by the Landau distribution. In order to avoid a possible bias in the fitted parameters, we have varied the upper limit to which each experimental distribution was fitted. The best fits had χ^2 -probabilities of the order of 50% or better and were obtained by fitting up to $\epsilon_{max} = \epsilon_{mp} + \alpha W$, where W is the full width at half maximum and the values of α are between 1.5 and 2.5. However, the fitted values of σ , ξ and ϵ_{mp} determined at the best χ^2 did not differ, within the error of their determination, from those of the complete fit.

In fig 1 the fitted values of the parameter ξ are shown. The continuous line is the calculated curve for incoming protons, in 300 μ m of silicon. The agreement between experiment and eq(1) seen in fig 1 indicates that our calibration of the energy scale for the data is correct, in average, to within 4%.

Table 1 gives a list of the fitted values of ξ , σ and the most probable energy-loss of the Landau distribution, ϵ_{mp} . The observed values of ϵ_{mp} are in general agreement, within the experimental errors, with those calculated from eq(7) following the Landau theory, provided the values of $\langle \epsilon \rangle$ are computed using the Bethe-Bloch formula given in eq(6). The most probable energy-loss, E_{mp} , of the overall straggling distribution defined by eq(2), is about 3% higher than ϵ_{mp} due to the folding of the Gaussian distribution (Table 1).

In fig 2, we show a plot of the values obtained for the parameter σ in this experiment and for protons and pions from[3]. The lines are the corresponding theoretical predictions made by Shulek et al. (eq.3). The quoted errors take into account the uncertainty in determining the noise contribution to σ_t .

4. Multiparticle Production

4.1. Experimental set-up

Multiparticle energy-loss measurements have been conducted in the H_3 beam of the CERN SPS using the experimental arrangement sketched in fig 3. The intensity of the incoming beam was 2×10^4 p/burst at a momentum of 115 GeV/c.

The geometrical acceptance (see fig 3) is defined by 16 mm diameter hole in a scintillator A (with an area of $40 \times 40 \text{ cm}^2$) mounted 20 cm downstream from the 5 mm thick lead target. This scintillator is placed in anti-coincidence with one upstream and two downstream ones, respectively S_1 and PM_1 and PM_2 . The half-angle of the forward acceptance cone subtended by the hole is 2.3° at the center of the target. The three latter scintillators have an area of $1 \times 1 \text{ cm}^2$. The restriction of our acceptance to a forward cone of 2.3° half-angle reduces the background of non-relativistic protons² to less than a few percent.

Two ion-implanted Si-detectors, of active area $1 \times 1 \text{ cm}^2$ and $(300 \pm 5) \mu\text{m}$ thickness, are located downstream from the target. Pulses from Si_B are accepted only when they correspond to energy-losses greater than the most probable energy-loss of a relativistic particle. A trigger defined by $T = S_1 \cdot PM_1 \cdot PM_2 \cdot Si_B \cdot \bar{A}$ establishes that a beam particle enters the target and that at least one charged particle emerges in the very forward direction.

The microstrip detector is a $300 \mu\text{m}$ thick ion-implantation device with 50 strips of $200 \mu\text{m}$ pitch disposed horizontally, with an active area $1 \times 3 \text{ cm}^2$. The multiplicity of an interaction is found by examining the height of the pulse formed by linearly summing the discriminated signals from all the strips of the microstrip detector. An output pulse, M , is produced when the input pulse height corresponds to a multiplicity lower than four. Hence $T \cdot M$ allows to trigger on an event in which up to three secondary particles traverse the silicon detector Si_A . The energy-loss in the detector is recorded at different multiplicities by a 2249A Lecroy ADC using such a

² The momentum range of slow protons is between 300 and 1000 MeV/c. They have a roughly flat angular distribution between 40° around the beam direction.

trigger. The special purpose electronics associated with Si_A and Si_B have been described in [3].

The above trigger can select interactions where two or three particles are produced in forward direction. However the single beam particle can give a trigger, provided that its energy-loss is sufficiently large in Si_B . In this way the parameters of the single particle, for eq(9), can be fitted in the same data sample containing energy loss from two and three relativistic particles. A contamination from four- and even five-particle production is expected due to the finite 200 μ m resolving distance for two particles in the microstrip detector and to a few unconnected strips.

4.2. Data analysis

Data samples were taken by using the trigger described above. The percentage of triggers due to beam particles could be modified by varying the threshold of the discriminator connected to the detector Si_B .

The energy-loss spectra were fitted to a function

$$L(\epsilon) = \sum_{n=1,5} P(n) \Lambda(\epsilon, n) \quad (10)$$

where $P(n)$ is the percentage of events in which n relativistic particles traverse the silicon detector Si_A . The free parameters of the fit are $P(n)$, $n=1,5$, σ and ξ .³ The value of ϵ_{mp} ⁴ for a single relativistic particle has been assumed to be 84keV[3]. The gaussian noise contribution was measured before, during and after the data taking. Its value was found to be 4.3 ± 0.2 keV.

The large amount (between 12000 and 32000 pulse heights per data sample) and high resolution of data provide a very sensitive test of the energy-loss model, but also require considerable care in the fitting procedure

³ As already discussed in sec 2, the parameters σ and ξ are those of the single relativistic particle.

⁴ The relationship between $\epsilon_{n,mp}$ (i.e. the most probable energy-loss of the Landau distribution for n -relativistic particles) and ϵ_{mp} is discussed in sec 2.

in order not to introduce numerical and statistical biases. The details of how we detected and eliminated these biases are given in [3], where the same fitting procedures were used. The entire fitting procedure has been tested by generating Monte Carlo data with known parameter values and fitting it through the same procedures. Good fits were obtained for all data samples.

Table II gives the fitted values of the parameters $P(n)$ $n=1,5$, ξ and σ for the three data samples with different thresholds of Si_B . The three energy-loss spectra detected by the Si_A detector are shown in fig 4a, 4b and 4c respectively. The energy-loss distributions have been fitted between 60 and 500keV. The χ^2 -probabilities associated with the fits varied between 45 and 85%. The determined values of ξ and σ do not have any systematic dependence on the set of the $P(n)$ values. This is the expected behaviour when both the fitting procedure is correct and the energy-loss distribution is properly taken into account. The expected values for ξ (from eq(1)) and σ (from eq(3)) are 5.35 and 5.76 keV respectively and are in agreement with the fitted ones within the errors, although the fitted values of ξ seem to be systematically higher. For the different data samples, $P(n=1)$ varied between 0.08 and 0.52.

When the data were fitted over a range from 50 to 400keV, only a slight variation of the values of the fitted parameters ξ and σ was observed. In particular the parameter ξ was systematically lowered. However the χ^2 -probabilities never exceeded 15% in this case.

5. Conclusion

The fitted values of the parameter σ for incoming proton kinetic energies between 220 and 850MeV agree with those found in a previous experiment in the same energy range[3]. Thus they confirm that the collision-loss theories of Landau and Vavilov have to be modified to take into account the atomic binding of electrons. For a single charged particle this effect seems to be properly accounted for by eq(2), provided that σ is computed by the Shulek et al. formula (eq(3)).

The energy-loss spectrum of n -relativistic and spatially separated particles, traversing a silicon detector of thickness $x=0.03\text{cm}$, is well described by the improved energy-loss distribution. The parameters $\epsilon_{n,mp}$, ξ_n and σ_n agree with those computed by scaling the parameters of a single

relativistic particle traversing a thickness nx (i.e. eq(9)), although the fitted values of ξ here are systematically ($\approx 10\%$) higher than those obtained in the single particle data of reference[3]. This may be explained partly by uncertainties in calibration (evaluated at $\approx 6\%$ in reference[3]), and partly by contamination of non-relativistic particles in the data samples (which causes a broadening of the spectra).

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TABLE I

Values of ξ , σ , ε_{mp} and E_{mp} vs the incoming proton kinetic energy

Kin.En. MeV	ξ kev	σ kev	ε_{mp} kev	E_{mp} kev
220	15.8 $\pm$ 0.8	9.7 $\pm$ 1.2	206.8 $\pm$ 6.2	210.2 $\pm$ 6.2
350	11.9 $\pm$ 0.8	8.7 $\pm$ 1.3	149.8 $\pm$ 4.5	152.3 $\pm$ 4.5
600	8.5 $\pm$ 0.4	8.2 $\pm$ 0.7	111.0 $\pm$ 3.4	113.5 $\pm$ 3.4
700	8.6 $\pm$ 0.5	5.0 $\pm$ 0.9	105.6 $\pm$ 3.2	108.1 $\pm$ 3.2
850	8.2 $\pm$ 0.4	4.8 $\pm$ 0.9	99.5 $\pm$ 3.0	101.2 $\pm$ 3.0

TABLE II

Fitted values of $P(n)$, ξ and σ

	P(1) %	P(2) %	P(3) %	P(4) %	P(5) %	ξ keV	σ keV
a)	52.2 $\pm$ 0.6	13.5 $\pm$ 0.4	25.5 $\pm$ 0.6	7.8 $\pm$ 0.5	1.0 $\pm$ 0.4	5.9 $\pm$ 0.4	4.9 $\pm$ 0.6
b)	40.9 $\pm$ 0.5	18.6 $\pm$ 0.5	30.5 $\pm$ 0.6	8.7 $\pm$ 0.6	0.8 $\pm$ 0.5	6.1 $\pm$ 0.6	6.6 $\pm$ 1.7
c)	7.6 $\pm$ 0.4	25.2 $\pm$ 0.8	49.3 $\pm$ 1.3	15.6 $\pm$ 1.1	2.3 $\pm$ 0.9	6.2 $\pm$ 0.5	6.6 $\pm$ 1.2

FIGURE CAPTIONS

FIG 1 : Determined values of the parameter ξ . The continuous and broken curves are computed from eq(1) for protons and pions respectively. The general agreement between the fitted value and the predicted one indicates that the energy calibration had been correctly performed.

FIG 2 : Determined values of σ . The continuous and broken lines are the Shulek et al. predictions for protons and pions respectively, computed from eq(3). The pion ($\bullet$) and proton (x) data are from [3]. The proton (o) data are the present experiment.

FIG 3 : Experimental lay-out for the multiparticle energy-loss measurements.

FIG 4 : Energy-loss spectra including both single and several relativistic particles. The percentages, $P(n)$, of the number of particles produced in figg a,b and c are listed in TABLE II. The continuous lines are the fitted curves.

Fig. 1

Fig. 2

Fig.3

Fig.4

Fig. 4

b)

Fig.4