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NATURALLY VANISHING COSMOLOGICAL CONSTANT
IN $N = 1$ SUPERGRAVITY

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A B S T R A C T

For $N = 1$ supergravity theories we show that the choice of a particular class of Einstein spaces for the Kähler manifold of the hidden sector leads to a vanishing cosmological constant without unnatural fine tuning. The total scalar potential from the hidden and physical sector is positive definite. The resulting low energy softly broken global supersymmetry for the matter fields is thus the same as in the case of factorized superpotential models with a flat Kähler metric.

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The almost vanishing value of the cosmological constant is an old, interesting, but unsolved problem in gravity and more recently in supergravity theories. The experimental upper limit on that quantity is found to be extremely small ($\Lambda \sim 10^{-50} \text{ GeV}^4$). However, no convincing theoretical or symmetry argument is known to explain the almost vanishing of the cosmological constant. It is the purpose of the current paper to study that problem in the context of $N = 1$ local supersymmetric theories.

Some successful phenomenological implications seem to follow from supersymmetric theories¹⁾. For instance, the stability under radiative corrections of the well-known hierarchy between the two mass scales (M_X, M_W) appearing in grand unified theories has a natural explanation in supersymmetric theories due to the non-renormalization theorems²⁾. The splitting between the doublet and triplet parts of 5 Higgs in $SU(5)$ theory can be also understood in the framework of supergravity theories³⁾. Furthermore, the M_X, M_W hierarchy is naturally realized, assuming a radiative breaking of the $SU(2) \times U(1)$ subgroup^{4),5)}. It turns out that the radiative breaking is the simplest mechanism which does not destabilize the doublet-triplet hierarchy when the $SU(2) \times U(1)$ is embedded into a grand unification group⁶⁾. Moreover, another serious hierarchy problem seems to find its natural explanation in the framework of a supersymmetric theory, such as the smallness of non-perturbative effects in QCD which are parametrized by θ_{QCD} ($\theta_{\text{QCD}} < 10^{-9}$!)⁷⁾.

Although the GUT hierarchy problems may be naturally solved in supergravity theories, it is not possible to shed any light on the cosmological constant problem, at least in the case where the minimal super-Higgs mechanism⁸⁾ is assumed. In this work, we focus our attention on that specific problem. We will show that if a suitable non-minimal structure is introduced for the hidden sector (chiral supermultiplets which are related to the supersymmetry breaking and decouple in the flat limit), a vanishing value for the cosmological constant is automatically ensured. To this non-minimal structure of the hidden sector there corresponds a particular class of non-trivial Kähler manifolds which are actually Einstein spaces. Hence no fine-tuning of the value of the cosmological constant is necessary. In fact, the total potential (from the hidden sector and the usual matter sector) turns out to be positive definite and its value at the minimum is identically equal to zero.

In order to explain the main idea, let us first consider only one chiral multiplet (z, X) coupled to supergravity⁸⁾. In this simple case, the scalar potential is given by

$$V(z, z^+) = e^G [G_z G_{z z^+}^{-1} G_{z^+} - 3] \quad (1)$$

where G is a real function of z and z^+ , and

$$G_z \equiv \frac{\partial}{\partial z} G, \quad G_{z^+} \equiv \frac{\partial}{\partial z^+} G, \quad G_{zz^+} \equiv \frac{\partial}{\partial z} \frac{\partial}{\partial z^+} G$$

The minimal kinetic term corresponds to the case where ^{*})

$$G_{zz^+} = 1 \quad (2a)$$

or

$$G = zz^+ + h(z) + h^+(z^+) \quad (2b)$$

The mass of the gravitino is given by

$$m_{3/2} = e^{G/2} \Big|_{\text{at the minimum}} \quad (3)$$

and the condition for a vanishing cosmological constant implies the well-known fine tuning through the relation

$$G_z G_{z^+} - 3 = 0 \Big|_{\text{at the minimum}} \quad (4)$$

Note that the Kähler manifold curvature in the minimal case is zero ⁹⁾

$$R_{zz^+} \equiv \frac{\partial}{\partial z} \frac{\partial}{\partial z^+} \log G_{zz^+} = 0 \quad (5)$$

Therefore the minimal kinetic term case corresponds to a flat Kähler space with the following (mass)² sum rule ^{8), 9)}

$$m_A^2 + m_B^2 - 4m_{3/2}^2 = -2e^G \frac{G_z G_{z^+}}{(G_{zz^+})^2} R_{zz^+} = 0$$

^{*}) We work in units of $M = M_P/\sqrt{8\pi} = 2.4 \cdot 10^{18} \text{ GeV} \equiv 1/K$. When it is necessary, we will return to conventional units. The connection between the Cremmer et al. ⁸⁾ notation and the present one is given by $G \rightarrow -G$, $K/\sqrt{2} z \rightarrow -z$ and $\log g(z) \rightarrow h(z)$, where $g(z)$ is the superpotential. However, the separation of G into a superpotential part and a kinetic part is not necessary. These two parts always appear in the particular combination defined by G ⁸⁾.

because of the absence of curvature contributions. The same $(\text{mass})^2$ sum rule is valid for all Kähler functions G such that $R_{zz^+} = 0$, which corresponds also to the minimal case $G_{z^+z^+} = 1$ after a field redefinition⁸⁾

$$z' = f(z) \quad \tilde{h}(z') = h(z) \quad (7)$$

There are interesting cases where the scalar potential given by Eq. (1) is identically equal to zero, although the supersymmetry may be broken. Rewriting the scalar potential of Eq. (1) as⁸⁾

$$V = g e^{\frac{4}{3}G} G_{zz^+}^{-1} \mathcal{I}_z \mathcal{I}_{z^+} e^{-\frac{1}{3}G} \quad (8)$$

it follows immediately that the potential is identically equal to zero if

$$G = -\frac{3}{2} \log \left(f(z) + f^+(z^+) \right)^2 \quad (9)$$

Contrary to the minimal kinetic term case, there the Kähler manifold is not flat ($R_{zz^+} = 0$), since using Eq. (5) one finds that

$$R_{zz^+} = \frac{2}{3} G_{zz^+} \quad (10)$$

Therefore, this defines an Einstein-Kähler manifold. However, relation (10) is not sufficient to ensure the vanishing of the potential.

Note that in the positive kinetic energy domain defined by

$$\mathcal{D} : G_{zz^+} > 0 \quad (11)$$

supersymmetry is spontaneously broken and the gravitino mass is different from zero:

$$m_{3/2}^2 = e^G = \frac{1}{|f(z) + f^+(z^+)|^3}, \quad \forall z \in \mathcal{D} \quad (12)$$

From the explicit expression for G_{zz^+} , it follows that

$$\mathcal{D} : |f_z| \neq 0 \quad (f + f^+)^2 \neq \infty \quad (13)$$

So, the "gravitino mass" is zero only at the boundary points $(f+f^+) = \infty$. The fact that the scalar potential is degenerate ($V \equiv 0$) reflects an arbitrariness for the value of the gravitino mass. This ambiguity can be easily removed if one demands the flatness condition ($V \equiv 0$) only locally, around a point $z_0 \in \mathcal{D}$.

$$V \underset{z \rightarrow z_0}{\simeq} C |z - z_0|^{2n}, \quad n > 2 \quad (14)$$

In fact we can build a large class of positive definite potentials demanding the relation

$$\partial_z \partial_{z^+} e^{-\frac{1}{3}G} = \varphi_{zz^+}(z, z^+) \quad (15)$$

where the real function $\varphi(z, z^+)$ satisfies the conditions

$$\varphi_{zz^+}(z, z^+) \geq 0, \quad \forall z \in \mathcal{D} \quad (16a)$$

with $\mathcal{D}$ the positive kinetic energy domain ($G_{zz^+} > 0$), and

$$\varphi_{zz^+}(z_0, z_0^+) = 0 \quad (16b)$$

The general solution of Eq. (15) is then

$$G = -\frac{3}{2} \log(f + f^+ + \varphi)^2 \quad (17a)$$

with

$$G_{zz^+} = 3 \frac{|f_z + \varphi_z|^2 - \varphi_{zz^+}(f + f^+ + \varphi)}{(f + f^+ + \varphi)^2} > 0 \quad (17b)$$

Equation (17b) defines the positive kinetic energy domain $\mathcal{D}$.

The corresponding scalar potential is positive definite in $\mathcal{D}$, provided that $\varphi_{zz^+} \geq 0$ and $f + f^+ + \varphi > 0$, as can be easily seen from its analytic expression

$$V_0 = 3 \frac{\varphi_{zz^+}(f + f^+ + \varphi)}{|f + f^+ + \varphi|^3 [|f_z + \varphi_z|^2 - \varphi_{zz^+}(f + f^+ + \varphi)]} \quad (18)$$

At the minimum ($z = z_0$), the potential vanishes identically and the gravitino mass is well defined. If the potential is locally flat at z_0 , the fourth derivative of φ at $z = z_0$ must also vanish:

$$\varphi_{zzz^+z^+}(z_0, z_0^+) = 0 \quad (19)$$

In that case, the relation (10) between the curvature R_{zz^+} and the Kähler metric G_{zz^+} is still valid locally ($z \sim z_0$), and the curvature contribution to the $(\text{mass})^2$ sum rule⁹⁾ is minus four times the gravitino mass squared:

$$\Delta m^2 = -2 e^G \frac{G_z G_{z^+}}{(G_{zz^+})^2} R_{zz^+} \Big|_{z=z_0} = -4 m_{3/2}^2 \quad (20)$$

One may then understand how the $(\text{mass})^2$ sum rule can be satisfied with a non-vanishing gravitino mass (signal of supersymmetry breaking) and vanishing scalar masses, as is obvious from the local flatness of the potential at the minimum.

Let us now examine the physical interesting case where the usual matter fields are also coupled to supergravity. In general, the positivity properties of the potential are destroyed when the matter fields are coupled in an arbitrary way. The solution to this problem is to assume that G is the sum of two uncoupled functions $G(z, z^+)$, as in Eq. (17) and $\bar{G}(y^i, y_i^+)$, so that the potential becomes^{*)}

$$V = e^{G_T} \left[\frac{G_z G_{z^+}}{G_{zz^+}} - 3 \right] + e^{G_T} \bar{G}_i (\bar{G}^{-1})^i_j \bar{G}^j + \frac{1}{2} D^\alpha D_\alpha \quad (21a)$$

with

$$G_T = G(z, z^+) + \bar{G}(y^i, y_i^+) \quad (21b)$$

and, therefore, remains positive definite provided that $\bar{G}^i_j$ is a Hermitian positive matrix; the latter being a necessary condition for ensuring a meaningful kinetic term.

^{*)} We denote by y_i the scalar component of the usual matter superfields and by $\bar{G}_i = (\partial/\partial y_i)G$, $\bar{G}^i = (\partial/\partial y_i^+)G$ and $\bar{G}^i_j = (\partial/\partial y_j)(\partial/\partial y_i^+)\bar{G}$. $(\bar{G}^{-1})^i_j$ is the inverse of the metric $\bar{G}^i_j$. The D^α terms are the usual D term with $D^\alpha = g_{\alpha i} (T^\alpha)^i_j y^j$.

The vanishing minimum arises for $z = z_0$ as previously and

$$\begin{cases} \bar{G}_i (\bar{G}^{-1})^i_j \bar{G}^j = 0 \\ D^\alpha = 0 \end{cases} \quad (22)$$

The choice of minimal kinetic terms for the matter sector (y^i) leads to the following positive definite potential

$$V(z, y^i) = e^{\bar{G}} V_0(z, z^+) + e^{G_T} \left[\sum_i |h_i + y_i^+|^2 \right] + \frac{1}{2} D^\alpha D^\alpha \quad (23)$$

where $V_0(z, z^+)$ is given by Eq. (18). The total G_T function reads

$$G_T = -\frac{3}{2} \log(f + f^+ + \varphi)^2 + y^i y_i^+ + h(y^i) + h^+(y_i^+) \quad (24)$$

The absolute minimum of the potential (23) is zero and occurs at $z = z_0$ and $y_i^+ = -h_i$ (if there are solutions). Note that the condition $y_i^+ = -h_i$ implies automatically the vanishing of D terms because of the invariance of h under the internal group⁸⁾. The vanishing of the cosmological constant is thus automatically satisfied without any unnatural fine tuning^{*)}. The gravitino mass in that case is given by

$$m_{3/2}^2 = e^{G_T(z, z^+, y^i, y_i^+)} \Big|_{\text{at the minimum}} \quad (25)$$

In the flat limit, the hidden sector decouples and the scalar particle z remains massless for locally flat potentials. The potential for the matter fields becomes

$$V(y^i, y_i^+) = \sum_i \left| \frac{\partial g(y^i)}{\partial y^i} + m_{3/2} y_i^+ \right|^2 + \frac{1}{2} D^\alpha D^\alpha \quad (26)$$

^{*)} In Ref. 10), a particular G function is used for the hidden sector which gives rise to a flat potential. Namely, it corresponds to the particular choice of a function $f(z)$ which appears in Eq. (9), $f(z) = (\sqrt{3} + z)/(\sqrt{3} - z)$. However the authors of that reference do not discuss the ambiguity in the definition of the gravitino mass. Furthermore, the way the matter fields are coupled re-introduces the problem of the fine tuning of the cosmological constant.

where we have rescaled $h(y^i) = (1/M^2 m_{3/2}) g(y^i)$ in order to have a residual effect. The D term in that case is given by

$$D^\alpha = y^i (g^\alpha T^\alpha)_i^j y_j^\dagger \quad (27)$$

The form of the potential presented in Eq. (26) no longer depends on the specific form of the hidden sector, and gives rise to soft global supersymmetric breaking terms of the same form as in the $A = 3$ case or the case of a factorized form of the superpotential¹¹⁾, and therefore the supertrace formula for the matter fields remains the same as in the minimal case^{8),11)}:

$$\sum_B m_B^2 - 2 \sum_F m_F^2 = 2 N_Y m_{3/2}^2 - 2 \sum_i r_i m_{\lambda_i}^2 \quad (28)$$

where N_Y is the number of chiral multiplets in the Y sector and r_i is the dimension of the internal gauge group and m_{λ_i} the gaugino masses coming from a non-minimal choice of f_{AB} ^{8),11)}. In the gravity hidden sector, the bosonic degrees of freedom are massless, although their fermionic partners obtain masses equal to the gravitino mass ($m_{3/2}$). To better appreciate this fact, let us generalize our results to the case of N chiral superfields (z_a, χ_a) in the hidden sector. In that case, the corresponding scalar potential of Eq. (1) becomes

$$V = e^G [G_a (G^{-1})^a_b G^b - 3] \quad (29)$$

and may be rewritten as

$$V = \frac{9}{N^2} e^{\frac{N+3}{3}G} (G^{-1})^a_b \partial_a \partial^b e^{-\frac{N}{3}G} \quad (30)$$

The flatness of the potential in that case implies a particular G , so that

$$(G^{-1})^a_b \partial_a \partial^b e^{-\frac{N}{3}G} = 0 \quad (31)$$

For simplicity, we only examine the case of N chiral superfields which are singlets under the internal group. An obvious particular solution of Eq. (31) is

$$G = \sum_{a=1}^N -\frac{3}{2N} \log (f_a(z^a) + f_a^\dagger(z_a^\dagger))^2 \quad (32)$$

where $f_a(z^a)$ is a function of the z^a field only. The potential is identically zero, as in the one-superfield case. Here also, one can realize the locally flat potential requirement ($z_a \sim z_a^0$) by modifying G to

$$G = \sum_{a=1}^N -\frac{3}{2N} \log \left(f_a(z^a) + f_a^+(z_a^+) + \varphi_a(z^a, z_a^+) \right)^2 \quad (33a)$$

with

$$\begin{aligned} \partial_a \partial^a \varphi_a &\geq 0 \quad \forall z_a \in \mathcal{D} \\ \partial_a \partial^a \partial_a \partial^a \varphi_a &= 0 \quad \text{at } z_a^0 \end{aligned} \quad (33b)$$

All scalar fields are then massless, contrary to their fermionic partners and the gravitino whose masses satisfy the following (mass)² formula⁹⁾

$$\begin{aligned} -2 \sum_{F=1}^{N-1} m_F^2 - 4 m_{3/2}^2 &= -2 e^G G_a (G^{-1})^a_b G^c (G^{-1})^d_c R_d^b \\ &\quad + 2 (N-1) m_{3/2}^2 \end{aligned} \quad (34)$$

where the curvature tensor R_a^b is given by⁹⁾

$$R_b^a = \partial^a \partial_b \log \text{Det} [G_n^m] \quad (35)$$

The particular structure of the function G in Eq. (33) implies the following non-trivial R_a^b :

$$R_a^b = \frac{2N}{3} G_a^b \quad (36)$$

Here also the Kähler space is an Einstein manifold.

Using Eq. (36) one finds

$$e^G G_a (G^{-1})^a_b G^c (G^{-1})^d_c R_d^b = 2N m_{3/2}^2 \quad (37)$$

and finally obtains, using Eqs. (34) and (36), the following mass formula

$$2 \sum_{F=1}^{N-1} m_F^2 = 2 (N-1) m_{3/2}^2 \quad (38)$$

which means that every fermionic degree of freedom acquires a mass equal to the gravitino mass. Their bosonic partners are massless. Note the reverse role of bosonic and fermionic degrees of freedom. In fact, contrary to the minimal kinetic term case⁸⁾, the extra mass contribution originating in the supersymmetry breaking is distributed among the fermionic degrees of freedom, while their bosonic partners remain massless.

Just as in the case of one chiral superfield in the hidden sector, the matter fields must be coupled in such a way that the properties of the hidden sector are not destroyed. One must choose the total G function as the sum of the hidden sector $G(z^a, z_a^+)$ and the matter one $G(y^i, y_i^+)$. The resulting potential for the y sector is the same as previously.

In conclusion, we have shown that when making the hidden scalar particle potential (locally) flat, a positive definite non-trivial Kähler metric is needed (non-minimal kinetic terms), giving rise to a Kähler curvature of the form of Eqs. (10) and (36). The cosmological constant is zero before and after the flat limit is taken without unnatural fine tuning. Introducing a non-minimal kinetic term in the hidden sector does not lead to complications, a fact which may possibly be related to the extended supergravity forces. For example, we note that the scalar fields belonging to extended supergravity multiplets always have non-minimal kinetic terms as required by the underlying non-compact symmetries in extended supergravity theories. Since our model possesses a (locally) flat potential in the hidden sector, and since the matter sector corresponds to the $A = 3$ case, there are indications that the radiative gravitational corrections are milder¹²⁾. The resulting matter potential is degenerate and definite positive, just as in normal global supersymmetric theories. This situation is very different from the minimal supergravity where minima with a negative cosmological constant may appear¹³⁾. The supergravity-induced soft supersymmetric breaking terms have the particular form of the $A = 3$ case without any other restriction on the form of the matter superpotential. All successful phenomenological implications of the $N = 1$ supergravity theory remain valid. For instance, the stability of the gauge hierarchy in grand unified theories, the generation of the doublet-triplet hierarchy³⁾, and the natural $M_X - M_W$ hierarchy introduced by the radiative $SU(2) \times U(1)$ breaking scenario^{4),5)} persist in the current model.

The restriction to the $A = 3$ case, as a consequence for the vanishing of the cosmological constant, has some interesting experimental consequences^{11),14)}, for example, the existence of a rather "light" ($\leq M_W$) low energy supersymmetric particle spectrum. In fact, it seems to reject the possibility of a tree-level

breaking of the $SU(2) \times U(1)$ type models, since it is difficult for the colour and/or electromagnetism to remain unbroken^{15),16)}. On the other hand, for the radiative breaking of $SU(2) \times U(1)$ type models, only those with a D term close to zero⁵⁾ have a chance to maintain a vanishing value for the cosmological constant, which means that supersymmetry must be almost unbroken around the weak interaction scale (~ 200 GeV) and, consequently, implies a "light" ($\leq M_W$) particle spectrum. Also, for the small D terms radiative breaking models, the $A = 3$ case avoids any kind of destabilization^{5b)} of the scalar potential and relaxes any unnatural constraint on the top quark mass^{5b)}.

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NOTE ADDED IN PROOF

To better emphasize the geometric relevance of global flatness it is worth noticing that Eq. (9) is equivalent to the unique solution

$$G = -\frac{3}{2} \log (z + z^*)^2$$

up to field redefinition $z \rightarrow f(z)$. The scalar Lagrangian takes the form

$$3\sqrt{-g} \ g^{\mu\nu} \frac{\partial_\mu z \partial_\nu z^*}{(z + z^*)^2}$$

This Lagrangian is the same as the scalar sector of the $N = 4$ supergravity theory [E. Cremmer, J. Scherk and S. Ferrara, Phys. Lett. 74B (1978) 61] and it describes a non-linear σ model with an $SU(1,1)$ non-compact symmetry

$$z \longrightarrow \frac{\alpha z + i\beta}{i\gamma z + \delta}$$

$\alpha, \beta, \gamma, \delta$ real and $\alpha\delta + \beta\gamma = 1$.

We also note that through the particular redefinition

$$z \longrightarrow \frac{z + \sqrt{3}}{z - \sqrt{3}}$$

the kinetic term takes the form of the conformal scalar coupling [R. Barbieri, S. Ferrara, D.V. Nanopoulos and K.S. Stelle, Phys. Lett. 113B (1983) 219]

$$\gamma_{zz^*} \partial_\mu z \partial_\nu z^* g^{\mu\nu} \sqrt{-g}$$

with

$$\gamma = -3 \log(1 - \frac{1}{3} z z^*)$$