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SU(2) SYMMETRY BREAKING FROM QCD SUM RULES

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ABSTRACT

We discuss the determination of the (flavour) SU(2) symmetry breaking due to the u and d quark vacuum condensate ratio, $1 - \langle \bar{d}d \rangle / \langle \bar{u}u \rangle \approx 0.022 - 0.027$, from QCD sum rules. We also comment on the SU(2) breaking for the $\omega\rho\pi$ couplings and its relevance in $\omega, \phi \rightarrow \pi^+\pi^-\pi^0$ and $\omega, \rho \rightarrow \pi\gamma$ decays.

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1. - INTRODUCTION

Flavour SU(2) symmetry breaking is characterized essentially by three different effects:

- (a) the "current" quark mass difference $m_d - m_u$, which appears as an explicit symmetry breaking parameter in the QCD Lagrangian, and which is believed to be related to some mass-generating mechanism through spontaneous breakdown of gauge symmetries;
- (b) the ratio of the quark vacuum condensates $\langle \bar{d}d \rangle / \langle \bar{u}u \rangle$, which characterizes isospin breaking in the QCD non-perturbative vacuum; and
- (c) electromagnetic effects, which are of the order of the QED coupling constant α/π .

Particular attention has been paid to the determination of the SU(2) breaking parameters from meson and baryon phenomenology since the first results indicating¹⁾ $m_d \approx 1.8 m_u$. By now it is widely accepted that SU(2) symmetry, although good at the hadronic level, should be very badly broken at the quark level. In a series of papers²⁾⁻⁴⁾, SU(3) and SU(3) $\times$ SU(3) symmetry breaking have been studied as an application of the Laplace transform QCD sum rules to the correlation functions of the vector and the axial-vector current divergences. As a result, one has related "current" quark masses and quark vacuum condensates to the masses and couplings of the lowest-lying $J^{PC} = 0^{++}$ and 0^{-+} mesons. Of particular interest seems to us the result⁴⁾ $\langle \bar{s}s \rangle / \langle \bar{u}u \rangle \approx 0.6$, to be compared with $\langle \bar{s}s \rangle / \langle \bar{u}u \rangle = 1$ used in standard current algebra applications, which indicates a somewhat large SU(3) breaking in the non-perturbative vacuum. In this paper, we would like to extend the considerations given in Refs. 3) and 4) to the case of SU(2) breaking, focusing in particular on the determination of the ratio of vacuum condensates $\langle \bar{d}d \rangle / \langle \bar{u}u \rangle$ from the mass and the coupling constant of the $\delta(980)$, $J^{PC} = 0^{++}$, non-strange meson. Finally, we will also consider, as an application of our results and in the more speculative formalism of three-point function sum rules recently proposed in Ref. 5), the isospin violations induced in the ratio of coupling constants $g_{\omega^0 \pi^0 \rho^0} / g_{\omega^0 \pi^- \rho^+}$ and the possibility of an experimental detection.

2. - SUM RULES FOR $\langle \bar{u}u \rangle$ - $\langle \bar{d}d \rangle$

Since the formalism which we will use has already been presented in full detail in Refs. 3) and 4), we only need to sketch the main steps of the derivation and emphasize the most relevant points for the discussion of SU(2) breaking. The correlation function to be considered in this case is that of the $\Delta I = 1$ vector current divergence

$$\Psi(Q^2 = -q^2 > 0)_u^d = i \int d^4x e^{iqx} \langle 0 | T \partial V(x)^{1-i2} \partial V(0)^{1+i2} | 0 \rangle \quad (1)$$

with

$$\partial V^{1-i2} = i(m_d - m_u) : \bar{d}u : . \quad (2)$$

Using current algebra Ward identities, one obtains the renormalization group invariant expression for Ψ_u^d at $Q^2 = 0$:

$$\Psi(0)_u^d = -(m_d - m_u) \langle \bar{d}d - \bar{u}u \rangle . \quad (3)$$

Notice that we are neglecting electromagnetic effects, so that the SU(2) breaking we are discussing is a "tadpole-like" mechanism analogous to that generated by the unequal u and d "current" quark masses.

As emphasized in Refs. 3) and 4), one can derive a sum rule which is sensitive to the value of the divergence's correlation function at $Q^2 = 0$. In the case of Ψ_u^d one can write:

$$\Psi(0)_u^d \simeq \int_{t_0}^{\infty} \frac{dt}{t} e^{-t/M^2} \frac{1}{\pi} \text{Im} \Psi(t)_u^d \left[1 - \frac{t}{M^2} \left(1 + 2 \frac{\bar{\alpha}_s(M)}{\pi} \right) \right] \quad (4)$$

The advantage of considering Eq. (4) lies in its very simple expression; one can notice, among other things, that the contribution from the gluon vacuum condensate drops out automatically. Also, the QCD radiative correction is smaller in this sum rule than in the Laplace transform of the sum rule for^{3),4)} $\frac{1}{\pi} \frac{1}{t} \text{Im} \psi(t)$.

In order to saturate the spectral function $\text{Im} \psi(t)$ of the sum rule, we take into account the contribution coming from the $\delta(980)$ resonance in the zero width approximation, and an additional contribution due to the continuum whose threshold has been fixed, for definiteness, at $t_c = (1.2 \text{ GeV})^2$ just above the $\delta(980)$ mass. Therefore, one has:

$$\frac{1}{\pi} \text{Im} \psi(t)_u^d = 2 M_\delta^4 f_\delta^2 \delta(t - M_\delta^2) + \frac{3}{8\pi^2} (\bar{m}_d - \bar{m}_u)^2 t \Theta(t - t_c) \quad (5)$$

where

$$\langle 0 | \partial V^{1-i2} | \delta^+ \rangle = i\sqrt{2} M_\delta^2 f_\delta \quad (6)$$

and $\bar{m}_{u,d}$ is the running quark mass defined to one loop in the $\overline{\text{MS}}$ renormalization scheme⁶⁾, as $\bar{m}_{u,d} = \hat{m}_{u,d} (\log M/\Lambda)^{-4/9}$ with $\hat{m}_{u,d}$ being the renormalization group invariant mass⁷⁾, and $\Lambda \approx 100$ MeV is the commonly accepted value of the QCD scale within the $\overline{\text{MS}}$ renormalization scheme. Information on the coupling constant f_δ can only be obtained indirectly. The simplest possibility, adopted in Refs. 2) and 8), is to relate it to the K^0 - K^+ non-electromagnetic mass difference (the conventional tadpole mechanism), and to the $\delta K\bar{K}$ coupling, which can be deduced from the $\delta \rightarrow \eta\pi$ decay width and SU(3) relations (neglecting, for simplicity, the small η - η' mixing). In this way, one finds^{2),8)}

$$f_\delta \approx (m_{K^0}^2 - m_{K^+}^2)_{\text{tad}} / \sqrt{6} M_\delta \approx 1.8 \text{ MeV}, \quad (7)$$

where, according to the discussion of Ref. 2), we have used $(m_{K^0}^2 - m_{K^+}^2)_{\text{tad}} = 0.006 \text{ GeV}^2$ as a mean value of various estimates⁹⁾.

From Eqs. (3) - (6), one immediately obtains

$$\begin{aligned} \psi(0)_u^d &= - (m_d - m_u) \langle \bar{d}d - \bar{u}u \rangle \approx \\ &\approx 2 M_\delta^2 f_\delta^2 \left[1 - \frac{M_\delta^2}{M^2} (1 + 2 \frac{\bar{\alpha}_s}{\pi}) \right] e^{-M_\delta^2/M^2} - \\ &- \frac{3}{8\pi^2} (\bar{m}_d - \bar{m}_u)^2 \left[t_c + 2 \frac{\bar{\alpha}_s}{\pi} (t_c + M^2) \right] e^{-t_c/M^2} \end{aligned} \quad (8)$$

and the value of $\psi(0)_u^d / \psi_s(0)_u^d = -(m_d - m_u) (\langle \bar{d}d \rangle - \langle \bar{u}u \rangle) / 2m_\pi^2 f_\pi^2$ displayed, as a function of M , in Fig. 1 (solid curve). Also shown in Fig. 1 (dashed curve) is the result corresponding to the first term of the right-hand side of Eq. (8), i.e., to saturating the sum rule only with the δ resonance. In all these numerical evaluations, we have used $f_\delta = 1.8 \text{ MeV}$, $f_\pi = 93 \text{ MeV}$ and, in agreement with

several recent estimates^{1),2),10)}, the invariant quark masses

$$\lambda \equiv \frac{\hat{m}_d - \hat{m}_u}{\hat{m}_d + \hat{m}_u} \approx \frac{1}{3}, \quad \hat{m}_d \approx 2 \hat{m}_u \approx 20 \text{ MeV}. \quad (9)$$

In Fig. 1, the arrow at $M \approx 0.95 \text{ GeV}$ indicates the point where the continuum contribution becomes more important than the resonant one. Since the latter contains the dynamically relevant information (f_δ and M_δ), we will conservatively consider the values of M ranging from the minimum of the curve, where one expects to have optimal information, to 0.95 GeV . So, we finally deduce:

$$1 - \frac{\langle \bar{d}d \rangle}{\langle \bar{u}u \rangle} \approx -2 \frac{\psi(0)_u^d}{\psi_5(0)_u^d} = 0.022 \sim 0.027, \quad (10)$$

indicating a reasonable isospin violation of the order of 2%. One can notice that the effect of working with quark masses smaller than in Eq. (9) is to reduce the continuum contribution, i.e., to raise the solid curve in Fig. 1 up to the dashed one.

Our main result, Eq. (10), is not far from the estimate coming from the ω - ρ mixing analysis¹¹⁾

$$1 - \frac{\langle \bar{d}d \rangle}{\langle \bar{u}u \rangle} \approx 0.015, \quad (11)$$

whereas it appears appreciably larger than a more recent determination¹²⁾ leading to $1 - \langle \bar{d}d \rangle / \langle \bar{u}u \rangle \approx 0.006$, from baryon two-point function sum rules. It is interesting to observe that the above-mentioned SU(3) violation in the non-perturbative vacuum, $1 - \langle \bar{s}s \rangle / \langle \bar{u}u \rangle \approx 0.4$, estimated by the same formalism⁴⁾ used here, is about 20 times the SU(2) violation quoted in Eq. (10), which matches with the traditional expectations on SU(3) versus SU(2) breaking.

3. - SU(2) BREAKING FOR THE $\omega\rho\pi$ COUPLINGS

As an application of our analysis, we now proceed to estimate the effects of SU(2) breaking in the ratio of coupling constants $g_{\omega\rho^0\pi^0}/g_{\omega\rho^+\pi^-}$, whose absolute value is one in the good SU(2) limit. A reasonable value for the $\omega\rho\pi$ coupling has already been obtained⁵⁾ making use of the QCD sum rules formalism for three-point functions in the so-called symmetrical subtraction configuration ($Q^2 = -p_\omega^2 = -p_\rho^2 = -p_\pi^2 \gg \Lambda^2$), where eventual mass singularities in the evaluation of the QCD part are absent. Following the lines and notation of Ref. 5), the relevant part of the QCD amplitude for the $\omega \rightarrow \rho^0\pi^0$ and $\omega \rightarrow \rho^+\pi^-$ transitions turns out to be, respectively,

$$A_{00}^{QCD}(Q^2) = \frac{I_{xy}}{8\pi^2} \frac{m_u^2 + m_d^2}{Q^2} - \frac{m_u \langle \bar{u}u \rangle + m_d \langle \bar{d}d \rangle}{2(Q^2)^2},$$

$$A_{+-}^{QCD}(Q^2) = \frac{I_{xy}(m_u + m_d)^2}{8\pi^2 Q^2} - \frac{(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle}{2(Q^2)^2}, \quad (12)$$

where the first (second) terms in the right-hand side of Eq. (12) are the lowest order perturbative (the leading non-perturbative) contributions, and $I_{xy} \equiv \int_0^1 dx \int_0^{1-x} dy [x(1-x) + y(1-y) - xy]^{-1} \approx 2.34$. Under the usual assumption that the corresponding phenomenological expressions can be saturated by the lowest mass resonance in each channel and in the zero width approximation, one also finds⁵⁾

$$A_{00,+-}^{exp}(Q^2) \approx G_{00,+-} \frac{M_\omega^2}{2\gamma_\omega} \frac{M_\rho^2}{2\gamma_\rho} \sqrt{\pi} \frac{1}{Q^2 + M_\omega^2} \cdot \frac{1}{Q^2 + M_\rho^2} \cdot \frac{1}{Q^2 + m_\pi^2} \quad (13)$$

with

$$G_{00} \equiv m_{\pi^0}^2 g_{\omega \rho^0 \pi^0}$$

$$G_{+-} \equiv 2 m_{\pi^\pm}^2 g_{\omega \rho^\pm \pi^\pm} \quad (14)$$

In Eq. (13), one can safely neglect the mass splittings inside the ρ and π isotriplets due to the large ρ mass and the large Q^2 values involved. Similarly, we have identified $f_{\pi^0} = f_{\pi^\pm}$ and $\gamma_{\rho^0} = \gamma_{\rho^\pm}$, as one can justify by simply observing that from the fulfilment of the analogous SU(3) relations, $f_\pi = f_K$ and $3\gamma_\rho = \gamma_\omega = -\sqrt{2}\gamma_\phi$, to a 10-20%, one has to expect an accuracy for their SU(2) analogues better than $\sim 1\%$.

Identifying the QCD amplitudes (12) with their corresponding phenomenological expressions (13) (or, equivalently, identifying their Laplace transformed versions) one obtains from Eq. (9) and $\langle \bar{u}u \rangle = -(0.18 \text{ GeV})^3$

$$\left| \frac{g_{\omega \rho^0 \pi^0}}{g_{\omega \rho^\pm \pi^\pm}} \right| \approx \frac{m_{\pi^+}^2}{m_{\pi^0}^2} \left\{ 1 - \frac{\langle \bar{u}u - \bar{d}d \rangle}{3\langle \bar{u}u + \bar{d}d \rangle} - \frac{I_{xy}}{24\pi^2} \frac{\hat{m}_u Q^2}{\langle \bar{u}u \rangle (\log Q/\Lambda)} \right\}^{8/9}$$

$$\approx \frac{m_{\pi^+}^2}{m_{\pi^0}^2} \approx 1.07. \quad (15)$$

where the final value comes entirely from the pion mass difference (whose numerical value is considerably enhanced by the smallness of the pion mass), since the other two corrections in the right-hand side of Eq. (15) have opposite signs and each one represents less than $\sim 1\%$ for any reasonable values of quark masses, quark condensates and Q^2 (or, equivalently, M^2) around 1 to 2 GeV^2 . The result quoted in Eq. (15) represents a rather huge (and somehow unexpected) violation of SU(2). In this sense, we are aware of the fact that a more detailed study of Eq. (15), including higher meson excitations, higher order QCD radiative and power terms, as well as possible electromagnetic effects, can become relevant. However, we notice that the two former effects tend to be reduced when performing the ratio (15) between two analogous coupling constants.

Obviously, a significant SU(2) breaking such as that predicted by Eq. (15) should not escape its experimental detection. That this seems to be the case can already be deduced from the experimental data¹³⁾ $\Gamma(\omega \rightarrow \pi\gamma) = 861 \pm 56 \text{ KeV}$, $\Gamma(\rho^- \rightarrow \pi^-\gamma) = 71 \pm 7 \text{ KeV}$ ¹⁴⁾ and¹³⁾ $\gamma_\omega = (3.07 \pm 0.20)\gamma_\rho^*$, which, in the context of vector-meson dominance (and taking into account the phase space correction factor, 0.97), lead to

$$\left| \frac{g_{\omega e^+e^-}}{g_{\omega e^+\pi^-}} \right| \simeq 0.97 \frac{\gamma_e}{\gamma_\omega} \left(\frac{\Gamma(\omega \rightarrow \pi\gamma)}{\Gamma(\rho^- \rightarrow \pi^-\gamma)} \right)^{1/2} \simeq 1.10 \pm 0.08 \quad (16)$$

in reasonable agreement with our prediction, Eq. (15). A second consequence of our SU(2) breaking result (15) could be the detection of small asymmetries in the $\omega \rightarrow \pi^+\pi^-\pi^0$ Dalitz plot, which is known to be described¹⁷⁾ through the decay chain $\omega \rightarrow \rho^0\pi^0 + \rho^\pm\pi^\mp \rightarrow \pi^+\pi^-\pi^0$, involving three (no longer symmetric) interfering amplitudes [this effect could be somewhat increased due to the SU(2) breaking in $g_{\rho^0\pi^+\pi^-}/g_{\rho^\pm\pi^\mp\pi^0} \simeq 1.02$, deducible from the small observed difference between the neutral and charged ρ decay widths]. Finally, the isospin violations in $\varphi \rightarrow \rho^\pm\pi^\mp$ and $\varphi \rightarrow \rho^0\pi^0$, which dominate¹⁸⁾ the $\varphi \rightarrow \pi^+\pi^-\pi^0$ decay with on-shell ρ mesons, could also be detected through an analysis similar to that of Ref. 18).

^{*}) The most recent data on $e^+e^- \rightarrow \omega \rightarrow 3\pi$ lead to¹⁵⁾ $(2\gamma_\omega)^2/4\pi = 22.2 \pm 1.7$. When this value is combined with the result $(2\gamma_\rho)^2/4\pi = 2.38 \pm 0.18$, coming from a global fit¹⁶⁾ of the Orsay and Novosibirsk data for $e^+e^- \rightarrow \rho^0 \rightarrow \pi^+\pi^-$, one gets $\gamma_\omega = (3.05 \pm 0.16)\gamma_\rho$.

4. - CONCLUSIONS

We have discussed some aspects of the SU(2) breaking in the framework of the QCD sum rules and obtained the following two results:

- i) working with the two-point correlation function for the $\Delta I = 1$ vector-current divergences, we have deduced $1 - \langle \bar{d}d \rangle / \langle \bar{u}u \rangle \simeq 0.022 - 0.027$ by saturating the phenomenological side of the sum rule with the δ meson (with $f_\delta = 1.8$ MeV) plus a continuum contribution. This SU(2) symmetry breaking of quark vacuum condensates fits nicely with its analogous result at the SU(3) level, $1 - \langle \bar{s}s \rangle / \langle \bar{u}u \rangle \simeq 0.4$, obtained with the same formalism in Ref. 4). Our result is not far from the estimate¹¹⁾ coming from an analysis of the ρ - ω mixing, but appreciably larger than the one obtained using baryon two-point functions¹²⁾.
- ii) More speculatively, we have also discussed the $\omega\rho\pi$ coupling constants in the context of three-point correlation functions obtaining a significant SU(2) violation, $|\xi_{\omega\rho^0\pi^0}/\xi_{\omega\rho^+\pi^-}| \simeq 1.07$, which should be detectable in $\omega, \varphi \rightarrow \pi^+\pi^-\pi^0$ decays. Moreover, such a violation seems to explain the large experimental value for the ratio $\Gamma(\omega \rightarrow \pi^0\gamma)/\Gamma(\rho^- \rightarrow \pi^0\gamma)$, which is hard to justify in other schemes.

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FIGURE CAPTION

$\psi(0)_{u\pi\pi}^d/2m_{\pi}^2 f_{\pi}^2$ as a function of M with $\psi(0)_u^d$ given by Eq. (8) (solid line).
The dashed line represents the resonant contribution coming only from the first
term in the right-hand side of Eq. (8).

Fig. 1