TH.3707: Properties of Susy Hadrons in a Bethe-Salpeter Model
A.N. Mitra and S. Ono.

TH.3724: How would the world look like if there were supersymmetric particles?
D.V. Nanopoulos, S. Ono and T. Yanagida

NOTE ADDED

For glueballon (g̃g-state) we quoted the selection rule \((-1)^{S+1} = -1\) derived by Zug et al., Phys. Rev. D28 (1983) 1706. However, this will be wrong since the intrinsic parity should not come into the Fermi symmetry condition. The correct selection rule will be \((-1)^{S+1} = +1\). Correct potential for g̃g-state should contain a factor two from exchange diagram, i.e., \(V_{g̃g}(r) = V_{g̃g}(r) = V_{qg}(r) = (9/4)V_{q̃q}(r)\). Thus, the decay width for the lowest g̃g will become, e.g.,

\[
\Gamma_{g̃g \rightarrow g̃g} = 7 \times 13.5 \times (1-5) \text{ MeV} = (100-500) \text{ MeV}
\]

One of the authors (S.O.) would like to thank H. Kühn for pointing out them. More detailed properties of g̃g-state are studied in the following paper: "Production and decay of gluino-gluino bound state", by J.H. Kühn and S., Ono, Orsay preprint, LPTHE Orsay 84/2.

Note Added
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PROPERTIES OF SUSY HADROS IN A BETHE-SALPETER MODEL

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ABSTRACT

We study the gluino gluon (gg) and hybrino (ggq) states in a Bethe-Salpeter model of confinement, characterized by a reduced spring constant and quark mass, which provides an excellent description of masses and couplings of qq and qqq hadrons. Our predictions for gg spectrum are completely different from the bag model results. Our model has the advantage that (i) in the m → 0 limit, the gg mass spectrum is smoothly connected to that for m_q = 0, (ii) the effect of the centre-of-mass motion is automatically removed. Properties of these new hadrons are discussed from the point of view of possible experimental detection.
1. - INTRODUCTION

Although supersymmetry (SUSY) has been extensively studied at the purely theoretical level, its experimental aspects, especially the investigation of the properties of SUSY partners of known hadrons, their modes of production and decay (which in turn depend on their mass patterns), are a comparatively new development\(^1\),\(^2\). Now by the very nature of supersymmetry, the interaction strengths of SUSY particles are governed by exactly the same laws as those of ordinary particles, so that the same CCF-oriented techniques that have been extensively employed in the recent years to study the bound states of \(qq\), \(qqq\) and \(gg\) systems can be employed for their SUSY partners without bringing in fresh parameters.

In the study of SUSY hadrons, or \(R\) hadrons for short, a central rôle is played by the gluino \((\tilde{g})\) which is believed to be light enough in many SUSY theories \((\sim 1\text{ GeV})\) for receiving the earliest attention at the phenomenological level. Being a coloured octet, it has a big enough coupling constant to any coloured particle, so that for \(m_{\tilde{g}}\text{(gluino mass)} \geq 1\text{ GeV}\), the gluino will be produced more than ten times as frequently as, e.g., quark pairs in \(p\bar{p}\) scattering\(^2\). Further, if \(m_{\tilde{g}}\) is less than \(m(t\bar{t})\), the gluino can also be produced in the decay of toponium, e.g., \(t\bar{t} \rightarrow \gamma\tilde{g}\tilde{g}\) which is also believed to have a reasonably large branching ratio\(^3\),\(^4\). Still another possibility is the decay \(Z^0 \rightarrow \tilde{g}\tilde{g}\) which can be seen in LEF\(^2\).

The gluino is believed to be relatively long lived \((10^{-8}-10^{-20}\text{ sec})\) so as to be experimentally identifiable without much difficulty. Indeed, glueballino states \((\tilde{gg})\) should be easier to identify experimentally than, e.g., glueball states which have a tendency to show strong mixing effects with \(qq\) states. In this respect, the glueball candidate \(1(1440)\)\(^5\) has been the subject of much controversy\(^6\). Indeed, shortly after the discovery\(^5\),\(^6\) of this resonance, Ono and Pene\(^7\) have shown that properties of this state can be explained alternatively as a radially excited \(u\bar{u} + d\bar{d}\) state. More recently, the Mark III group\(^8\) has detected the \(1 + \rho\gamma\) mode which rules out the pure \(gg\) assumptions and favours the \(qq\) interpretation\(^7\). On the other hand, \(gg\) states can be identified by checking the large missing transverse momenta carried away by the photino \((\tilde{\gamma})\).

In this paper we wish to discuss the spectral properties of what are believed to be the relatively light SUSY partner, viz. \(\tilde{g}\) and \(\tilde{g}\tilde{g}\) states \((q = u, d, s\text{ only})\) from the point of view of their qualitative experimental signatures. For this purpose we employ the Bethe-Salpeter equations for two-
three-body systems developed by one of us\(^9\) and applied to light \(q\bar{q}\) and \(qqq\) systems\(^{10\text{-}14}\) with considerable success. The model which is characterized by a universal spring constant \((\bar{G})\) and the quark mass \((m_q)\) of the flavour sector under study, provides an excellent description of meson\(^{10,12}\) and baryon\(^{10,11}\) spectra, electromagnetic properties of mesons\(^{12}\) and baryons\(^{14}\), as well as their pionic couplings\(^{13}\), all within an integrated framework.

We sketch the essential steps in the derivation of \(gg\) and \(g\bar{g}\) mass patterns in Section 2 and that of \(gq\bar{q}\) mass patterns in Section 3. Discussions and comparison with other model predictions are presented in Section 4. Conclusions are given in Section 5.

2. - THE \(gg\) (GLUEBALLING) AND \(g\bar{g}\) (GLUEBALLON) SPECTRA

The Bethe-Salpeter model\(^9\) to be used in this calculation is characterized by a common colour \((F_1 \cdot F_2)\) and spin \((\gamma_{\mu}^{(1)} \gamma_{\mu}^{(2)})\) dependence of both the confining and the one-gluon exchange (Coulomb) parts of the \(q\bar{q}\), \(qq\) and \(g\bar{g}\) interactions. (In this sense it is a vector, colour dependent model of confinement, of which more will be said in Section 4.) There are two basic ingredients to the calculation, the colour factors and the reduced spring constant. Now by the very logic of the SUSY concept, the over-all interaction strength of \(gg\), \(g\bar{g}\) and \(gg\) pairs are intimately related. Of these, the colour factors are unambiguous, viz.

\[
F_g \cdot F_{\bar{g}} = -3 \quad F_g \cdot F_g = F_{\bar{g}} \cdot F_{\bar{g}} = -3 \times 2
\]

(2.1)

where the factor 2 for the \(gg\) and \(g\bar{g}\) cases comes from the exchange effect.

As to the spring constant, the most successful predictions of hadron spectra\(^{12\text{-}14}\) and other properties\(^{12\text{-}14}\) have been based on the universality of the "reduced" spring constant \((\bar{G})\) defined through\(^{10}\)

\[
\omega_{g\bar{g}}^2 = \tau_{12} m_{12} \tilde{\omega}^2
\]

(2.2)

\[
m_{12} \equiv m_1 + m_2 \quad \tau_{12} \equiv 4 m_1 m_2 m_{12}^{-2}
\]

(2.3)

\[\text{[Eq. (2.2) was used in Ref. 12] without the \(\tau_{12}\) factor but it made no difference for \(m_1 = m_2\) and little difference to the kaon results as \(\tau_{12}\) was 0.99 for u, d and s masses used.]\]

Further, we have checked afresh the working of Eq. (2.2)
for the D, F and B meson regions using the formalism of Ref. 12) with
\(\alpha_s = 0.4\) and the following values of the masses are obtained (in GeV) :

\[
\tilde{\omega} = 0.14, \quad m_u, d = 0.28, \quad m_s = 0.38,
\]
\[
m_c = 1.71, \quad m_b = 5.66
\]

These are in conformity with the same values of \(\tilde{\omega}\) and \(m_{u,d}\) as used in
Refs. 10)-14). The results in MeV are

<table>
<thead>
<tr>
<th>D</th>
<th>D*</th>
<th>F</th>
<th>F*</th>
<th>B</th>
<th>B*</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M(TH))</td>
<td>1875</td>
<td>2009</td>
<td>1971</td>
<td>2103</td>
<td>5281</td>
</tr>
<tr>
<td>(M(EXP))</td>
<td>1865</td>
<td>2010</td>
<td>1970</td>
<td>2130?</td>
<td>5283</td>
</tr>
</tbody>
</table>

In obtaining this spectrum, an over-all short fall of one unit in the
zero point oscillator energy has been taken into account (i.e., \(N + \frac{3}{2} + N + \frac{1}{2}\)
as found in Ref. 11) and elsewhere\(^{16}\), to hold very nearly for all types of
hadrons \((\bar{q}q, qqq, q\bar{q}\bar{q})\). These results provide the necessary check that should
warrant the continued identification of \(\tilde{\omega}\) as the universal spring constant for the
interaction of SUSY partners of quarks and gluons, in accordance with the
ansatz (2.2). In this connection a word of caution needs to be added regarding
the status of the gluon mass which should vanish in the strict chiral limit.
However, as has been argued elsewhere\(^{15}\), it is not meaningful to speak of a
strictly massless "confined" gluon, though the problem of assuming a definite
value for the "constituent" gluon mass \((\bar{g}_n)\) was sought to be circumvented in
Ref. 15) through an alternative ansatz \(2\bar{g}_n = \omega^2 \) to set the mass scale for the
gluon scenario in the BS model\(^{15}\). In the present paper, on the other hand,
we shall continue to use the more logical ansatz (2.2), using a symbolic value
for the gluon mass \(\bar{g}_n\) (presumably small), but as will be seen, the final result
for the \(\bar{g}_n\) spectrum is insensitive to this parameter (and even has a well-
defined limit when \(\bar{g}_n \rightarrow 0\), as long as the gluino mass \((m_{\tilde{g}})\) is reasonably
large (\(\gtrsim 1\) GeV).

With these remarks the BS formalism for the \(g\tilde{g}\) system is a straight-
forward application of the (unequal mass) techniques described earlier\(^{10},12\) and
the three-dimensional (instantaneous) form of the BS equation reads as

\[
D_{\tau2} \Psi(\bar{q}) = \frac{9}{4} \omega_{\tilde{g}}^2 \left[ \frac{1}{4} \tau_{12} M_{12}^2 \bar{q}^2 + \frac{1}{4} \bar{q} \cdot \bar{q} + \frac{11}{2} - 2 \bar{q} \cdot \bar{q} \right] \Psi(\bar{q})
\]

(2.5)
where

\[ D_{12} = 2 M_{12} \left\{ q_{12}^2 - \frac{1}{4} \tau_{12} \left( M_{12}^2 - m_{12}^2 \right) \right\} \quad (2.6) \]

\( m_{12} \equiv M \) is the \( gg \) mass and all other symbols are as defined in Refs. 10 and 12. The solution of this equation, with the (perturbative) inclusion of the Coulomb term, may be expressed as\(^{10}\)

\[ F_{\text{HO}} (M) + F_{\text{Coul}} (M) = N + \frac{3}{2} \quad (2.7) \]

where

\[ \Omega_{gg} \cdot F_{\text{HO}} (M) = \tau_{12} \left( M^2 - m_{12}^2 \right) \]

\[ + \frac{9}{4} \omega_{gg}^2 M^{-1} \left( Q_N + \frac{11}{2} - 2 \frac{J \cdot S}{M} \right) \quad (2.8) \]

\[ \Omega_{gg} = 6 M \omega_{gg} \sqrt{2 M \tau_{12}} \quad (2.9) \]

\[ M \Omega_{gg} \cdot F_{\text{Coul}} (M) = 2 \langle 3 \alpha_s \tau_{12} M_{12}^2 / \mathcal{R}_{12} \rangle \quad (2.10) \]

and \( \langle \ldots \rangle \) is calculated for the quantum state under study. For the ground state\(^{10}\)

\[ \Psi_0 (\mathbf{r}) = \left( \frac{\beta}{\pi} \right)^{3/4} e^{-\frac{1}{2} \beta^2 r^2} \quad (2.11) \]

\[ \beta_4 = \frac{9}{8} \tau_{12} M \omega_{gg}^2 \quad (2.12) \]

Substitution of (2.2) in (2.8) leads to the simplified equation (after cancellation of the \( \tau_{12} \) factor in every term)

\[ M^2 - m_{12}^2 + \frac{18 \tilde{\omega}^2 m_{12}}{M} \left( Q_N + \frac{11}{2} - 2 \frac{J \cdot S}{M} \right) \]

\[ + \frac{12 \alpha_s}{\sqrt{\pi}} \beta \delta M = 6 \sqrt{2 m_{12} M} \tilde{\omega} \left( N + \frac{3}{2} \right) \quad (2.13) \]
where $\delta = 1$ for the ground state and $O(1)$ for others. Equation (2.13) already reveals the close proximity of $M$ to $m_{12}$ for large $m_1 = m_g$, thus an approximate solution being

$$M \approx m_{\tilde{g}} + m_{\tilde{g}} + 3\sqrt{2} \tilde{\omega} \left( N + \frac{1}{2} \right) - \frac{6 \alpha_s}{\sqrt{\pi}} \delta \left[ 3\sqrt{2} m_{\tilde{g}} \tilde{\omega} \right]^{1/2} + O \left( \frac{\tilde{\omega}^2}{m_{\tilde{g}}^2} \right),$$

(2.14)

where the replacement $N + \frac{1}{2} \rightarrow N + \frac{1}{2}$ in (2.12) has been effected in accordance with the remarks following Eq. (2.4), as explained in Refs. 11) and 16). Taking $m_g$ to be small, Eq. (2.14) gives the following estimate for the glueballino mass

$$M(\tilde{g}) \approx m_{\tilde{g}} + 300 \text{ MeV} \times (2N + 1),$$

(2.15)

using $\tilde{\omega} = 140$ MeV, consistently with $D$, $F$ spectra as well as earlier results.

For the sake of completeness, we give a brief sketch of the mass formula for the $\tilde{g}\tilde{g}$ spectrum which is more akin to $g\tilde{g}$ case$^{15}$, except for the much larger gluino mass. The BS equation is now of the simpler (spinor-spinor)$^9,10$ form

$$\left( m_{\tilde{g}}^2 + p_{1}^2 \right) \left( m_{\tilde{g}}^2 + p_{2}^2 \right) \overline{\Psi}(p, \tilde{g}) = i F_1 \cdot F_2 \int \frac{d^4 k}{(2\pi)^4} \times \left[ V_{1}^{(0)} V_{1}^{(2)} - \frac{1}{2} (q - k)^2 \right] \frac{3}{4} (2\pi)^4 \omega_{g\tilde{g}}^2 \times \nabla_{k}^2 \delta(q - k) \overline{\Gamma}(p, k)$$

(2.16)

where the subtracted term $\frac{1}{2}(q-k)^2$ corresponds to the $\tilde{g}\tilde{g}$ counterpart of the "gauge invariant" adaptation$^{15}$ of the $g\tilde{g}$ interaction arising out of the contact $g^*$ interaction in the glueball case$^{15}$. The allowed states in this case are governed by the selection rule

$$(-1)^{(L+S)} = -1$$

(2.17)

The final spectrum which takes account of the exchange effect through the colour factor (2.1), is again expressible in a form similar to (2.12), viz.,
\[ M^2 - m_{12}^2 + \frac{36 \tilde{\omega}^2 m_{12}}{M} (q_N + \frac{15}{4} - 2J^3) \]
\[ + \frac{24 \alpha_s \beta \delta M}{\sqrt{\pi}} = 12 \sqrt{m_{12}} m \tilde{\omega} \left( N + \frac{3}{2} \right) \]  \hspace{1cm} (2.18)

where
\[ 8 \beta^2 = \Omega_{\tilde{g} \tilde{g}} = 16 \tilde{\omega} \sqrt{M m_{12}} \]  \hspace{1cm} (2.19)

Equation (2.17) finally simplifies
\[ M \cong 2 m_{\tilde{g}} + 6 \tilde{\omega} (N + \frac{3}{2}) - \frac{12 \alpha_s \delta}{\sqrt{\pi}} \sqrt{3 m_{\tilde{g}} \tilde{\omega}} + O\left( \tilde{\omega}^2 \right) \]  \hspace{1cm} (2.20)

However, this time the negative Coulomb term is large because of the factor \( \sqrt{m_{\tilde{g}}} \) which is more than offsets the logarithmic decrease of \( q_N \) at \( m_{\tilde{g}} \). Thus unlike the \( g\bar{g} \) case, the \( \tilde{g}\tilde{g} \) system becomes tightly bound and the number of bound states below the \( \tilde{g}\tilde{g} + g\bar{g} + g\tilde{g} \) threshold increases, analogously to the heavy quarkonium case.

3. SPECTRA OF HYBRINO (\( \tilde{g}q\bar{q} \)) SYSTEMS

We discuss next the case of \( \tilde{g}q\bar{q} \) systems when \( q, \bar{q} \) are light (u,d,s) quarks. Since the system is a three-body one with unequal mass kinematics\(^9\), the algebra is now much more involved\(^{11}\) than, e.g., in a bag model calculation\(^{17}\). However, to bring out the main features of the spectrum within a colour-dependent model of confinement\(^9\), it is useful at the first step to make the simplifying assumption of equal masses \( m_2 = m_3 \) for the two light quarks which either limits the system of \( u, d \) quarks or implies an average mass if one of them is an s quark (which is not much heavier than \( u \) or \( d \) in our "constituent" model). More important, this assumption must be considered in the background of the much heavier gluino mass \( m_{\tilde{g}} \) which controls the main dynamics and facilitates several important simplifications based on the smallness of the ratio \( m_{\tilde{g}}/m_{\tilde{g}} \), so that it should not be difficult within such a framework to estimate corrections arising from \( m_2 \neq m_3 \) if and when the experimental situation so warrants.
The $gq\bar{q}$ formalism is basically similar to that of strange $qqq$ baryons\textsuperscript{11} involving unequal mass kinematics, except for two important differences. First, the colour factors $F_1 \cdot F_j$ involved in $gq\bar{q}$ are no longer $-2/3$ for each pair as in a baryon system but instead are given by the following results for the three different pairs to make an over-all colour singlet hadron

$$F_F \cdot F_2 = F_F \cdot F_F = - \frac{3}{2} ; \quad F_{gq} = \frac{1}{6} \quad (3.1)$$

Thus the main confinement is now expected to arise from the $gq$ and $g\bar{q}$ pairs, while the $qq$ pair contributes a small repulsion (much like in a helium-atom problem). Secondly, the inequality $m_1 > m_2$ now helps in greatly simplifying the algebra compared to the ($m_1 \geq m_2$) $qqq$ case\textsuperscript{11}). This last feature further reduces (in the ratio $m_2^2/m_1^2$) the already small effect due to the $q\bar{q}$ colour factor.

Thus in the first approximation it is permissible to neglect the $q\bar{q}$ part of the kernel from the three-body BS equation\textsuperscript{9}), an assumption which can be checked through a subsequent perturbation estimate if necessary.

With the simplification thus obtained, it is somewhat easier to write down the three-dimensional form of the BS equation for $gq\bar{q}$, on lines similar to the $qqq$ case\textsuperscript{9)-11}), but with the new colour combinations Eq. (3.1) and with the $q\bar{q}$ kernel omitted. For this purpose, the normalized internal momenta $\xi$, $\gamma$ are defined as\textsuperscript{11})

$$\sqrt{\frac{m_0}{m_1}} \xi = \vec{P}_3 - \vec{P}_2 , \quad m_0 \gamma = -2 m_2 \vec{P}_2 + m_1 (\vec{P}_2 + \vec{P}_3) \quad (3.2)$$

where

$$m_0 = m_1 + m_2 + m_3 = m_1 + 2 m_2 \quad (3.3)$$

The "master" equation\textsuperscript{11}) is now of the simpler form

$$[m_0^2 - \Delta^2 + A_\xi \xi^2 - B_\xi \nabla^2 + A_\gamma \gamma^2 - B_\gamma \nabla^2] \psi(\xi, \gamma)$$

$$= \left\{ \text{RHS} + \langle \text{COUL} \rangle \right\} \psi(\xi, \gamma) \quad (3.4)$$
where, using the ansatz (2.2)

\begin{align}
A_\xi, \eta &= \frac{1}{4} \frac{m_0^3}{m_1 m_2 m_1^2} + \frac{1}{4} \frac{m_0^2}{m_1^2} \\
B_\xi &= \frac{m}{m_0} \quad B_\eta = \frac{18 M m_1 m_2}{m_1^2} \quad \tilde{\omega}^2
\end{align}

\begin{align}
[RHS] &= \frac{9}{2} m_0^3 \tilde{\omega}^2 m_1^{-2} M^{-1} \left[ \frac{2}{m_1 m_2} \left( \frac{m_0^2}{m_1^2} \eta^2 + \frac{m_0 m_1}{m_1^2} \xi^2 \right) \\
&+ \left( \frac{2 m_0^2}{m_1^2} - \frac{1}{2} \right) Q_N \eta + \left( \frac{2 m_0^2}{m_1^2} - \frac{1}{2} \right) Q_N \xi + \frac{4 m_0 m_1}{m_1^2} Q_N \xi \eta \\
&- \frac{4}{m_1} \cdot \frac{1}{m_2} - \frac{4}{m_1} \cdot \frac{1}{m_3} - \frac{4 m_0}{m_1} \nabla_1 \cdot \left( \frac{1}{m_1} + \frac{1}{m_2} \right) \\
&- \frac{4 m_0}{m_1} \nabla_1 \cdot \left( \frac{1}{m_1} + \frac{1}{m_2} \right) \right]
\end{align}

\begin{align}
\langle COUL \rangle &= 3 \alpha_3 \frac{m_0 M}{m_1} \left\langle \frac{1}{\eta_{12}} + \frac{1}{\eta_{13}} \right\rangle
\end{align}

and the unexplained notations are the same as in Ref. 11). The symbol \( \langle COUL \rangle \) represents the shift in the \( M^2 \) value due to the one-gluon exchange interactions, calculated perturbatively, in the \( gg \) and \( gq \) pairs.

To calculate the mass spectrum, we note first that the term \( [RHS] \) in (3.4) will contribute momentum and spin-dependent corrections to the spectrum of \( O(\alpha_s^2/\mu_0) \) compared to main effects, exactly as in the \( gg \) case, Eq. (2.13), and may therefore be neglected. The final mass spectrum then works out \(11,16 \) straightforwardly as

\begin{align}
M^2 - m_0^2 &= \Omega_\xi (N_\xi + 1) + \Omega_\eta (N_\eta + 1) + \langle COUL \rangle
\end{align}

where

\begin{align}
\Omega_\xi &= 2 \sqrt{A_\xi B_\xi} \\
\Omega_\eta &= 2 \sqrt{A_\eta B_\eta}
\end{align}
\[ \langle \text{COUL} \rangle = -\frac{M m_0}{m_{12}} \frac{12 \hat{A}}{\pi} \left( \frac{m_1}{m_o} \right)^{1/4} \left( \beta_\xi \beta_\eta \right)^{1/2} \Gamma^2 (3/4), \quad (3.11) \]

\[ \beta_\xi^4 = \frac{B_\xi}{A_\xi}, \quad \beta_\eta^4 = \frac{B_\eta}{A_\eta} \quad (3.12) \]

Using the typical values (in GeV units)

\[ m_1 = 3.0, \quad m_2 = 0.3, \quad \tilde{w} = 0.14, \quad (3.13) \]

Eq. (3.9) yields the following M values for the L = 0 and L = 1 states (in GeV units)

\[ M(L = 0) = 3.60 + O(\tilde{w}^2/m_o) \]
\[ M(L = 1, L = 0) = 3.87 + O(\tilde{w}^2/m_o) \]
\[ M(L = 0, L = 1) = 3.93 + O(\tilde{w}^2/m_o) \quad (3.14) \]

Further, the universal length scales \( \beta_\xi \) and \( \beta_\eta \) for the ground state work out (in GeV units) as

\[ \beta_\xi = 0.543, \quad \beta_\eta = 0.547 \quad (3.15) \]

We end this section with a qualitative estimate of the hitherto neglected \( q \bar{q} \) interaction effect in the above calculation. For this purpose, it is convenient to express the three-body BS equation in the instantaneous approximation \(^9\)

in the symbolic form (in momentum space):

\[ \psi = \frac{K_{12} \psi}{D_{12}} + \frac{K_{13} \psi}{D_{13}} + \frac{K_{23} \psi}{D_{23}}, \quad (3.16) \]

where \( D_{ij} \) are energy denominators of the types described earlier \(^9\), \(^10\) and \( K_{ij} \)'s represent the complete effects of the pairwise kernels. In particular,

\[ D_{23} = 2 M_{23} \left[ \frac{1}{4} \frac{m_0}{m_1} \xi_2^2 + \frac{\eta_2^2}{4} + m_2^2 \left( 1 - M^2 m_o^{-2} \right) \right] \quad (3.17) \]
with \( K_{23} = 2m_2 M/m_0 \), in accordance with the theoretical framework\(^{9)}\)\(^{-11)}\). Now the effects of the first two terms in (3.16) are fully included in the basic Eq. (3.4); its simple-looking form reflecting the near equality of \( D_{12}, D_{13} \) (apart from some small \( \xi, \eta \) terms of opposite signs). Of the \( (23) \) term, its harmonic part is given by

\[
K_{23}/D_{23} \approx - \frac{m_2 m_1^2 \tilde{\omega}^2}{M} \left( \frac{\eta^2}{4} + \frac{m_0}{4m_1} \xi^2 \right)^{-1} \nabla_\xi^2
\]

(3.18)

where the neglect of the last term in (3.17) is justified by the twin effect of the small difference of \( M \) from \( m_0 \) as seen from Eq. (3.14) as well as the relative smallness of \( m_2^2 \) with respect to the \( (\xi^2, \eta^2) \) terms. Now a perturbative calculation of the expectation value of (3.18) in the "zero-order" ground state, viz., wave function determined from (3.9),

\[
\Psi_0(\xi, \eta) = (\pi \beta_\xi \beta_\eta)^{-3/2} \exp \left( -\frac{1}{2} \xi^2 \beta_\xi^{-2} - \frac{1}{2} \eta^2 \beta_\eta^{-2} \right)
\]

(3.19)

shows, without further analysis, that

\[
\langle K_{23}/D_{23} \rangle \approx \mathcal{O} \left( m_2^2 \tilde{\omega}^2 / \beta_\xi^4, \beta_\eta^4 \right) \lesssim 3\%
\]

(3.20)

which merely amounts to multiplication correction of ~3% to the first two terms on the right-hand side of Eq. (3.16).

4. - DISCUSSIONS AND COMPARISONS WITH OTHER MODEL PREDICTIONS

From Eqs. (2.14) and (3.14), we find energy levels of glueballino and hybrino states which are shown in Fig. 1. Our notation for hybrino is the following. If the hybrino is composed of one known quarkonium (e.g., \( \rho \)) state and one gluino, we put a suffix \( \bar{g} \) (e.g., \( \rho_{\bar{g}} \)). If the spins of \( q\bar{q} \) and \( \bar{g} \) are parallel we put an asterisk (e.g., \( \rho_{\bar{g}}^* \)) and if they are antiparallel we do not (\( \rho_{\bar{g}} \)). The suffix \( \bar{g} \) is necessary to distinguish from a \( q\bar{q} \) state where \( \bar{q} \) is the scalar partner of the quark. In principle, hybrino mixes with \( q\bar{q} \) states.
Some of the states in Fig. 1 fall to lower states by emitting a \( \pi \) or a \( \pi \pi \) pair, e.g.,

\[
\begin{align*}
\pi \tilde{f}^\pm &\rightarrow (g \tilde{f})_{J = \frac{3}{2}}^- + \pi \\
\tilde{p}_f^* &\rightarrow (g \tilde{f})_{J = \frac{1}{2}}^- + \pi \\
(g \tilde{f})_{1P} &\rightarrow (\pi \tilde{f}^0 \text{ or } \tilde{p}_f \text{ or } \tilde{p}_f^* + \pi) \\
(g \tilde{f})_{1P} &\rightarrow (g \tilde{f})_{1S} + \pi \pi \quad ; \quad \Gamma \sim \mathcal{O}(10) \text{ KeV}
\end{align*}
\]

(4.1)

As seen from Fig. 1, hybrino ground states with \( I = 0, \frac{1}{2} \) become stable against strong and electromagnetic decays. Especially \( K^*_e, K^*_\mu \) contain charged particles which are easier to see experimentally.

Decays of these metastable particles are dominated by free gluino decay by leaving other particles as spectators, whose diagrams are shown in Figs. 2a and 2b. Decay rates are given by\(^2\)

\[
\begin{align*}
\Gamma_{(2a)}^{(2a)} &= \sum_b \frac{\alpha_s \alpha_s}{96 \pi} \frac{e^2 M_{\tilde{g}}^{-5}}{\tilde{m}_L^{-4} + \tilde{m}_R^{-4}} \\
\Gamma_{(2b)}^{(2a)} &= \sum_{\text{flavours}} \frac{\alpha_s}{4} \left( \frac{\tilde{m}_R^{-2} - \tilde{m}_L^{-2}}{\tilde{m}_L^{-4} + \tilde{m}_R^{-4}} \right)^2 \\
&\sim \frac{1}{20} \quad (\text{in a model} \quad ^2)
\end{align*}
\]

(4.2)

Spectators do not have enough energy to produce jets. Thus, the decay 2b produces only one gluon jet with large missing \( p_T \) carried away by the photino which usually escapes from detection. The decay 2a produces two jets, also with large missing \( p_T \). The decay 2b is easier to identify than the decay 2a. If \( M_{\tilde{g}} \) is small enough and scalar quark masses \( \tilde{m}_L, \tilde{m}_R \) are large enough the life times of charged \( K^*, K^{*\pm} \) can become long enough so that they can be seen even in a bubble chamber. Meta-

stable neutral hybrino states will not be difficult to observe either, since they can hit protons whose recoil is observable.
We now discuss the model dependence of energy levels of glueballino and hybrino states.

A) Bag model

In the bag model, the lowest glueballino state is in a TE mode, i.e., $\sigma_{\text{TE}}$ with $J^P = \frac{1}{2}^-, \frac{3}{2}^-$. The energy level is given by $^{18}$

$$\frac{d}{dr} \left[ r \frac{d}{dr} (kr) \right]_{r=R} = 0 \tag{4.4}$$

(First zero is at $kr = 2.7437$ for $\ell = 1$.) If we use $R$ from Ref. 19, we get $M(\sigma_{\text{TE}}) = M(\sigma) + 480$ MeV. The boundary condition for the TM mode is given by

$$\frac{d}{dr} (kr) \bigg|_{r=R} = 0 \tag{4.5}$$

(First zero is at $kr = 4.4934$ for $\ell = 1$.)

Thus, the mass of the lowest $\sigma_{\text{TM}}$ state ($J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$) is $M(\sigma_{\text{TM}}) = M(\sigma) + 795$ MeV. Adding $q\bar{q}$ in a $J^P = 1^-$ state to a certain system will have a similar effect energetically to that of adding $\sigma_{\text{TM}}$. Thus $\frac{1}{2}^+$ and $\frac{3}{2}^+$ hybrino states have the mass $M(\sigma\bar{q}) \sim M(\sigma) + 800$ MeV. The negative parity state is around several hundred MeV higher than this. Thus, we find an important difference between the BS model (or any relativistic potential model) and bag model results: in the BS model the lowest hybrino states and lowest $\sigma\bar{q}$ states have the same parity, while in the bag model these states have opposite parities. In the BS model the lowest glueballino state is in an S wave, i.e., $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$. In the bag model, on the other hand, the motion of the gluino is neglected and the lowest gluon state is in the TE mode ($J^P = 1^+$) which corresponds to the P state, so that $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ glueballino states are the lowest. There is no such difference for a $\sigma\bar{q}$ state since two gluons are moving in a fixed cavity in the bag model and two negative parities arising from angular momentum compensate each other.

In principle, one can experimentally check on this difference in the respective predictions by measuring the relative parity between the ground states of glueballino and hybrino. If both $g$ and $\sigma$ are massive and heavy, it is almost evident that the $S$ state is the ground state. If we take the $m_\sigma \to 0$ limit, the BS model prediction goes smoothly to the one for $m_\sigma = 0$. On the other hand in the bag model if we take this limit the $S$ state and $P$ state
turn over suddenly at $m_g = 0$. In this respect, the BS model prediction would appear to be more natural than the bag model one.

B) Non-relativistic quark model (NRM)

Although the NRM for systems which include light quarks does not have a firm basis, it provides an excellent description for $q\bar{q}$ and $qqq$ systems\textsuperscript{20),21). In the NRM if the mass of the constituent particle increases, the binding energy increases. Bertlmann and Ono\textsuperscript{22) used the Feynman-Hellman theorem to find the range of mass of $m(t\bar{u})$ for any non-relativistic model with $m(t) = 20$ GeV

$$19.948 \leq M(t\bar{u}) \leq 20.017 \text{ GeV}$$

Thus the $u$ quark mass is almost cancelled by the increase of the binding energy, i.e., $\Delta E$ (binding energy) $\approx M(u)$.

In the case of a hybrino state the interaction between $q$ and $\bar{q}$ is weak due to the colour factor, thus it is neglected. Since $V_{q\bar{g}} \approx \frac{g}{8}qq^*$, $\Delta E$ (binding energy) per quark is larger than that for $t\bar{u}$ by a factor $\frac{g}{8}$. Thus one obtains the following estimate

$$M(\pi_q) \approx M(\tilde{g}) + 2M(u) - \frac{g}{8} \times 2 M(u)$$

$$\approx M(\tilde{g}) - 70 \text{ MeV}$$

Thus masses of hybrino states are much lower than the predictions of the BS model or the bag model and even the lowest $\tilde{g}g$ state might become unstable due to the inverse transition of Eq. (4.1). However, ambiguity remains on this conclusion since the NRM cannot predict the glueball mass spectrum and the masses defined in the NRM are the effective ones and need not be the same as those used in other models.

5. CONCLUSIONS

By using a BS model which is able to reproduce various properties of a few quark systems, we have computed $gg$ and $qqq$ spectra. We have found that some of the low lying states (including charged ones) become stable against strong
decay. The parities of the ground state glueballino and hybrino are the same in
the BS model, but opposite in the bag model. The level splitting between 1P
and 1S states for the glueballino is larger than that for the hybrino due to
the colour factors and due to the normal effects of relativistic two-body dynamics
versus three-body dynamics\textsuperscript{9),10)}. The non-relativistic quark model predicts re-
latively low masses for hybrino states. Thus, if glueballino and hybrino states
are found, they will not only constitute a decisive support for SUSY but also
provide a crucial test for BS, bag and non-relativistic potential models.

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REFERENCES

1) Proceedings of the CERN Workshop on SUSY Versus Experiment, organized by
D.V. Nanopoulos, A. Savoy-Navarro and Ch. Tao - CERN Preprint TH.3311/
EP.82/63-CERN (1982).

2) H.E. Haber and G.L. Kane - University of Michigan Preprint UMTH83-18 (July
1983).


6) D.L. Scharre - Talk at the International Symposium on Lepton and Photon
Interactions at High Energies, Bonn (August 1981) ;
E.D. Bloom - XXI International Conference on High Energy Physics, Paris
(July 1982).


8) C. Heusch - Private communication.


14) A.N. Mitra and A. Mittal - Delhi University Preprint (May 1983).


20) S. Ono - Lectures presented at the XXIII Cracow School of Theoretical


FIGURE CAPTIONS

**Fig. 1**  The glueballino (gg) and hybrino (gq̅q̅) spectra obtained by the BS model. The notation for hybrino is, e.g., $\pi^+ \equiv gud$, $S = \frac{1}{2}$. We assume $J^P = \frac{1}{2}^-$ for gluino.

**Fig. 2**  The free gluino decay
a) two-quark jets creation;
b) single-gluon jet creation.