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Note Added

TH.3707: Properties of Susy Hadrons in a Bethe-Salpeter Model

A.N. Mitra and S. Ono.

TH.3724: How would the world look like if there were supersymmetric particles?

D.V. Nanopoulos, S. Ono and T. Yanagida

NOTE ADDED

For glueballon ($\tilde{g}\tilde{g}$ -state) we quoted the selection rule $(-1)^{S+\lambda} = -1$ derived by Zug et al., Phys. Rev. D28 (1983) 1706. However, this will be wrong since the intrinsic parity should not come into the Fermi symmetry condition. The correct selection rule will be $(-1)^{S+\lambda} = +1$. Correct potential for $\tilde{g}\tilde{g}$ -state should contain a factor two from exchange diagram, i.e., $V_{\tilde{g}\tilde{g}}(r) = V_{\tilde{g}\tilde{g}}(r) = V_{\tilde{g}\tilde{g}}(r) = (9/4)V_{qq}(r)$. Thus, the decay width for the lowest $\tilde{g}\tilde{g}$ will become, e.g.,

$$\Gamma_{\tilde{S}_0 \rightarrow gg} = 7 \times 13.5 \times (1.5) \text{ MeV} = (100-500) \text{ MeV}$$

One of the authors (S.O.) would like to thank H. Kühn for pointing out them. More detailed properties of $\tilde{g}\tilde{g}$ -state are studied in the following paper: "Production and decay of gluino-gluino bound state", by J.H. Kühn and S. Ono, Orsay preprint, LPTHE Orsay 84/2.

Note Added

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PROPERTIES OF SUSY HADRONS IN A BETHE-SALPETER MODEL

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A B S T R A C T

We study the gluino gluon ($\tilde{g}g$) and hybrino ($\tilde{g}q\bar{q}$) states in a Bethe-Salpeter model of confinement, characterized by a reduced spring constant and quark mass, which provides an excellent description of masses and couplings of $q\bar{q}$ and qqq hadrons. Our predictions for $\tilde{g}g$ spectrum are completely different from the bag model results. Our model has the advantage that (i) in the $m_q \rightarrow 0$ limit, the $\tilde{g}g$ mass spectrum is smoothly connected to that for $m_q = 0$, (ii) the effect of the centre-of-mass motion is automatically removed. Properties of these new hadrons are discussed from the point of view of possible experimental detection.

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1. - INTRODUCTION

Although supersymmetry (SUSY) has been extensively studied at the purely theoretical level, its experimental aspects, especially the investigation of the properties of SUSY partners of known hadrons, their modes of production and decay (which in turn depend on their mass patterns), are a comparatively new development^{1),2)}. Now by the very nature of supersymmetry, the interaction strengths of SUSY particles are governed by exactly the same laws as those of ordinary particles, so that the same QCD-oriented techniques that have been extensively employed in the recent years to study the bound states of $q\bar{q}$, qqq and gg systems can be employed for their SUSY partners without bringing in fresh parameters.

In the study of SUSY hadrons, or R hadrons for short, a central rôle is played by the gluino ($\tilde{g}$) which is believed to be light enough in many SUSY theories ($\gtrsim 1$ GeV) for receiving the earliest attention at the phenomenological level. Being a coloured octet, it has a big enough coupling constant to any coloured particle, so that for $m_{\tilde{g}}$ (gluino mass) $\gtrsim 1$ GeV, the gluino will be produced more than ten times as frequently as, e.g., quark pairs in $p\bar{p}$ scattering²⁾. Further, if $m_{\tilde{g}}$ is less than $m(t\bar{t})$, the gluino can also be produced in the decay of toponium, e.g., $t\bar{t} \rightarrow \gamma\tilde{g}\tilde{g}$ which is also believed to have a reasonably large branching ratio^{3),4)}. Still another possibility is the decay $Z^0 \rightarrow \tilde{g}\tilde{g}$ which can be seen in LEP²⁾.

The gluino is believed to be relatively long lived (10^{-8} - 10^{-20} sec) so as to be experimentally identifiable without much difficulty. Indeed, glueballino states ($\tilde{g}g$) should be easier to identify experimentally than, e.g., glueball states which have a tendency to show strong mixing effects with $q\bar{q}$ states. In this respect, the glueball candidate $\iota(1440)$ ⁵⁾ has been the subject of much controversy⁶⁾. Indeed, shortly after the discovery^{5),6)} of this resonance, Ono and Pene⁷⁾ have shown that properties of this state can be explained alternatively as a radially excited $u\bar{u} + d\bar{d}$ state. More recently, the Mark III group⁸⁾ has detected the $\iota \rightarrow p\gamma$ mode which rules out the pure gg assumptions and favours the $q\bar{q}$ interpretation⁷⁾. On the other hand, $\tilde{g}\tilde{g}$ states can be identified by checking the large missing transverse momenta carried away by the photino ($\tilde{\gamma}$).

In this paper we wish to discuss the spectral properties of what are believed to be the relatively light SUSY partner, viz. $\tilde{g}g$ and $\tilde{g}q\bar{q}$ states ($q = u, d, s$ only) from the point of view of their qualitative experimental signatures. For this purpose we employ the Bethe-Salpeter equations for two- and

three-body systems developed by one of us⁹⁾ and applied to light $q\bar{q}$ and qqq systems¹⁰⁾⁻¹⁴⁾ with considerable success. The model which is characterized by a universal spring constant ($\tilde{\omega}$) and the quark mass (m_q) of the flavour sector under study, provides an excellent description of meson^{10),12)} and baryon^{10),11)} spectra, electromagnetic properties of mesons¹²⁾ and baryons¹⁴⁾, as well as their pionic couplings¹³⁾, all within an integrated framework.

We sketch the essential steps in the derivation of $\tilde{g}g$ and $\tilde{g}\tilde{g}$ mass patterns in Section 2 and that of $\tilde{g}q\bar{q}$ mass patterns in Section 3. Discussions and comparison with other model predictions are presented in Section 4. Conclusions are given in Section 5.

2. - THE $\tilde{g}g$ (GLUEBALLINO) AND $\tilde{g}\tilde{g}$ (GLUEBALLON) SPECTRA

The Bethe-Salpeter model⁹⁾ to be used in this calculation is characterized by a common colour ($F_1 \cdot F_2$) and spin ($\gamma_\mu^{(1)} \gamma_\mu^{(2)}$) dependence of both the confining and the one-gluon exchange (Coulomb) parts of the $q\bar{q}$, qq and gg ¹⁵⁾ interactions. (In this sense it is a vector, colour dependent model of confinement, of which more will be said in Section 4.) There are two basic ingredients to the calculation, the colour factors and the reduced spring constant. Now by the very logic of the SUSY concept, the over-all interaction strength of $\tilde{g}g$, $\tilde{g}\tilde{g}$ and gg pairs are intimately related. Of these, the colour factors are unambiguous, viz.

$$F_g \cdot F_{\tilde{g}} = -3, \quad F_g \cdot F_g = F_{\tilde{g}} \cdot F_{\tilde{g}} = -3 \times 2 \quad (2.1)$$

where the factor 2 for the gg and $\tilde{g}\tilde{g}$ cases comes from the exchange effect. As to the spring constant, the most successful predictions of hadron spectra¹²⁾⁻¹⁴⁾ and other properties¹²⁾⁻¹⁴⁾ have been based on the universality of the "reduced" spring constant ($\tilde{\omega}$) defined through¹⁰⁾

$$\omega_{g\tilde{g}}^2 = \tau_{12} m_{12} \tilde{\omega}^2 \quad (2.2)$$

$$m_{12} \equiv m_1 + m_2, \quad \tau_{12} \equiv 4 m_1 m_2 m_{12}^{-2} \quad (2.3)$$

[Eq. (2.2) was used in Ref. 12) without the τ_{12} factor but it made no difference for $m_1 = m_2$ and little difference to the kaon results as τ_{12} was 0.99 for u , d and s masses used.] Further, we have checked afresh the working of Eq. (2.2)

for the D, F and B meson regions using the formalism of Ref. 12) with $\alpha_s = 0.4$ and the following values of the masses are obtained (in GeV) :

$$\begin{aligned} \tilde{\omega} &= 0.14, \quad m_{u,d} = 0.28, \quad m_s = 0.38, \\ m_c &= 1.71, \quad m_b = 5.66 \end{aligned} \quad (2.4)$$

These are in conformity with the same values of $\tilde{\omega}$ and $m_{u,d}$ as used in Refs. 10)-14). The results in MeV are

	D	D*	F	F*	B	B*
M(TH)	1875	2009	1971	2103	5281	5335
M(EXP)	1865	2010	1970	2130?	5283	?

In obtaining this spectrum, an over-all short fall of one unit in the zero point oscillator energy has been taken into account (i.e., $N + \frac{3}{2} \rightarrow N + \frac{1}{2}$) as found in Ref. 11) and elsewhere¹⁶⁾, to hold very nearly for all types of hadrons ($q\bar{q}$, qqq , $qq\bar{q}\bar{q}$). These results provide the necessary check that should warrant the continued identification of $\tilde{\omega}$ as the universal spring constant for the interaction of SUSY partners of quarks and gluons, in accordance with the ansatz (2.2). In this connection a word of caution needs to be added regarding the status of the gluon mass which should vanish in the strict chiral limit. However, as has been argued elsewhere¹⁵⁾, it is not meaningful to speak of a strictly massless "confined" gluon, though the problem of assuming a definite value for the "constituent" gluon mass (m_g) was sought to be circumvented in Ref. 15) through an alternative ansatz $2\omega_{gg}^2 = \omega_{q\bar{q}}^2$ to set the mass scale for the glueball scenario in the BS model¹⁵⁾. In the present paper, on the other hand, we shall continue to use the more logical ansatz (2.2), using a symbolic value for the gluon mass m_g (presumably small), but as will be seen, the final result for the $\tilde{g}g$ spectrum is insensitive to this parameter (and even has a well-defined limit when $m_g \rightarrow 0$), as long as the gluino mass ($m_{\tilde{g}}$) is reasonably large (≥ 1 GeV).

With these remarks the BS formalism for the $\tilde{g}g$ system is a straightforward application of the (unequal mass) techniques described earlier^{10),12)} and the three-dimensional (instantaneous) form of the BS equation reads as

$$D_{12} \psi(\vec{q}) = 9\omega_{\tilde{g}\tilde{g}}^2 \left[\frac{1}{4} \tau_{12} M_{12}^2 \nabla_{\vec{q}}^2 + \frac{1}{4} \hat{Q}_{\vec{q}} + \frac{11}{2} - 2 \vec{J} \cdot \vec{S} \right] \psi(\vec{q}) \quad (2.5)$$

where

$$D_{12} = 2 M_{12} \left\{ \vec{q}_{12}^2 - \frac{1}{4} \tau_{12} (M_{12}^2 - m_{12}^2) \right\} \quad (2.6)$$

$M_{12} \equiv M$ is the gg mass and all other symbols are as defined in Refs. 10) and 12). The solution of this equation, with the (perturbative) inclusion of the Coulomb term, may be expressed as¹⁰⁾

$$F_{HO}(M) + F_{COUL}(M) = N + \frac{3}{2} \quad (2.7)$$

where

$$\begin{aligned} \Omega_{g\tilde{g}} F_{HO}(M) = & \tau_{12} (M^2 - m_{12}^2) \\ & + \frac{9}{4} \omega_{g\tilde{g}}^2 M^{-1} (Q_N + \frac{11}{2} - 2 \vec{J} \cdot \vec{S}) \end{aligned} \quad (2.8)$$

$$\Omega_{g\tilde{g}} = 6 M \omega_{g\tilde{g}} \sqrt{2 M \tau_{12}} \quad (2.9)$$

$$M \Omega_{g\tilde{g}} F_{COUL}(M) = 2 \langle 3 \alpha_s \tau_{12} M_{12}^2 / r_{12} \rangle \quad (2.10)$$

and $\langle \dots \rangle$ is calculated for the quantum state under study. For the ground state¹⁰⁾

$$\psi_0(\vec{r}) = (\beta^2/\pi)^{3/4} e^{-\frac{1}{2}\beta^2 r^2} \quad (2.11)$$

$$\beta^4 = \frac{9}{8} \tau_{12} M \omega_{g\tilde{g}}^2 \quad (2.12)$$

Substitution of (2.2) in (2.8) leads to the simplified equation (after cancellation of the τ_{12} factor in every term)

$$\begin{aligned} M^2 - m_{12}^2 + \frac{18 \tilde{\omega}^2 m_{12}}{M} \left(Q_N + \frac{11}{2} - 2 \vec{J} \cdot \vec{S} \right) \\ + \frac{12 \alpha_s}{\sqrt{\pi}} \beta \delta M = 6 \sqrt{2 m_{12} M} \tilde{\omega} \left(N + \frac{3}{2} \right), \end{aligned} \quad (2.13)$$

where $\delta = 1$ for the ground state and 0(1) for others. Equation (2.13) already reveals the close proximity of M to m_{12} for large $m_1 = m_{\tilde{g}}$, thus an approximate solution being

$$M \approx m_{\tilde{g}} + m_g + 3\sqrt{2} \tilde{\omega} \left(N + \frac{1}{2}\right) - \frac{6\alpha_s}{\sqrt{\pi}} \delta [3\sqrt{2} m_g \tilde{\omega}]^{1/2} + O\left(\frac{\tilde{\omega}^2}{m_{\tilde{g}}}\right), \quad (2.14)$$

where the replacement $N + \frac{3}{2} \rightarrow N + \frac{1}{2}$ in (2.12) has been effected in accordance with the remarks following Eq. (2.4), as explained in Refs. 11) and 16). Taking $m_{\tilde{g}}$ to be small, Eq. (2.14) gives the following estimate for the glueballino mass

$$M(g\tilde{g}) \approx m_{\tilde{g}} + 300 \text{ MeV} \times (2N + 1), \quad (2.15)$$

using $\tilde{\omega} = 140 \text{ MeV}$, consistently with D, F spectra as well as earlier results.

For the sake of completeness, we give a brief sketch of the mass formula for the $\tilde{g}\tilde{g}$ spectrum which is more akin to gg case¹⁵⁾, except for the much larger gluino mass. The BS equation is now of the simpler (spinor-spinor)^{9),10)} form

$$\begin{aligned} (m_{\tilde{g}}^2 + p_1^2)(m_{\tilde{g}}^2 + p_2^2)\Psi(p, q) &= i F_1 \cdot F_2 \int \frac{d^4 k}{(2\pi)^4} \\ &\times [V_\mu^{(1)} V_\mu^{(2)} - \frac{1}{2}(q-k)^2] \frac{3}{4} (2\pi)^4 \omega_{\tilde{g}\tilde{g}}^2 \\ &\times \nabla_k^2 \delta(q-k) \bar{\Psi}(p, k) \end{aligned} \quad (2.16)$$

where the subtracted term $\frac{1}{2}(q-k)^2$ corresponds to the $\tilde{g}\tilde{g}$ counterpart of the "gauge invariant" adaptation¹⁵⁾ of the gg interaction arising out of the contact g^4 interaction in the glueball case¹⁵⁾. The allowed states in this case are governed by the selection rule

$$(-1)^{L+S} = -1 \quad (2.17)$$

The final spectrum which takes account of the exchange effect through the colour factor (2.1), is again expressible in a form similar to (2.12), viz.,

$$M^2 - m_{12}^2 + \frac{36 \tilde{\omega}^2 m_{12}}{M} \left(Q_N + \frac{15}{4} - 2 \vec{J} \cdot \vec{S} \right) + \frac{24 \alpha_s \beta \delta M}{\sqrt{\pi}} = 12 \sqrt{m_{12} M} \tilde{\omega} \left(N + \frac{3}{2} \right), \quad (2.18)$$

where

$$8 \beta^2 = \Omega_{\tilde{g}\tilde{g}} = 16 \tilde{\omega} \sqrt{M m_{12}} \quad (2.19)$$

Equation (2.17) finally simplifies

$$M \cong 2 m_{\tilde{g}} + 6 \tilde{\omega} \left(N + \frac{3}{2} \right) - \frac{12 \alpha_s \delta}{\sqrt{\pi}} \sqrt{3 m_{\tilde{g}} \tilde{\omega}} + O\left(\frac{\tilde{\omega}^2}{m_{\tilde{g}}}\right) \quad (2.20)$$

However, this time the negative Coulomb term is large because of the factor $\sqrt{m_{\tilde{g}}}$ which is more than offsets the logarithmic decrease of α_s at $m_{\tilde{g}}$. Thus unlike the $\tilde{g}\tilde{g}$ case, the $\tilde{g}\tilde{g}$ system becomes tightly bound and the number of bound states below the $\tilde{g}\tilde{g} \rightarrow \tilde{g}g + g\tilde{g}$ threshold increases, analogously to the heavy quarkonium case.

3. - SPECTRA OF HYBRINO ($\tilde{g}q\bar{q}$) SYSTEMS

We discuss next the case of $\tilde{g}q\bar{q}$ systems when $q, \bar{q}$ are light (u,d,s) quarks. Since the system is a three-body one with unequal mass kinematics⁹⁾, the algebra is now much more involved¹¹⁾ than, e.g., in a bag model calculation¹⁷⁾. However, to bring out the main features of the spectrum within a colour-dependent model of confinement⁹⁾, it is useful at the first step to make the simplifying assumption of equal masses ($m_2 = m_3$) for the two light quarks which either limits the system of u, d quarks or implies an average mass if one of them is an s quark (which is not much heavier than u or d in our "constituent" model). More important, this assumption must be considered in the background of the much heavier gluino mass m_1 which controls the main dynamics and facilitates several important simplifications based on the smallness of the ratio m_2/m_1 , so that it should not be difficult within such a framework to estimate corrections arising from $m_2 \neq m_3$ if and when the experimental situation so warrants.

The $\bar{g}q\bar{q}$ formalism is basically similar to that of strange qqq baryons¹¹⁾ involving unequal mass kinematics, except for two important differences. First, the colour factors $F_i \cdot F_j$ involved in $gq\bar{q}$ are no longer $-\frac{2}{3}$ for each pair as in a baryon system but instead are given by the following results for the three different pairs to make an over-all colour singlet hadron

$$F_{\bar{g}} \cdot F_g = F_{\bar{g}} \cdot F_{\bar{g}} = -\frac{3}{2} ; F_{g\bar{g}} = \frac{1}{6} \quad (3.1)$$

Thus the main confinement is now expected to arise from the $\bar{g}q$ and $\bar{g}\bar{q}$ pairs, while the $q\bar{q}$ pair contributes a small repulsion (much like in a helium-atom problem). Secondly, the inequality $m_1 \gg m_2$ now helps in greatly simplifying the algebra compared to the $(m_1 \gtrsim m_2)$ qqq case¹¹⁾. This last feature further reduces (in the ratio m_2^2/m_1^2) the already small effect due to the $q\bar{q}$ colour factor.

Thus in the first approximation it is permissible to neglect the $q\bar{q}$ part of the kernel from the three-body BS equation⁹⁾, an assumption which can be checked through a subsequent perturbation estimate if necessary.

With the simplification thus obtained, it is somewhat easier to write down the three-dimensional form of the BS equation for $\bar{g}q\bar{q}$, on lines similar to the qqq case⁹⁾⁻¹¹⁾, but with the new colour combinations Eq. (3.1) and with the $q\bar{q}$ kernel omitted. For this purpose, the normalized internal momenta $\vec{\xi}$, $\vec{\eta}$ are defined as¹¹⁾

$$\sqrt{\frac{m_0}{m_1}} \vec{\xi} = \vec{p}_3 - \vec{p}_2, \quad m_0 \vec{\eta} = -2 m_2 \vec{p}_2 + m_1 (\vec{p}_2 + \vec{p}_3) \quad (3.2)$$

where

$$m_0 = m_1 + m_2 + m_3 = m_1 + 2 m_2 \quad (3.3)$$

The "master" equation¹¹⁾ is now of the simpler form

$$\begin{aligned} & [m_0^2 - M^2 + A_{\xi} \xi^2 - B_{\xi} \nabla_{\xi}^2 + A_{\eta} \eta^2 - B_{\eta} \nabla_{\eta}^2] \psi(\vec{\xi}, \vec{\eta}) \\ & = \{ [RHS] + \langle COUL \rangle \} \psi(\vec{\xi}, \vec{\eta}), \end{aligned} \quad (3.4)$$

where, using the ansatz (2.2)

$$A_{\xi, \eta} = \frac{1}{4} \frac{m_0^3}{m_1 m_2 m_{12}} + \frac{1}{4} \frac{m_0^2}{m_{12}^2} \quad , \quad (3.5)$$

$$B_{\xi} = \frac{m_1}{m_0} B_{\eta} = \frac{18 M m_1^2 m_2^2}{m_{12}^2} \tilde{\omega}^2 \quad , \quad (3.6)$$

$$\begin{aligned} [RHS] = & \frac{9}{2} m_0^3 \tilde{\omega}^2 m_{12}^{-2} M^{-1} \left[\frac{2}{m_1 m_2} \left(\frac{m_0^2}{m_{12}^2} \eta^2 + \frac{m_0 m_1}{m_{12}^2} \xi^2 \right) \right. \\ & + \left(\frac{2 m_0^2}{m_{12}^2} - \frac{1}{2} \right) Q_{N\eta} + \left(\frac{2 m_1^2}{m_{12}^2} - \frac{1}{2} \right) Q_{N\xi} + \frac{4 m_0 m_1}{m_{12}^2} Q_{N\xi\eta} \\ & - 4 \vec{\sigma}_1 \cdot \vec{\sigma}_2 - 4 \vec{\sigma}_1 \cdot \vec{\sigma}_3 - \frac{4 m_0}{m_{12}} \vec{L}_{\eta} \cdot \left(\vec{S} + \frac{\vec{\sigma}_1}{2} \right) \\ & \left. - \frac{4 m_1}{m_{12}} \vec{L}_{\xi} \cdot \left(\vec{S} + \frac{\vec{\sigma}_1}{2} \right) \right] \quad , \quad (3.7) \end{aligned}$$

$$\langle COUL \rangle = 3 d_s \frac{m_0 M}{m_{12}} \left\langle \frac{1}{r_{12}} + \frac{1}{r_{13}} \right\rangle \quad , \quad (3.8)$$

and the unexplained notations are the same as in Ref. 11). The symbol $\langle COUL \rangle$ represents the shift in the M^2 value due to the one-gluon exchange interactions, calculated perturbatively, in the $\tilde{g}q$ and $\tilde{g}\bar{q}$ pairs.

To calculate the mass spectrum, we note first that the term $[RHS]$ in (3.4) will contribute momentum and spin-dependent corrections to the spectrum of $O(\tilde{\omega}^2/m_0)$ compared to main effects, exactly as in the $\tilde{g}g$ case, Eq. (2.13), and may therefore be neglected. The final mass spectrum then works out^{11), 16)} straightforwardly as

$$M^2 - m_0^2 = \Omega_{\xi} (N_{\xi} + 1) + \Omega_{\eta} (N_{\eta} + 1) + \langle COUL \rangle \quad (3.9)$$

where

$$\Omega_{\xi} = 2 \sqrt{A_{\xi} B_{\xi}} \quad , \quad \Omega_{\eta} = 2 \sqrt{A_{\eta} B_{\eta}} \quad (3.10)$$

$$\langle \text{COUL} \rangle = - \frac{M m_0}{m_{12}} \frac{12 \alpha_s}{\pi} \left(\frac{m_1}{m_0} \right)^{1/4} (\beta_\xi \beta_\eta)^{1/2} \Gamma^2(3/4), \quad (3.11)$$

$$\beta_\xi^4 = \frac{B_\xi}{A_\xi}, \quad \beta_\eta^4 = \frac{B_\eta}{A_\eta} \quad (3.12)$$

Using the typical values (in GeV units)

$$m_1 = 3.0, \quad m_2 = 0.3, \quad \tilde{\omega} = 0.14, \quad (3.13)$$

eq. (3.9) yields the following M values for the $L = 0$ and $L = 1$ states (in GeV units)

$$\begin{aligned} M(L=0) &= 3.60 + O(\tilde{\omega}^2/m_0) \\ M(L_\xi=1, L_\eta=0) &= 3.87 + O(\tilde{\omega}^2/m_0) \\ M(L_\xi=0, L_\eta=1) &= 3.93 + O(\tilde{\omega}^2/m_0) \end{aligned} \quad (3.14)$$

Further, the universal length scales β_ξ and β_η for the ground state work out (in GeV units) as

$$\beta_\xi = 0.543, \quad \beta_\eta = 0.547 \quad (3.15)$$

We end this section with a qualitative estimate of the hitherto neglected $q\bar{q}$ interaction effect in the above calculation. For this purpose, it is convenient to express the three-body BS equation in the instantaneous approximation⁹⁾ in the symbolic form (in momentum space) :

$$\psi = \frac{K_{12} \psi}{D_{12}} + \frac{K_{13} \psi}{D_{13}} + \frac{K_{23} \psi}{D_{23}}, \quad (3.16)$$

where D_{ij} are energy denominators of the types described earlier^{9),10)} and K_{ij} 's represent the complete effects of the pairwise kernels. In particular,

$$D_{23} = 2 M_{23} \left[\frac{1}{4} \frac{m_0}{m_1} \xi^2 + \frac{\eta^2}{4} + m_2^2 (1 - M^2 m_0^{-2}) \right] \quad (3.17)$$

with $M_{23} = 2m_2 M/m_0$, in accordance with the theoretical framework⁹⁾⁻¹¹⁾. Now the effects of the first two terms in (3.16) are fully included in the basic Eq. (3.4); its simple-looking form reflecting the near equality of D_{12} , D_{13} (apart from some small ξ, η terms of opposite signs). Of the (23) term, its harmonic part is given by

$$K_{23}/D_{23} \approx - \frac{m_1 m_2^2 \tilde{\omega}^2}{M} \left(\frac{\eta^2}{4} + \frac{m_0}{4m_1} \xi^2 \right)^{-1} \vec{\nabla}_\xi^2 \quad (3.18)$$

where the neglect of the last term in (3.17) is justified by the twin effect of the small difference of M from m_0 as seen from Eq. (3.14) as well as the relative smallness of m_2^2 with respect to the (ξ^2, η^2) terms. Now a perturbative calculation of the expectation value of (3.18) in the "zero-order" ground state, viz., wave function determined from (3.9),

$$\psi_0(\xi, \eta) = (\pi \beta_\xi \beta_\eta)^{-3/2} \exp\left(-\frac{1}{2} \xi^2 \beta_\xi^{-2} - \frac{1}{2} \eta^2 \beta_\eta^{-2}\right) \quad (3.19)$$

shows, without further analysis, that

$$\langle K_{23}/D_{23} \rangle = O(m_2^2 \tilde{\omega}^2 / \beta_{\xi, \eta}^4) \lesssim 3\% \quad (3.20)$$

which merely amounts to multiplication correction of $\sim 3\%$ to the first two terms on the right-hand side of Eq. (3.16).

4. - DISCUSSIONS AND COMPARISONS WITH OTHER MODEL PREDICTIONS

From Eqs. (2.14) and (3.14), we find energy levels of glueballino and hybrino states which are shown in Fig. 1. Our notation for hybrino is the following. If the hybrino is composed of one known quarkonium (e.g., ρ) state and one gluino, we put a suffix $\tilde{g}$ (e.g., $\rho_{\tilde{g}}$). If the spins of $q\bar{q}$ and $\tilde{g}$ are parallel we put an asterisc (e.g., $\rho_{\tilde{g}}^*$) and if they are antiparallel we do not ($\rho_{\tilde{g}}$). The suffix $\tilde{g}$ is necessary to distinguish from a $\tilde{q}\bar{q}$ state where $\tilde{q}$ is the scalar partner of the quark. In principle, hybrino mixes with $\tilde{q}\bar{q}$ states.

Some of the states in Fig. 1 fall to lower states by emitting a π or a $\pi\pi$ pair, e.g.,

$$\left. \begin{aligned}
 \pi_{\tilde{g}} , \rho_{\tilde{g}} &\rightarrow (g\tilde{g})_{J=3/2^-} + \pi \\
 \rho_{\tilde{g}}^* &\rightarrow (g\tilde{g})_{J=1/2^-} + \pi \\
 (g\tilde{g})_{1P} &\rightarrow (\pi_{\tilde{g}} \text{ or } \rho_{\tilde{g}} \text{ or } \rho_{\tilde{g}}^*) + \pi
 \end{aligned} \right\} \Gamma \sim \text{several hundred MeV}$$

$$(g\tilde{g})_{1P} \rightarrow (g\tilde{g})_{1S} + \pi\pi \quad ; \quad \Gamma \sim O(10) \text{ KeV}$$

(4.1)

As seen from Fig. 1, hybrino ground states with $I = 0, \frac{1}{2}$ become stable against strong and electromagnetic decays. Especially $K_{\tilde{g}}, K_{\tilde{g}}^*$ contain charged particles which are easier to see experimentally.

Decays of these metastable particles are dominated by free gluino decay by leaving other particles as spectators, whose diagrams are shown in Figs. 2a and 2b. Decay rates are given by²⁾

$$\Gamma_{\tilde{g}}^{(2a)} = \sum_{\tilde{g}} \frac{\alpha_s \alpha e_g^2 M_{\tilde{g}}^5}{96 \pi} \left(\frac{1}{\tilde{m}_L^4} + \frac{1}{\tilde{m}_R^4} \right)$$

(4.2)

$$\Gamma^{(2b)} / \Gamma^{(2a)} = \sum_{\text{flavours}} \frac{\alpha_s}{4} \frac{(\tilde{m}_R^2 - \tilde{m}_L^2)^2}{\tilde{m}_L^4 + \tilde{m}_R^4}$$

$$\sim \frac{1}{20} \quad (\text{in a model}^2)$$

(4.3)

Spectators do not have enough energy to produce jets. Thus, the decay 2b produces only one gluon jet with large missing P_T carried away by the photino which usually escapes from detection. The decay 2a produces two jets, also with large missing p_T . The decay 2b is easier to identify than the decay 2a. If $M_{\tilde{g}}$ is small enough and scalar quark masses $\tilde{m}_L, \tilde{m}_R$ are large enough the life times of charged $K_{\tilde{g}}, K_{\tilde{g}}^*$ can become long enough so that they can be seen even in a bubble chamber. Metastable neutral hybrino states will not be difficult to observe either, since they can hit protons whose recoil is observable.

We now discuss the model dependence of energy levels of glueballino and hybrino states.

A) Bag model

In the bag model, the lowest glueballino state is in a TE mode, i.e., $\tilde{g}g_{TE}$ with $J^P = \frac{1}{2}^-, \frac{3}{2}^-$. The energy level is given by¹⁸⁾

$$\frac{d}{dr} [r j_\ell(kr)]_{r=R} = 0 \quad (4.4)$$

(First zero is at $kR = 2.7437$ for $\ell = 1$.) If we use R from Ref. 19), we get $M(\tilde{g}g_{TE}) = M(\tilde{g}) + 480$ MeV. The boundary condition for the TM mode is given by

$$j_\ell(kr) \Big|_{r=R} = 0 \quad (4.5)$$

(First zero is at $kR = 4.4934$ for $\ell = 1$.)

Thus, the mass of the lowest $\tilde{g}g_M$ state ($J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$) is $M(\tilde{g}g_{TM}) = M(\tilde{g}) + 795$ MeV. Adding $q\bar{q}$ in a $J^P = 1^-$ state to a certain system will have a similar effect energetically to that of adding g_{TM} . Thus $\frac{1}{2}^+$ and $\frac{3}{2}^+$ hybrino states have the mass $M(\tilde{g}q\bar{q}) \sim M(\tilde{g}) + 800$ MeV. The negative parity state is around several hundred MeV higher than this. Thus, we find an important difference between the BS model (or any relativistic potential model) and bag model results : in the BS model the lowest hybrino states and lowest $\tilde{g}g$ states have the same parity, while in the bag model these states have opposite parities. In the BS model the lowest glueballino state is in an S wave, i.e., $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$. In the bag model, on the other hand, the motion of the gluino is neglected and the lowest gluon state is in the TE mode ($J^P = 1^+$) which corresponds to the P state, so that $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ glueballino states are the lowest. There is no such difference for a gg state since two gluons are moving in a fixed cavity in the bag model and two negative parities arising from angular momentum compensate each other.

In principle, one can experimentally check on this difference in the respective predictions by measuring the relative parity between the ground states of glueballino and hybrino. If both g and $\tilde{g}$ are massive and heavy, it is almost evident that the S state is the ground state. If we take the $m_g \rightarrow 0$ limit, the BS model prediction goes smoothly to the one for $m_g = 0$. On the other hand in the bag model if we take this limit the S state and P state

turn over suddenly at $m_g = 0$. In this respect, the BS model prediction would appear to be more natural than the bag model one.

B) Non-relativistic quark model (NRQM)

Although the NRQM for systems which include light quarks does not have a firm basis, it provides an excellent description for $q\bar{q}$ and qqq systems^{20),21)}. In the NRQM if the mass of the constituent particle increases, the binding energy increases. Bertlmann and Ono²²⁾ used the Feynman-Hellman theorem to find the range of mass of $m(t\bar{u})$ for any non-relativistic model with $m(t) = 20 \text{ GeV}$

$$19.948 \leq M(t\bar{u}) \leq 20.017 \text{ GeV} \quad (4.6)$$

Thus the u quark mass is almost cancelled by the increase of the binding energy, i.e., $\Delta E (\text{binding energy}) \approx M(u)$.

In the case of a hybrino state the interaction between q and $\bar{q}$ is weak due to the colour factor, thus it is neglected. Since $V_{q\tilde{g}} \approx \frac{9}{8} V_{q\bar{q}}$, ΔE (binding energy) per quark is larger than that for $t\bar{u}$ by a factor $\frac{9}{8}$. Thus one obtains the following estimate

$$\begin{aligned} M(\pi_{\tilde{g}}) &\approx M(\tilde{g}) + 2M(u) - \frac{9}{8} \times 2M(u) \\ &\approx M(\tilde{g}) - 70 \text{ MeV} \end{aligned} \quad (4.7)$$

Thus masses of hybrino states are much lower than the predictions of the BS model or the bag model and even the lowest $\tilde{g}g$ state might become unstable due to the inverse transition of Eq. (4.1). However, ambiguity remains on this conclusion since the NRQM cannot predict the glueball mass spectrum and the masses defined in the NRQM are the effective ones and need not be the same as those used in other models.

5. - CONCLUSIONS

By using a BS model which is able to reproduce various properties of a few quark systems, we have computed $\tilde{g}\tilde{g}$ and $\tilde{g}q\bar{q}$ spectra. We have found that some of the low lying states (including charged ones) become stable against strong

decay. The parities of the ground state glueballino and hybrino are the same in the BS model, but opposite in the bag model. The level splitting between $1P$ and $1S$ states for the glueballino is larger than that for the hybrino due to the colour factors and due to the normal effects of relativistic two-body dynamics versus three-body dynamics^{9),10)}. The non-relativistic quark model predicts relatively low masses for hybrino states. Thus, if glueballino and hybrino states are found, they will not only constitute a decisive support for SUSY but also provide a crucial test for BS, bag and non-relativistic potential models.

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FIGURE CAPTIONS

Fig. 1 The glueballino ($\tilde{g}g$) and hybrino ($\tilde{g}q\bar{q}$) spectra obtained by the BS model. The notation for hybrino is, e.g., $\pi_{\tilde{g}}^+ \equiv \tilde{g}u\bar{d}$, $S = \frac{1}{2}$. We assume $J^P = \frac{1}{2}^-$ for gluino.

Fig. 2 The free gluino decay
a) two-quark jets creation ;
b) single-gluon jet creation.

Mass - M(g)

- Figure 1 -

(a)

(b)

- Figure 2 -