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RHO MESON COUPLING TO THE NUCLEON IN THE CHIRAL BAG MODEL

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A B S T R A C T

The vector and tensor couplings of the ρ meson to the nucleon are calculated by assuming the indirect coupling of the ρ to the nucleon, through the pion cloud of the chiral bag model. Qualitative agreement with experiment is found at reasonable bag radii for both versions of the model, the CBM with pions in the interior, and the standard model with pions excluded from the bag.

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1. INTRODUCTION

Chiral bag models in their different versions¹⁻⁷⁾ have increasingly gained recognition as phenomenological tools that can provide a fair description of the physical properties of the nucleon and other baryons. In all these models the pion is introduced in order to restore the continuity of the axial-vector current, which has a discontinuity on the surface in the original MIT bag model⁸⁾. The pion is coupled to the surface of the bag. Some versions exclude the pions from the interior of the bag²⁻³⁾ (we will call this version the standard model), whilst others allow the pions to penetrate into the interior^{1,4-6)} [we will refer to this version by its usual name of cloudy bag model, CBM^{4,5)}].

Besides a residual colour interaction due to gluon exchange, the only ingredients in these bag models are the quarks, and the Goldstone boson (the pion), which appears as a consequence of the chiral symmetry breaking. With these ingredients alone, we may start asking questions about the interpretation of the empirical coupling of other mesons to the nucleons, such as the ρ meson, which is relatively well known. The ρ meson plays an important role in many pionic processes in nuclei⁹⁾, and the interpretation of its coupling to the nucleon in the bag picture should be of interest at a time when the quark degrees of freedom are invoked in order to interpret some peculiarities of nuclear reactions and properties.

A weak point in these chiral bag models is that they are unstable unless other ingredients are introduced into the theory. Among some mechanisms invoked to stabilize the bag, the idea of de Kam and Pirner¹⁰⁾ -- of considering pion finite size effects -- looks appealing because of its simplicity and because it does not introduce foreign elements to the quark and the Goldstone pion. Another factor in its favour is that consideration of pion finite size effects eliminates the disturbing divergences that one finds in the evaluation of nucleon pionic self-energies^{11,12)}.

The idea of the present paper is to respect the simplicity of the chiral bag model by keeping only the quarks and the pions as the essential ingredients. Since we explicitly want to avoid a direct coupling of the ρ to the nucleon constituent quarks, the coupling of the ρ meson to the nucleon has necessarily to go via the virtual pion cloud attached to the nucleon, as shown in fig. 1. Another way of looking at it is to say that the ρ meson decouples into two pions (by far the most important decay channel), which later couple to the quarks of the nucleon according to the rules of the chiral bag model.

The paper proceeds as follows. In Section 2 the model for the coupling of the ρ meson to the nucleon through the pion cloud is outlined. In Section 3 the specific contribution from intermediate nucleon and Δ states is calculated. In Section 4 we derive the general formula for the contribution of excited intermediate states, whilst the contribution from the states of negative energy is discussed in Section 5. Finally, in Section 6 we show the numerical results and discuss their implications. Section 7 is a brief summary of the work.

2. THE MODEL FOR THE ρ -MESON COUPLING

As we stated in the Introduction, the idea of the present work is to couple the ρ meson to the nucleon indirectly, through the meson cloud that surrounds the nucleon. In order to do so we do not need to invoke a bag model for the hadrons. The pions couple to the nucleons and other hadrons, considered as elementary particles, and this coupling can be taken into account phenomenologically by means of effective interaction Lagrangians, complemented by form factors. Diagrammatically the coupling is given by Fig. 1, where B can be any intermediate hadronic state that can be excited by the pion coupling. The evaluation of the contribution when $B = N, \Delta$ is straightforward, and we show the details in Section 3. However, if we start introducing other excited hadronic states, N^*, Δ^* , etc., we run into problems because of our ignorance of the hadronic wave functions as well as the transition Hamiltonians $H_{\pi NB}$. One has to rely upon some model, and it is here where we introduce the chiral bag model

which offers simultaneously an approximate picture for the excited baryonic states as well as for the $\pi N B$ vertices.

For the $\rho\pi\pi$ vertex we use the conventional interaction Lagrangian

$$\mathcal{L}_I(x) = f_\rho \vec{\rho}^\mu (\vec{\phi} \times \partial_\mu \vec{\phi}) , \quad (1)$$

where $\vec{\rho}^\mu$ and $\vec{\phi}$ are the ρ -meson and pion fields, respectively. This Lagrangian contributes a factor for the vertex of Fig. 2 given by

$$V_{\rho\pi\pi} = - f_\rho (p_2 - p_1)^\alpha \varepsilon_\alpha^*(q) \varepsilon_{\lambda\mu\nu} , \quad (2)$$

where $\varepsilon_\alpha(q)$ is the ρ -meson polarization vector and λ, μ, ν are the isospin indices for the ρ meson and the two pions.

Since we are interested in the coupling of a ρ meson of zero frequency, we have taken for f_ρ the value given by the KSRF relation¹³⁾, based upon current algebra and the assumption of a universal coupling of the ρ meson to the isovector current

$$\frac{f_\rho^2}{4\pi} = \frac{1}{2} \frac{g^2}{4\pi} \left(\frac{m_\rho}{M g_A} \right)^2 = 3.1 \quad (3)$$

where $g^2/4\pi$ is the πNN coupling constant and $g_A = 1.25$ the axial-current coupling constant. This value is essentially the same as the one that we derive from the ρ -meson width, $f_\rho^2/4\pi = 3.0$, in a different kinematical situation for the pions and the ρ meson, and very close to other values derived from the πN s-wave scattering lengths, K^* decay and other decays¹⁴⁾. Hence we will use the coupling of Eq. (3) in the evaluation of the ρNN coupling.

3. N AND Δ CONTRIBUTION

In terms of the conventional πNN and $\pi N \Delta$ couplings⁹⁾, the contribution from Figs. 4a and 4b to the ρNN coupling is given by

$$V_{\rho NN}^{(N)}(q) = \sum_{s'', t''} \int \frac{d^4 K}{(2\pi)^4} \frac{f^2}{\mu^2} \langle s' | \vec{\sigma}(\vec{k}-\vec{q}) | s'' \rangle \langle s'' | \vec{\sigma}(-\vec{k}) | s \rangle$$

$$u(\vec{k}) u(\vec{k}-\vec{q}) \langle t' | \vec{\tau}^\mu | t'' \rangle \langle t'' | \vec{\tau}^\nu | t \rangle \epsilon_{\lambda\mu\nu} \cdot$$

$$\cdot (-) f_\rho (2k-q)^\mu \epsilon_\mu^* \frac{i}{k^0 2 - \vec{k}^2 - \mu^2 + i\epsilon} \frac{i}{(k^0 - q^0)^2 - (\vec{k}-\vec{q})^2 - \mu^2 + i\epsilon}$$

$$\frac{i}{p^0 - k^0 - \epsilon(\vec{p}-\vec{k}) + i\epsilon} \quad (4)$$

and a similar contribution for the Δ case by substituting $f \rightarrow f^*$, $\vec{\sigma}(\vec{k}-\vec{q}) \rightarrow \vec{S}(\vec{k}-\vec{q})$, $\vec{\sigma}\vec{k} \rightarrow \vec{S}\vec{k}$, $\tau^\mu \rightarrow T^\mu$, $\tau^\nu \rightarrow T^{\nu\dagger}$, where $\vec{S}$ and $\vec{T}$ are the spin and isospin transition operators. The function $u(\vec{k})$ is the πNN vertex form factor.

By carrying out the spin and isospin sums and the k^0 integration, taking into account the analytical properties of the integrand in the complex plane, we have

$$V_{\rho NN}^{(N)}(q) = 2 \int \frac{d^3 k}{(2\pi)^3} \frac{f^2}{\mu^2} \left\{ (-i)(\vec{k}-\vec{q}) \vec{k} + \epsilon_{ij\epsilon} k_i q_j \sigma_\epsilon \right\} \vec{\sigma}^\lambda$$

$$u(\vec{k}) u(\vec{k}-\vec{q}) f_\rho \left\{ \epsilon^{\alpha*} F_1(k, q, p, \epsilon) - (2\vec{k}-\vec{q}) \vec{\epsilon}^* F_2(k, q, p, \epsilon) \right\}$$

(5)

where the functions $F_i(k, q, p, \epsilon)$ are given by

$$F_1(k, q, p, \epsilon) = \frac{1}{p} \left\{ \omega^2 (2\omega' + q^0) + \omega [2\omega'^2 - q^0(p^0 - \epsilon)] \right.$$

$$\left. + \omega' q^0 (p^0 - \epsilon - \omega' - q^0) \right\}$$

$$F_2(k, q, p, \epsilon) = \frac{1}{p} \left\{ \omega^2 + \omega (2\omega' - p^0 + \epsilon) + \omega'^2 + \omega' q^0 \right.$$

$$\left. - \omega' p^0 + \omega' \epsilon \right\}$$

with

$$P = 2 \omega \omega' (\omega - q^0 + \omega' - i\epsilon) (\omega + q^0 + \omega' - i\epsilon) (p^0 - \omega - \epsilon + i\epsilon) \cdot (p^0 - q^0 - \omega' - \epsilon + i\epsilon)$$

and

$$\begin{aligned} \omega &= \omega(\vec{k}) \\ \omega' &= \omega(\vec{k} - \vec{q}) \\ \epsilon &= \epsilon(\vec{p} - \vec{k}) \end{aligned} \quad (6)$$

The function P^{-1} reflects the analytic structure of Fig. 4a with poles which correspond to having either two pions or one pion and one nucleon on shell. In the kinematical regime which we study, we will not encounter these singularities. The vertex function of Eqs. (2) and (5) already have the factor $(-i)$ from perturbation theory included.

We would now like to compare this result with the phenomenological coupling of a ρ meson to the nucleon given by the vertex function¹⁵⁾

$$V_{\rho NN}(q) = (-i) \bar{u}(p) \left\{ G_{NN\rho}^V \gamma^\mu + \frac{G_{NN\rho}^T}{2i m_p} \sigma^{\mu\nu} q_\nu \right\} \epsilon_\mu^* u(p) \vec{\sigma} \cdot \hat{p} \quad (7)$$

corresponding to the vertex of Fig. 5, with $\hat{p}$ the isospin unit vector for the ρ meson and $q = p - p'$.

It is useful to write Eq. (7) in the Breit frame, $q^0 = 0$, in terms of Pauli spinors, in order to facilitate the comparison with the results from Eq. (5).

For a ρ meson of isospin λ we have

$$V_{\rho NN}(q) = \chi_f \left\{ -i G_{NN\rho}^E(q) \epsilon^{0*} + \frac{G_{NN\rho}^M(q)}{2 m_p} (\vec{\sigma} \times \vec{q}) \cdot \vec{\epsilon}^* \right\} \vec{\sigma} \cdot \hat{\chi}_i \quad (8)$$

where, by analogy to the electromagnetic form factors, we have introduced

$G_{NN\rho}^E$ and $G_{NN\rho}^M$ given by

$$G_{NNP}^E(q) = G_{NNP}^V - \frac{\vec{q}^2}{4m_p^2} G_{NNP}^T$$

$$G_{NNP}^M(q) = G_{NNP}^T + G_{NNP}^V$$
(9)

Thus, by looking at Eqs. (5) and (8) we get

$$G_{NNP}^{E(N)}(q) = 2 \int \frac{d^3k}{(2\pi)^3} f_p \frac{f^2}{\mu^2} (\vec{k}-\vec{q}) \cdot \vec{k} u(\vec{k}) u(\vec{k}-\vec{q}) F_1(k, q, p, E)$$
(10)

For the tensor part of the coupling, we have the non-vanishing terms

$$-2\delta^{\lambda} \int \frac{d^3k}{(2\pi)^3} \frac{f^2}{\mu^2} f_p \epsilon_{ije} k_i q_j 2k_m \epsilon_m^* \sigma_e u(\vec{k}) u(\vec{k}-\vec{q}) F_2(k, q, p, E)$$
(11)

where spatial contravariant components are understood.

We will have

$$\epsilon_{ije} q_j \epsilon_m^* \sigma_e \int d^3k \dots k_i k_m = (A \delta_{im} + B q_i q_m) \epsilon_{ije} q_j \epsilon_m^* \sigma_e$$
(12)

but the B term will not contribute to formula (12). By taking the trace (δ_{im}) in formula (12) and multiplying by $q_i q_m$ we get two equations that allow us to get the coefficient A.

This term contributes directly to the $(\vec{\sigma} \times \vec{q}) \cdot \vec{e}^*$ term of (8). Hence we obtain

$$\frac{G_{NNP}^{M(N)}}{2m_p} = 2 \int \frac{d^3k}{(2\pi)^3} \frac{f^2}{\mu^2} f_p u(\vec{k}) u(\vec{k}-\vec{q}) F_2(k, q, p, E) \left\{ \vec{k}^2 - \frac{1}{q^2} (\vec{k} \cdot \vec{q})^2 \right\}$$
(13)

In a completely analogous way, we find for the contribution of the Δ intermediate state

$$G_{NNP}^{E(\Delta)} = \frac{-4}{9} \int \frac{d^3k}{(2\pi)^3} \frac{f^{*2}}{\mu^2} f_p u(\vec{k}) u(\vec{k}-\vec{q}) F_1(k, q, p, E = M_\Delta - M_N + T_\Delta) (\vec{k}-\vec{q}) \cdot \vec{k}$$
(14)

$$\frac{G_{NN\pi}^{M(\Delta)}}{2m_p} = \frac{2}{9} \int \frac{d^3k}{(2\pi)^3} \frac{f^{*2}}{\mu^2} f_p u(\vec{k}) u(\vec{k}-\vec{q}) F_2(k, q, p, E = M_\Delta - M_N + T_0) \cdot \left\{ \vec{k}^2 - \frac{1}{q^2} (\vec{k} \cdot \vec{q})^2 \right\} \quad (15)$$

Note also that the explicit calculation of these terms allows, apart from the use of the physical coupling constants, the use of the proper energy for the Δ state, which in the bag model would be degenerate with the nucleon, unless pionic and gluonic corrections are considered in addition to the pure bag energies.

4. OTHER INTERMEDIATE-STATE CONTRIBUTIONS TO THE ρ NN COUPLING

We would now like to consider the contribution to the ρ NN coupling from baryonic intermediate states other than the nucleon and Δ which we have already taken into account. We could include the contribution from the first excited baryonic states, but soon the procedure would prove impracticable since we would be running into states where we would have little or no information about their wave functions and the coupling to the pion.

At this point the MIT bag model^{1,2,8)} would prove a useful tool that would provide a first approximation to the excited states and a complete set of them. When the pion field is included in the model, as is done in the chiral bag models, we also obtain, in a straightforward way, the transition Hamiltonians needed in our calculation.

However, in classical chiral bag models¹⁻³⁾ we are restricted to static pions and quarks in their ground state, and hence we get stuck at the same point where we are now, i.e. contribution from N and Δ as intermediate states. The easiest way to go further is to assume that the Hamiltonian operator which we have in the classical model not only connects diagonal quark states but is also responsible for non-diagonal transitions. This statement is equivalent to quantizing the model, as has been done for the CBM^{4,5)} or for the standard bag model¹¹⁾. It has been suggested¹¹⁾ and proved analytically¹²⁾ that this model

contains divergences in the nucleon self-energy, and other physical quantities which are not easily removable owing to the lack of translational invariance of the model. As stated in the Introduction, inclusion of extra form factors due to the consideration of the finite size of the pion removes these divergences and at the same time solves the problem of the collapse of the classical chiral bag. By doing so, we sacrifice some of the simplicity of the original chiral bag model where the pion is a point-like field, while here it is an extended source. The coupling of the pion to the bag becomes non-local and the axial current also contains non-localities close to the bag surface.

Obviously this quantum mechanical extrapolation of the classical chiral bag is not unique but it is a simple one to start with. In this way a new phenomenological model is constructed which allows us to face new problems. The ability of the model to give a fair description of different physical processes will then be a measure of the approximation of the model to a more realistic and complicated dynamics based ultimately on QCD.

The two models used here, the SM and the CBM, could be considered as two extremes of a more complicated approach to reality. The CBM allows the pions to propagate in all the space, whilst the interaction takes place only in the surface. The SM strictly forbids the pions from the bag interior forcing the coupling of the pions to the bag surface. However, as we shall see, our results in both models are very similar (25% difference) and we should expect similar results from other more refined models.

Let us now briefly introduce the elements of the chiral bag model which we will need in our calculation.

The effective quark-pion interaction in the chiral bag model is given by the Hamiltonian⁵⁾

$$H_{int} = (2\pi)^{-3/2} \sum_{\alpha, \beta, i} \int d^3p \frac{1}{[2\omega(\vec{p})]^{1/2}} v_{\vec{p}, i}^{\alpha, \beta} \alpha^\dagger \beta a_{\vec{p}, i} + h.c. \quad (16)$$

where α and β are quark eigenstates annihilation operators, and $a_{\vec{p}, i}$ is the corresponding annihilation operator for a pion of momentum $\vec{p}$, energy $\omega(\vec{p})$, and isospin i . The vertex function in the version of the CBM⁴⁻⁵), where pions are allowed in the interior of the bag, is given by

$$v_{\vec{p}, i}^{\alpha, \beta} = \frac{i}{2f_\pi} \int d^3x e^{i\vec{p}\cdot\vec{x}} \delta(r-R) \langle \alpha | \bar{q}(\vec{x}) \gamma_5 q(\vec{x}) | \beta \rangle \quad (17)$$

where $q(\vec{x})$ is the quark field and f_π the pion decay constant. Equation (17) reflects the coupling on the surface and assumes plane waves for the pion. Later on we will impose restrictions on the pion wave in order to perform the calculations in the standard model with pions excluded from the bag interior.

The general expression for a massless quark spinor in the spherical cavity of a bag of radius R is given by¹⁶⁾

$$\psi_{n, \kappa}(\vec{x}) = N_{n, \kappa} \left\{ \begin{array}{l} j_e(\omega_{n\kappa} r/R) \\ -i \operatorname{sgn}(\kappa) j_{e'}(\omega_{n\kappa} r/R) \vec{\sigma} \cdot \hat{r} \end{array} \right\} \chi_\kappa^{\eta} \quad (18)$$

where χ_κ^η are the spin-spherical harmonics, and ℓ' and $N_{n, \kappa}$ are given by

$$j = |\kappa| - \frac{1}{2} \quad (19)$$

$$\ell = j + \frac{1}{2} \operatorname{sgn}(\kappa) ; \ell' = j - \frac{1}{2} \operatorname{sgn}(\kappa)$$

$$N_{n, \kappa} = R^{-3/2} \left[\frac{\omega_{n\kappa}}{2(\omega_{n\kappa} + \kappa)} \right]^{1/2} \frac{1}{|j_e(\omega_{n\kappa})|} \quad (20)$$

The eigenfrequencies $\omega_{n\kappa}$ can be calculated numerically by imposing the linear boundary condition, as is done in Ref. 16.

A straightforward calculation of the vertex function of Eq. (17) with the quark wave functions of (18) gives, for the case where α is the ground-state quark, $n=1$, $\kappa = -1$, and β is a general quark eigenstate n, κ , the following expression

$$V_{\vec{p}, i}^{\alpha, \beta} = - \frac{C_{n\kappa}}{2f_\pi} \sum_m C(\ell \ 1/2 \ j; m, \eta - m) \sum_{\lambda \ell} 4\pi R^2 i^\lambda j_\lambda(pR) Y_{\lambda \ell}(\hat{p}) \left[\frac{2\ell+1}{4\pi(2\lambda+1)} \right]^{1/2} C(1 \ell \lambda; -m+\ell, m) C(1 \ell \lambda; 000) \quad (21)$$

$$(-1)^{m-\ell} \langle 1/2, m_s | \sigma_{m-\ell} | 1/2, \eta - m \rangle \langle 1/2, m_t | \zeta_i | 1/2, m''_t \rangle$$

where

$$C_{n\kappa} = 2 N_{1,-1} N_{n,\kappa} j_0(\omega_{1-1}) j_\ell(\omega_{n\kappa}) \quad (22)$$

The space of intermediate hadronic wave functions will consist of all possible bag states, colour singlet, with the quarks excited to any one of the eigenstates of the spherical bag of radius R . This is a set of mutually orthogonal states, which is complete in the space where the quarks are confined. Of course, since the pion coupling of Eq. (16) involves only a one-body operator, only those states with one quark in an excited state will contribute. Since we will only consider the bag energies, all states of three quarks in a given configuration of the quark eigenstates will be degenerate. We can thus take a complete set of intermediate states represented by a product of single-quark wave functions. However, the initial and final nucleon wave functions will be properly antisymmetrized. Hence for a proton with spin $+1/2$ we have

$$|p \uparrow\rangle = \frac{1}{\sqrt{2}} \frac{1}{6} \left[4 u d u \uparrow \downarrow \uparrow - 2 u d u \downarrow \uparrow \uparrow - 2 u d u \uparrow \uparrow \downarrow - 2 d u u \uparrow \downarrow \uparrow + 4 d u u \downarrow \uparrow \uparrow - 2 d u u \uparrow \uparrow \downarrow - 2 u u d \uparrow \downarrow \uparrow - 2 u u d \downarrow \uparrow \uparrow + 4 u u d \uparrow \uparrow \downarrow \right] \quad (23)$$

Thus in order to calculate $\langle p^\uparrow | H_{\text{int}} | B \rangle$ we will take a term of (22) for the $p^\uparrow$ spin and isospin wave function, together with the corresponding spatial wave function and the wave function of $|B\rangle$. Let us assume that the third quark of $|B\rangle$ is in an excited state. Then the first two single-particle wave functions of the term of $|p^\uparrow\rangle$ and $|B\rangle$ must coincide. The same will now happen for the matrix element $\langle B | H_{\text{int}} | p^\uparrow \rangle$. Thus the product of these two matrix elements will be reduced to the product of two single-particle matrix elements, whilst the other two quark wave functions in the initial, final, and intermediate states must be identical. This is depicted diagrammatically in Fig. 3a. The same, of course, can be said if the final state is not a $|p^\uparrow\rangle$ but any other combination of spin-isospin. Since the excited state can run over any of the three quarks of the intermediate state, this will provide an over-all factor of three, given the symmetry of the initial and final wave functions. If the intermediate state had been the ground state we would have now got a sum of single-particle matrix elements in $\langle p^\uparrow | H_{\text{int}} | B \rangle$ and another sum for $\langle B | H_{\text{int}} | p^\uparrow \rangle$, meaning that the pion can couple to different quark states in the initial or final nucleon. This is depicted in Fig. 3b, and it is taken into account implicitly in the calculation of Section 3.

According to Fig. 1 we will need to calculate $\sum_{\eta} V_{\vec{k}-\vec{q},\mu}^{\alpha,\beta} V_{\vec{k},\nu}^{\beta,\alpha^*}$ with V given by Eq. (21). The m sum of the Clebsch-Gordan coefficients in $V_{\vec{k}-\vec{q},\mu}^{\alpha,\beta}$ and $V_{\vec{k},\nu}^{\beta,\alpha}$ can be expressed in terms of Racah coefficients. Analogously we can express the product of the two spherical harmonics in terms of irreducible tensors or rank zero and one, the only two allowed here, and express the η sum of the remaining Clebsch-Gordan coefficient in terms of another Racah coefficient. After this standard exercise in angular momentum algebra, in which we can also prove that λ in Eq. (21) is equal to ℓ' [the rank of the Bessel function in the second component of the quark wave function, Eq. (18)], we can write

$$\begin{aligned}
 \sum_{\eta, m''_t} V_{\vec{k}-\vec{q}, \mu}^{\alpha, \beta} V_{\vec{k}, \nu}^{\beta, \alpha *} \epsilon_{\lambda \mu \nu} = \\
 : A_0 \left[Y_{e'}(\hat{k}-\hat{q}) \otimes Y_{e'}(\hat{k}) \right]_0^0 \langle m_t | \tau^\lambda | m'_t \rangle \\
 + A_1 \frac{1}{\sqrt{3}} i \langle m_s | \sigma_0 | m_s \rangle \left[Y_{e'}(\hat{k}-\hat{q}) \otimes Y_{e'}(\hat{k}) \right]_0^1 \\
 \cdot \langle m_t | \tau^\lambda | m'_t \rangle
 \end{aligned} \quad (24)$$

where A_0 is given by

$$\begin{aligned}
 A_0 = \left(\frac{1}{2f_\pi} \right)^2 C_{\pi\pi}^2 6 \cdot 4\pi R^4 (2j+1)(2\ell+1) \\
 j_{e'}(1\vec{k}-\vec{q}|R) j_{e'}(kR) (-1)^{j-\frac{1}{2}} W(j \ 1/2 \ e' \ 1; \ell \ 1/2)^2 \\
 C(1\ell\ell; 000)^2 [2(2j+1)]^{1/2} W(1/2 \ e' \ 1/2 \ e'; j \ j)
 \end{aligned} \quad (25)$$

The operators σ and τ in Eq. (24) are acting on the space of the quarks. Our aim is to rewrite it in terms of the same operators acting on the spin-isospin space of the nucleons considered as elementary particles. For simplicity we have assumed in Eq. (24) that our initial and final nucleons have the same spin, although the arguments that follow are completely general. Thus, since two of the quark wave functions in the initial and final wave functions must have the same spin, the third one, on which the operators act, must also have the same spin. Hence only $\langle m_s | \sigma_0 | m_s \rangle$ appears in Eq. (24), where m_s is the spin of the quark involved in the transition. Of course m_s is not fixed as can be seen in Eq. (23) which simply means that expectation values of this operator for the wave functions of Eq. (23) and others are to be considered. The same arguments can be applied to the isospin, and again, for simplicity, we will assume the case of a ρ^0 coupled to an initial and final state $|p\uparrow\rangle$, which means $\lambda = 0$ and $m_t = m'_t$ in Eq. (24). If the expectation values are calculated we obtain

$$\begin{aligned}
 A_0 \tau_q^0 &\rightarrow 3 A_0 \frac{1}{2} \frac{1}{36} \left\{ 40 \langle u\uparrow | \tau^0 | u\uparrow \rangle + 8 \langle u\downarrow | \tau^0 | u\downarrow \rangle \right. \\
 &\quad \left. + 8 \langle d\uparrow | \tau^0 | d\uparrow \rangle + 16 \langle d\downarrow | \tau^0 | d\downarrow \rangle \right\} \quad (26) \\
 &\rightarrow A_0 \tau_N^0
 \end{aligned}$$

$$\begin{aligned}
 A_1 \tau_q^0 \sigma_q^0 &\rightarrow 3 A_1 \frac{1}{2} \frac{1}{36} \left\{ 40 \langle u\uparrow | \tau^0 \sigma^0 | u\uparrow \rangle \right. \\
 &\quad \left. + 8 \langle u\downarrow | \tau^0 \sigma^0 | u\downarrow \rangle + 8 \langle d\uparrow | \tau^0 \sigma^0 | d\uparrow \rangle \right. \\
 &\quad \left. + 16 \langle d\downarrow | \tau^0 \sigma^0 | d\downarrow \rangle \right\} \quad (27) \\
 &\rightarrow \frac{5}{3} A_1 \sigma_N^0 \tau_N^0
 \end{aligned}$$

where τ_q^0 and σ_q^0 means τ^0 and σ^0 acting on the quarks, whilst τ_N^0 and σ_N^0 are supposed to act on the nucleons as spin $1/2$, isospin $1/2$ objects.

Equation (24) clearly shows the separation into a scalar part, the one with coefficient A_0 and a vector part, the other with coefficient A_1 , which will contribute, respectively, to the electric and magnetic parts of the coupling of the meson to the nucleon. In order to write Eq. (24) in a way that explicitly shows the term $[(\vec{k}-\vec{q}) \times \vec{k}]$ as in Eq. (5), we can rewrite it in an equivalent form

$$\begin{aligned}
 \sum_{\ell, m_\ell} V_{\vec{k}-\vec{q}, \mu}^{\alpha, \beta} V_{\vec{k}, \nu}^{\beta, \alpha *} \epsilon_{\lambda \mu \nu} &\rightarrow i A_0 \left[Y_{\ell'}(\hat{k}-\hat{q}) \otimes Y_{\ell'}(\hat{k}) \right]_0^0 \tau_N^\lambda \\
 &+ \frac{5}{3} \sqrt{\frac{2}{3}} A_1 C(e' e' 1; 10) \tilde{Y}_{\ell' 1}((\hat{k}-\hat{q}) \cdot \hat{k}) \frac{1}{\sin \theta_{\hat{k}-\hat{q}, \hat{k}}} \\
 &\cdot \left(\frac{2\ell'+1}{4\pi} \right)^{1/2} \frac{(\vec{k}-\vec{q}) \times \vec{k}}{|\vec{k}-\vec{q}| |\vec{k}|} \vec{\sigma}_N \tau_N^\lambda \quad (28)
 \end{aligned}$$

where $\tilde{Y}$ means that the angle ϕ has to be taken equal to zero in the spherical harmonic.

Now, by following the same steps as in Section 3, we can explicitly write down the contribution from these states to the electric and magnetic parts of the ρ coupling. We have

$$G_{NN\rho}^E = \sum_{n,\kappa} \int \frac{d^3k}{(2\pi)^3} A_0 \left[Y_{e'}(\hat{\kappa}-\hat{q}) \otimes Y_{e'}(\hat{k}) \right]_0^0 f_\rho \cdot F_1(k, q, p, E_{n\kappa}) \quad (29)$$

$$\begin{aligned} \frac{G_{NN\rho}^M}{2m_p} = & - \sum_{n,\kappa} \int \frac{d^3k}{(2\pi)^3} A_1 \frac{5}{3} \sqrt{\frac{2}{3}} c(e'e'1;10) \tilde{Y}_{e'}((\hat{\kappa}-\hat{q}), \hat{k}) \\ & \frac{1}{\sin \theta_{\hat{\kappa}-\hat{q}, \hat{k}}} \left(\frac{2\ell'+1}{4\pi} \right)^{1/2} f_\rho F_2(k, q, p, E_{n\kappa}) \\ & \cdot \left\{ \vec{k}^2 - \frac{1}{\vec{q}^2} (\vec{k} \cdot \vec{q})^2 \right\} \frac{1}{|\vec{k}-\vec{q}| |\vec{k}|} \end{aligned} \quad (30)$$

where $\varepsilon_{n\kappa}$ are the energies of the quark eigenstates given by $\omega_{n\kappa}/R$ (we will take $p^0 = \omega_{1-1}/R$).

5. CONTRIBUTION FROM NEGATIVE ENERGY QUARK STATES

In order to account for the contribution of the negative energy quark eigenstate corresponding to n, κ we can use exactly the same formalism, but the quark propagator $(p^0 - k^0 - \varepsilon_{n\kappa} + i\varepsilon)^{-1}$, which appears in Eq. (4), has now to be replaced by $(p^0 - k^0 + \varepsilon_{n\kappa} - i\varepsilon)^{-1}$. This finally leads to different functions $\bar{F}_i(k, q, p, \varepsilon)$ which can be directly obtained from $F_i(\cdot)$ of Eq. (6) by means of

$$\begin{aligned} \bar{F}_1(k, q, p, \varepsilon) &= F_1(k, -q^0, \vec{q}, -p^0, \vec{p}, \varepsilon) \\ \bar{F}_2(k, q, p, \varepsilon) &= -F_2(k, -q^0, \vec{q}, -p^0, \vec{p}, \varepsilon) \end{aligned} \quad (31)$$

It is also important to note that the quark wave function of the negative energy state corresponding to n, κ essentially permutes the upper and lower components but also changes χ_{κ}^{η} to $\chi_{-\kappa}^{-\eta}$. This means that the angular coefficient will be changed. The argument ℓ' that appears in the formulas will now be ℓ instead. Hence it is better to combine the contribution of the quark of positive energy corresponding to n, κ with the contribution from the quark of negative energy corresponding to $n, -\kappa$. The contribution from the latter will be like the one which we have calculated so far, with the only prescription of changing

$$F_i(\cdot, \varepsilon_{n\kappa}) \rightarrow \bar{F}_i(\cdot, \varepsilon_{n-\kappa})$$

and replacing $C_{n\kappa}^2$ by $C_{n-\kappa}^2$ in A_{ρ} of Eq. (25).

It is important to include the contribution from the negative energy states because, as was shown in Refs. 11 and 12 in connection with the nucleon pionic self-energy, the sum over the principal quantum number n for fixed κ diverges. However, when one sums over the positive and negative energy eigenstates, there are cancellations that make the series go as $1/n^2$ and thus converge. In spite of the convergence of the sum in n , there is still an unpleasant feature in the self-energy because the sum over κ diverges linearly in κ . The contribution of each κ can be proved to grow approximately like κ . However, one can also prove that there are further cancellations between the contribution of κ and $-\kappa+1$ ¹²⁾, leading finally to a linearly divergent series. We should note here that in Ref. 11 the cancellation between the contributions of κ and $-\kappa+1$ was not noticed, and the linear character of the divergence was accidentally claimed based upon incorrect numerical results which gave an essentially constant contribution to the self-energy for each κ .

In the present case for $q^0 = 0$, F_1 of Eq. (6) would go like $1/\varepsilon_{n\kappa}^2$ for large values of n, κ , and hence as $1/n^2$ for large values of n , causing no problems of convergence in the n sum. On the other hand, both F_1 in the case $q^0 \neq 0$ or F_2 in any case would go as $1/\varepsilon_{n\kappa}$. But, because of the properties of Eq. (31) for the

negative energy states, there would again be cancellations which would make the n sum convergent. However, the κ sum is still divergent, in much the same way as in the case of the self-energy. A numerical study suggests a linear divergence in κ , as in the self-energy case.

We would also like to note here that although the unphysical sharp boundary bears some of the blame for these divergences, they are not removed when one uses a continuous confining potential⁷⁾ with a gradual coupling of the pions to the quarks^{17,18)}.

As we have mentioned before, at this point we borrow the idea of de Kam and Pirner¹⁰⁾ and implement an extra form factor related to the finite size of the bare pion which couples to the chiral bag. In order to minimize the degrees of freedom, we have chosen the same parameters as in Ref. 10 which are related to the pion decay constant¹⁹⁾. For the Gaussian pion wave function that they use, the corresponding form factor is given by

$$g_{\pi}(q) = \exp(-q^2/4\alpha^2) \quad (32)$$

with

$$\alpha^2 = 34.6 \text{ fm}^{-2}$$

Consideration of the finite size effects leads, in our case, to the implementation of this extra factor in each of the pion vertices. After this is done, all divergences are removed. The results are, however, quite stable under reasonable changes of the size parameter.

6. NUMERICAL RESULTS AND DISCUSSION

We have carried out the numerical calculations in the standard chiral bag model and the CBM. The formulas, as they have been derived, stand for the CBM. The same formulas can, however, be used to calculate the results in the standard model by substituting the pion wave functions, i.e. the Bessel functions

$j_{\ell'}(|\vec{k}-\vec{q}|R)$, $j_{\ell'}(kR)$ of Eq. (25), by the corresponding angular wave functions for pions excluded from the bag interior, which can be found in Ref. 11,

$$j_{\ell}(kR) \rightarrow \phi_{\ell}(x) = \frac{1}{x^2} \frac{1}{N_{\ell}(x)} \quad (33)$$

with

$$x \equiv kR$$

$$N_{\ell}^2(x) = [j'_{\ell}(x)^2 + y'_{\ell}(x)^2]$$

The contribution of $n = 1$, $\kappa = -1$ of the general formulas (29) and (30) is subtracted and replaced by the explicit contribution from N and Δ intermediate states, studied in Section 3. The form factor $u(\vec{k})$ has been taken from the corresponding model as

$$\begin{aligned} u(k) &= \frac{3 j_1(kR)}{kR} && \text{for the CBM,} \\ u(k) &= \frac{2 \phi_1(kR)}{kR} && \text{for the standard model.} \end{aligned} \quad (34)$$

The coupling constants f and f^* are taken as $f^2/4\pi = 0.08$, $f^{*2}/4\pi = 0.36$, and f_{π} is slightly modified from its free value, 93 MeV, so as to get the appropriate πNN coupling in the CBM⁵⁾ by means of the relation $(5/18 f_{\pi}) [\omega_{1-1}/(\omega_{1-1} - 1)] = f/\mu$.

We first show the total results as a function of the bag radius and later discuss the details of the contributions.

In Fig. 6 we show the calculated value of $G_{NN\rho}^E$, at $\vec{q} = 0$, $q^0 = 0$, as a function of R , the bag radius, and compare it with the experimental value of Ref. 15. [Note, however, that the absence of a factor $1/2$ in our definition (7), which is present in Ref. 15, makes our G^V and G^T half the values generally quoted there.] The values that we have extracted after some averaging of different experiments

are

$$\begin{aligned} G_{NN\rho}^{V, \text{exp}} &= 2.9 \pm 0.3 \\ \frac{G_{NN\rho}^{T, \text{exp}} \cdot m_{\pi}}{2m_p} &= 1.35 \pm 0.12 \end{aligned} \quad (35)$$

In Fig. 6 we can see the results on $G_{NN\rho}^E$ for different values of the pion size parameter α^2 . The results are shown for $\alpha^2 = 34.6 \text{ fm}^{-2}$ and $\alpha^2 = 18 \text{ fm}^{-2}$. It can be seen that changes in the size parameter of 100% in α^2 result in changes of around 30% in $G_{NN\rho}^E$, which shows a certain stability of the results under moderate changes of α^2 . For the same value of R the values of $G_{NN\rho}^E$ calculated with the CBM or the SM differ by about 25%, with lower values for the CBM. In Fig. 7 we show the results for $G_{NN\rho}^M$ with similar results as before, although it can be observed that the magnetic coupling constant is not so strongly dependent on R as for $G_{NN\rho}^E$. It can also be seen that there are difficulties in obtaining a simultaneous good fit for $G_{NN\rho}^E$ and $G_{NN\rho}^M$. Whilst for $G_{NN\rho}^E$ we would get better results for values of R around 0.9-1.0 fm, for $G_{NN\rho}^M$ we would obtain better results in the region around $R \approx 0.6$ fm. A good qualitative agreement is obtained around $R = 0.8-0.9$ fm for the CBM with $\alpha^2 = 34.6 \text{ fm}^{-2}$ and around the same radii for the SM with $\alpha^2 = 18 \text{ fm}^{-2}$. Increasing the pion size parameter α^2 leads to better results for $G_{NN\rho}^M$ but worse results for $G_{NN\rho}^E$.

It is interesting to note here that the model developed for the ρ coupling to the nucleon can be extended in a straightforward way to account for the coupling of a photon to the nucleon through the pion cloud. This contribution, together with the contribution from the coupling of the photon to the quarks, should account for the electric and magnetic form factors of the nucleon; $G_{NN\gamma}^{E(\pi \text{ cloud})}$ at $\vec{q} = 0$ is a measure of the charged-pion content of the nucleon; $G_{NN\gamma}^{E(\pi \text{ cloud})}$ would then be given by $G_{NN\rho}^E/f_\rho$ and hence, with a model that reproduces $G_{NN\rho}^E$, we would get $G_{NN\gamma}^{E(\pi \text{ cloud})}(\vec{q} = 0) \approx 0.4$. In a recent study of the electromagnetic properties of the nucleon²⁰, a very reasonable agreement with the proton and neutron charge distributions has been found, with values of $G_{NN\gamma}^{E(\pi \text{ cloud})}(\vec{q} = 0) \approx 0.3-0.4$ and values of the bag radius around $R \approx 0.8-0.9$ fm.

The tensor coupling of the ρ to the nucleon is often used in nuclear physics because of the interference with the pion exchange in many pionic processes. We have calculated the coupling at $q^0 = 0, \vec{q} = 0$. However, it is interesting to find what is the $\vec{q}$ dependence of this coupling. We have performed the calculations

for different values of $\vec{q}$ and found the fall-out compatible with a monopole form factor of the type $\Lambda_\rho^2/(\Lambda_\rho^2 + \vec{q}^2)$, with $\Lambda_\rho \approx 750\text{--}800\text{ MeV}$. However, the lack of Lorentz invariance of the bag model and the large mass of the ρ meson does not allow a direct comparison with invariant form factors of the type $(\Lambda_\rho^2 - m_\rho^2)/(\Lambda_\rho^2 - q^2)$ used in the literature, where Λ_ρ is taken much larger than 800 MeV to overcome the m_ρ^2 term.

After this discussion on the results, we now come back to some details of the calculation. We take the results of the CBM as reference, with analogous conclusions for the SM. If we look at the contribution as a function of n , we can see that the sum of all contributions for positive energy states with $\kappa = -1$ and negative states with $\kappa = 1$, excluding the N and Δ ($n = 1$, $\kappa = -1$, positive energy), is only 20% of the N plus Δ contribution for the electric coupling and 13% for the magnetic coupling. The terms with $n = 1$ and κ negative contribute somewhat, to both G^E and G^M . All together, for the electric coupling, the N and Δ terms contribute 28% of the total, whilst for the tensor coupling they contribute 40%. However, the convergence in κ is fast. Values of $|\kappa| \geq 4$ contribute only 20% to the total values of G^E and G^M .

As we can see, a non-negligible contribution comes from states other than N and Δ , 72% of G^E and 60% of G^T . Analysis of this problem in terms of mean field theory does not allow for intermediate states other than the ground state, and implicitly assumes static propagators²¹⁾. Thus the results are rather different from those obtained here. The hypothesis of a direct coupling of the ρ -mesons to the quarks, in addition to the coupling to the pion cloud, which was invoked in Ref. 21 in order to obtain agreement with experiment, is found unnecessary here, thus preserving the simplicity of the chiral bag model with only pions coupling directly to the quarks.

However, the fact that a large part of the contribution comes from states for which the bag model offers only a rough approximation should make us consider the approximately good agreement with experiment as qualitative only.

7. SUMMARY

We have studied the effective coupling of a ρ meson to the nucleon in the chiral bag model. With the explicit aim of keeping the simplicity of the chiral bag model, with quarks as primary constituents and pions coupling to them, the coupling of the ρ meson to the nucleon was done through the pion cloud. Furthermore, we considered a finite size for the bare pions, which was found in Ref. 10 to be a natural way of stabilizing the chiral bag. With these ingredients we found a good qualitative agreement of our results with experiment, for both the CBM and the standard model, for values of the bag radius commonly used in the literature.

The similarity of this work with that on the electromagnetic properties of a nucleon is also interesting. We showed that if the coupling of the ρ meson to the nucleon is given through the coupling of the ρ to the pion cloud, as assumed here, it implies a contribution to the proton charge from the pion cloud of around 0.4. A later study of the proton and neutron charge distributions along similar lines shows good agreement with experiment for values of this magnitude around 0.3-0.4 and similar bag radii.

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Figure captions

- Fig. 1 : ρ -meson coupling to the nucleon through the pion cloud.
- Fig. 2 : Diagrammatic representation of meson decay into two pions.
- Fig. 3 : Two ways of coupling the pion cloud to the quarks of the nucleon.
- Fig. 4 : ρ -meson coupling to the nucleon with intermediate states of a) N and b) Δ .
- Fig. 5 : Diagrammatic representation of the empirical coupling of a ρ meson to the nucleon.
- Fig. 6 : The ρNN electric coupling constant as a function of the bag radius.
C34: CBM with $\alpha^2 = 34.6 \text{ fm}^{-2}$. S34: standard model with $\alpha^2 = 34.6 \text{ fm}^{-2}$.
S26: idem, $\alpha^2 = 26 \text{ fm}^{-2}$. S18: idem, $\alpha^2 = 18 \text{ fm}^{-2}$. Horizontal shaded region, experimental values.
- Fig. 7 : Same as Fig. 6 for the ρNN magnetic coupling.

Fig. 1

Fig. 2

a)

b)

Fig. 3

a)

b)

Fig. 4

Fig. 5

Fig. 6

Fig. 7