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NON-LINEAR DENSITY DEPENDENCE OF THE Δ SELF-ENERGY IN
NUCLEAR MATTER AND ITS IMPLICATIONS IN PIONIC ATOMS *)

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A B S T R A C T

The $\Delta(3/2,3/2)$ self-energy in a nuclear medium is shown to be highly non-linear in the density, when proper care is taken of the virtual meson propagation in the medium and retardation effects. As a consequence the p wave absorptive potential for pionic atoms diverges appreciably from the standard ρ^2 form. A fit to the existing pionic atoms data is carried out with the new functional of the density and turns out to be as good as those with the ρ^2 functional. The successful fits with such different density functionals is due to a very narrow range of nuclear effective densities felt by the pion in the observed pionic atoms. The influence of these effects in related problems is discussed along with a suggestion to widen the range of nuclear densities felt by the pions by looking at other nuclear phenomena.

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1. - INTRODUCTION

The new brand of exotic atoms with Σ particles, together with the corresponding Σ hypernuclei, have recently become a fashionable field of research which manifests some peculiarities. One of the peculiar features of these states is the apparent contradiction in the information that one obtains for the Σ nucleus interaction from the atomic or hypernuclear states. Indeed, a phenomenological analysis of the Σ atoms¹⁾ provides a Σ nucleus optical potential by means of which the calculated width of the Σ hypernuclei²⁾ is three to four times larger than the experimental one³⁾. A variety of possible explanations have appeared in recent literature⁴⁾ which essentially emphasize the role of the Pauli blocking in the Σ self-energy. While the reduction from this effect does not seem to be too drastic in most cases, another problem still remains, which is the apparent constancy of the width ($\Gamma \sim 5$ MeV) for all states and nuclei measured³⁾.

A recent interpretation⁵⁾ differs essentially from all others in the fact that the $\Sigma N \rightarrow \Lambda N$ transition amplitude is shown to be drastically modified in a nuclear medium with respect to its free form. Indeed, one pion exchange is an important ingredient in this transition, and, in the kinematical regime where this virtual pion appears, it is appreciably renormalized by the interaction of the pion with the medium. As a consequence, the imaginary part of the Σ self-energy appears to be approximately linear in the density at low densities ($\rho \approx 0.1 \rho_0$), but becomes highly non-linear when the density increases, showing signs of saturation around $\rho \approx 0.7 \rho_0$. In terms of this new Σ self-energy one can now interpret simultaneously both the atomic and hypernuclear widths⁵⁾.

The essential ingredient in this approach has been the interaction of the virtual pion with the nuclear medium. Therefore, one may think that similar results would appear if one studied the self-energy of a $\Delta(3/2, 3/2)$ resonance in the nuclear medium. Here again, the virtual pion from the transition $\Delta N \rightarrow NN$ would be renormalized in a similar way, thus modifying a channel that is related to pion absorption in π nucleus scattering⁶⁾⁻⁸⁾. The relationship is depicted in Fig. 1. Figure 1c, based upon Δ -h excitation by the pion source, plays an important role in the p wave absorptive optical potential for pionic atoms, because of the difficulty to excite p-h components in nuclei with zero momentum transfer as a consequence of the Pauli exclusion principle. In addition, there is some cancellation between direct and crossed p-h terms⁹⁾.

In Fig. 1a, because of isospin conservation, the mesons exchanged between the two vertices necessarily have isospin $T = 1$, which restricts the mediators of this interaction, in the one boson exchange picture, to π and ρ , in complete analogy with the $\Sigma N \rightarrow \Lambda N$ transition.

One of the aims of the present work is to study the modification to the pion optical potential from Fig. 1c when one allows the mesons to interact with the medium. This will serve us as a guideline to observe what happens in other pieces of the pion self-energy and the repercussions that this has on pionic atoms.

2. - Δ SELF-ENERGY INSIDE NUCLEAR MATTER

We will be essentially interested in the imaginary part of the pionic optical potential, hence will pay attention to the decay channel of the Δ , Fig. 2a, and its modification in the medium, Fig. 2 b-e. As we have stated before, we will consider π and ρ meson exchange as the source of the interaction in this case.

If we turn our attention to Fig. 1 we can evaluate the self-energy in terms of the effective Hamiltonians:

$$\begin{aligned} H_{\pi N \Delta}(x) &= \frac{f^*}{\mu} \psi_N^+(x) S_i \partial_i \phi^\lambda(x) T^\lambda \psi_\Delta(x) + h.c. \\ H_{\pi N N}(x) &= \frac{f}{\mu} \psi_N^+(x) \sigma_i \partial_i \phi^\lambda(x) \tau^\lambda \psi_N(x) \\ H_{\rho N \Delta}(x) &= \frac{f_\rho^*}{m_\rho} \psi_N^+(x) (\vec{S} \times \vec{\nabla}) \cdot \vec{\rho}^\lambda(x) T^\lambda \psi_\Delta(x) + h.c. \\ H_{\rho N N}(x) &= \frac{f_\rho}{m_\rho} \psi_N^+(x) (\vec{\sigma} \times \vec{\nabla}) \cdot \vec{\rho}^\lambda(x) \tau^\lambda \psi_N(x) \end{aligned} \quad (1)$$

where S_i, T^λ are the spin and isospin transition operators defined by

$$\begin{aligned} \langle 3/2 \ M_s | S_\mu^+ | 1/2 \ m_s \rangle &= G(1/2, 1, 3/2; m_s, \mu, M_s) \\ \langle 3/2 \ M_t | T_\lambda^+ | 1/2 \ m_t \rangle &= G(1/2, 1, 3/2; m_t, \lambda, M_t) \end{aligned} \quad (2)$$

which defines S, T as rank one tensors with reduced matrix element $\langle 3/2 || S^+ || 1/2 \rangle = 1$, and the same for T^+ . Some interesting properties of these operators are:

$$\sum_{M_s} S_i | M_s \rangle \langle M_s | S_j^+ = \delta_{ij} - \frac{1}{3} \sigma_i \sigma_j \quad (3a)$$

$$\begin{aligned} \sum_{m_s} \langle M'_s | S^+ \vec{q} | m_s \rangle \langle m_s | \vec{S} \vec{q} | M_s \rangle &= \frac{1}{3} \vec{q}^2 \delta_{M_s M'_s} \\ &- \frac{1}{3} \vec{q}^2 \sqrt{4\pi} G(3/2, 2, 3/2; M'_s, M_s - M'_s) Y_{2, M_s - M'_s}(\hat{q}) \end{aligned} \quad (3b)$$

Use of the standard Feynman rules with these Hamiltonians gives for Fig. 2a and one pion exchange

$$\begin{aligned} \langle M'_s, M'_t | -i \Sigma(K) | M_s, M_t \rangle &= \sum_{m_s, m_t, \lambda} \int \frac{d^4 q}{(2\pi)^4} \frac{f^{*2}(q)}{\mu^2} \\ &\cdot \langle M'_s | \vec{S}^+ \vec{q} | m_s \rangle \langle m_s | \vec{S}(-\vec{q}) | M_s \rangle \\ &\cdot \langle M'_t | T^{+\lambda} | m_t \rangle \langle m_t | T^\lambda | M_t \rangle i G_0(K-q) i D_0(q) \end{aligned} \quad (4)$$

where G_0 and D_0 are the nucleon and pion propagators respectively.

The isospin sum can be carried out immediately by means of (3a) with the result

$$\sum_{m_t, \lambda} \langle M'_t | T^{+\lambda} | m_t \rangle \langle m_t | T^\lambda | M_t \rangle = \delta_{M_t M'_t} \quad (5)$$

On the other hand, Eq. (3b) shows that $\vec{S}^+ \vec{q} \vec{S} \vec{q}$ contains a scalar and a tensor part which would finally lead to a scalar and a tensor part in $\Sigma(k)$ in the variable $\vec{k}$, but as we shall see later on, the contribution of the tensor part of $\Sigma(k)$ to the pion self-energy can be neglected for pionic atoms and only the scalar part is relevant. Hence by dropping the spin and isospin dependence in $\Sigma^S(k)$ ($\delta_{M_s M'_s} \delta_{M_t M'_t}$) and using (3) we get

$$\Sigma^S(k) = i \frac{1}{3} \delta_{ij} \int \frac{d^4 q}{(2\pi)^4} \frac{f^{*2}(q)}{\mu^2} q_i q_j G_0(K-q) D_0(q) \quad (6)$$

Note that now $q_i q_j D_0(q)$ gives the q dependence of the one pion exchange potential which we can define, for the N-N interaction, as:

$$V_{ij}(q) = \frac{f^2(q)}{\mu^2} q_i q_j \frac{1}{q^2 - \vec{q}^2 - \mu^2 + i\epsilon} \quad (7)$$

Note also that we are including form factors in the vertex, chosen of a monopole type

$$f_i(q) = f_i F_i(q) = f_i \left(\frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - q^2} \right) \quad (8)$$

which are extracted from the study of the NN scattering data¹⁰⁾, the $\bar{p}p$ and np charge exchange reactions¹¹⁾, etc.

However, we will not only need one pion exchange but also ρ exchange, conveniently modulated by the presence of the short range nuclear correlations, which modify the interaction at short distances. This is a standard procedure used in pion nucleus scattering, pionic excitations in nuclei and related processes^{12),13)}.

By means of a simple approximation to the correlation function we can write the nuclear effective interaction in this channel in the following way¹⁴⁾

$$\begin{aligned}\tilde{V}_{ij}^{\pi}(q) &= \frac{f^2}{\mu^2} \left\{ q_i q_j D_0^{\pi}(q) F_{\pi}^2(q) - (q_i q_j + \frac{1}{3} q_c^2 \delta_{ij}) \tilde{D}_0^{\pi}(q) \tilde{F}_{\pi}^2(q) \right\} \\ \tilde{V}_{ij}^{\rho}(q) &= \frac{f^2}{\mu^2} C_{\rho} \left\{ \left[\frac{2}{3} \vec{q}^2 \delta_{ij} - (q_i q_j - \frac{1}{3} \vec{q}^2 \delta_{ij}) \right] D_0^{\rho}(q) F_{\rho}^2(q) \right. \\ &\quad \left. - \left[\frac{2}{3} (\vec{q}^2 + q_c^2) \delta_{ij} - (q_i q_j - \frac{1}{3} \vec{q}^2 \delta_{ij}) \right] \tilde{D}_0^{\rho}(q) \tilde{F}_{\rho}^2(q) \right\}\end{aligned}\quad (9)$$

where q_c is a characteristic correlation parameter, chosen equal to the ω meson mass, C_{ρ} is the ratio

$$C_{\rho} = \frac{f_{\rho}^2}{m_{\rho}^2} : \frac{f^2}{\mu^2} \quad (10)$$

and $\tilde{D}_0^i, \tilde{F}_i(q)$ are the corresponding π and ρ propagator and form factor obtained by substituting $\vec{q}^2$ by $\vec{q}^2 + q_c^2$.

In order to obtain the proper Δ self-energy in the medium, the bare one pion exchange interaction (7) should be substituted in (6) by $\tilde{V}_{ij}(q)$ where

$$\tilde{V}_{ij}(q) = \tilde{V}_{ij}^{\pi}(q) + \tilde{V}_{ij}^{\rho}(q) \quad (11)$$

On the other hand, $G_0(k-q)$ appearing in (6) refers to the nucleon propagator in the uncorrelated Fermi sea of nuclear matter and is given by

$$\begin{aligned}G_0(k^0, \vec{k}) &= \frac{n(\vec{k})}{k^0 - \vec{k}^2/2m - i\epsilon} + \frac{1 - n(\vec{k})}{k^0 - \vec{k}^2/2m + i\epsilon} \\ &= \frac{1}{k^0 - \vec{k}^2/2m + i\epsilon} + 2\pi i n(\vec{k}) \delta(k^0 - \vec{k}^2/2m)\end{aligned}\quad (12)$$

with $n(\vec{k})$ the occupation number. Equation (12) shows a free nucleon propagator piece and a genuine many body correction (the δ piece). The term with the δ function in (12), when substituted in (6), takes care of the subtraction to the Δ width due to the blocking of the decay of the Δ into the occupied states of the nucleus, according to the Pauli exclusion principle. It is depicted by Fig. 2b.

However, as we can see in Fig. 2, there are more pieces contributing to the Δ self-energy in this channel. Note that all of them can be considered, as the corresponding Fig. 1a, but with the pion or ρ meson conveniently renormalized because of the interaction of the mesons with the nuclear medium. Indeed, such a picture with p-h excitation, Δ -h excitation and the iteration of these pieces in a random phase approximation (RPA) expansion, is the standard microscopic model for pion nucleus interaction^{9),13),15)}.

Hence, instead of $\tilde{V}_{ij}(q)$ we should put

$$\tilde{V}_{ij} + \tilde{V}_{ie} U \tilde{V}_{ej} + \tilde{V}_{ie} U \tilde{V}_{em} U \tilde{V}_{mj} + \dots \quad (13)$$

where U is the Lindhard function for p-h and Δ -h excitation

$$U(q) = U_N(q) + U_\Delta(q) \quad (14)$$

for which explicit expressions can be found in Ref. 16) and in the appendix of Ref. 17).

Note the structure of (11) which can be cast in the form

$$\tilde{V}_{ij}(q) = a \delta_{ij} + b q_i q_j \quad (15)$$

where a and b can immediately be obtained from (11) and (9)

$$\begin{aligned} a &= \frac{f^2}{\mu^2} \left\{ \vec{q}^2 D_o^p(q) F_p^2(q) C_p - \frac{1}{3} q_c^2 \tilde{D}_o^\pi(q) \tilde{F}_\pi^2(q) \right. \\ &\quad \left. - \left[\vec{q}^2 + \frac{2}{3} q_c^2 \right] \tilde{D}_o^p(q) \tilde{F}_p^2(q) C_p \right\} \\ b &= \frac{f^2}{\mu^2} \left\{ D_o^\pi(q) F_\pi^2(q) - D_o^p(q) F_p^2(q) C_p \right. \\ &\quad \left. - \tilde{D}_o^\pi(q) \tilde{F}_\pi^2(q) + \tilde{D}_o^p(q) \tilde{F}_p^2(q) C_p \right\} \end{aligned} \quad (16)$$

and then the geometric sum in (13) can be immediately performed with the result :

$$\tilde{V}_{ie} [1 - \mathcal{U} \tilde{V}]_{ej}^{-1} = \frac{1}{1 - \mathcal{U}a} \left(a \delta_{ij} + \frac{b q_i q_j}{1 - \mathcal{U}a - \mathcal{U}b \vec{q}^2} \right) \quad (17)$$

Only a last step remains, the subtraction of the free self-energy pieces that go into the renormalization of a Δ particle in free space. This contribution can be obtained from our formula in the limit when the nuclear density goes to zero and hence $\mathcal{U} \rightarrow 0$ and $G_0 \rightarrow G_0^{\text{free}}$. Thus the genuine many body correction to the Δ self-energy is given by

$$\begin{aligned} \Sigma^S(K) = & i \frac{1}{3} \left(\frac{f^*}{f} \right)^2 \int \frac{d^4 q}{(2\pi)^4} \left\{ G_0(K-q) \frac{1}{1 - \mathcal{U}a} \left(3a + \frac{b \vec{q}^2}{1 - \mathcal{U}a - \mathcal{U}b \vec{q}^2} \right) \right. \\ & \left. - \frac{1}{K^0 - q^0 - \mathcal{E}(\vec{K} - \vec{q}) + i\epsilon} (3a + b \vec{q}^2) \right\} \quad (18) \end{aligned}$$

which implicitly assumes the same form factors for the coupling of the meson to NN or $N\Delta$.

From the technical point of view it is extremely useful to perform a Wick rotation for the q^0 integration. Details of this can be found in the same Ref. 17), where these ideas are used to evaluate the nucleon self-energy in connection with the effective mass.

The final result up to a constant is the same one found there, and we also use the same coupling constants and form factor as in this work. The result for the Δ self-energy is then

$$\begin{aligned} \Sigma^S(K) = & - \frac{1}{3} \frac{1}{(2\pi)^2} \left(\frac{f^*}{f} \right)^2 \int_0^\infty q^2 dq \\ & \cdot \left[\int_0^\infty \frac{dx}{2\pi} \frac{m}{kq} \ell_n \frac{(K^0 - (K-q)^2/2m)^2 + x^2}{(K^0 - (K+q)^2/2m)^2 + x^2} M(q^0, q) \right]_{q^0 = ix} \\ & - \int_{-1}^1 d\cos\varphi \theta(K^0 - \mathcal{E}(\vec{K} - \vec{q})) M(q^0, q) \Big|_{q^0 = K^0 - \mathcal{E}(\vec{K} - \vec{q})} \\ & + \int_{-1}^1 d\cos\varphi n(\vec{K} - \vec{q}) \frac{1}{1 - \mathcal{U}a} \left\{ 3a + \frac{b \vec{q}^2}{1 - \mathcal{U}a - \mathcal{U}b \vec{q}^2} \right\} \Big|_{q^0 = K^0 - \mathcal{E}(\vec{K} - \vec{q})} \quad (19) \end{aligned}$$

where

$$M(q^0, \vec{q}) = \frac{U}{1 - Ua - Ub\vec{q}^2} \left\{ 3a^2 + 2ab\vec{q}^2 + b^2\vec{q}^4 - \frac{2Ua^2b\vec{q}^2}{1 - Ua} \right\} \quad (20)$$

Out of the three terms in (19), the first one is a real background. The second one, where the intermediate nucleon has been set on shell ($k^0 - q^0 = \epsilon(\vec{k} - \vec{q})$), gives an imaginary part from the imaginary part of U (which appears when $q^0, \vec{q}$ can give rise to a physical p-h or Δ -h excitation). Hence the corresponding piece in the pion optical potential, Fig. 1c, will contain an imaginary part corresponding to pion absorption exciting two particle two holes, with the particles ejected to the continuum, (the main channel for pion absorption in pionic atoms). The last term is the Pauli correction mentioned before. This last piece does not provide an appreciable imaginary part for $k^0 \approx \mu$ (pionic atoms situation) since there is little phase space available for $N\pi$ decay.

Note also that although (19) contains the full induced interaction coupled to the Δ , we can easily get the contribution of diagrams 2c and 2d by simply taking into account the terms linear in U in the function $M(q^0, \vec{q})$ of (20). This allows comparison of the full calculation with the lowest order calculation and of the latter with previous calculations⁽⁶⁾⁻⁸⁾ which are also done in lowest order.

3. - PION OPTICAL POTENTIAL

If we go back to Fig. 1c but consider the full Δ self-energy, we can evaluate the pion self-energy. Use of the Feynman rules now gives

$$-i\Pi(q) = \sum_{\substack{M_s \\ M'_s \\ M_t}} \sum_{\substack{m_s \\ m_t}} \int \frac{d^3K}{(2\pi)^3} \frac{f^{*2}(q)}{\mu^2} \langle m_s | \vec{S}(-\vec{q}) | M'_s \rangle n(\vec{K}) \\ \langle M'_s | \frac{i}{q^0 - \omega_R - \epsilon_\Delta(\vec{K} + \vec{q}) + \epsilon_N(\vec{K}) - \Sigma_\Delta(K+q) + i\epsilon} | M_s \rangle \quad (21)$$

$$\langle M_s | \vec{S}^+ \vec{q} | m_s \rangle \langle m_t | T^\lambda | M_t \rangle \langle M_t | T^{+\lambda} | m_t \rangle$$

where $\omega_R = M_\Delta - M_N = 2.1\mu$ and $n(\vec{k})$ is the nucleon occupation number. We neglect the crossed Δ term, since it would not contribute to the imaginary part and only 10% to the real part of (23). The difference between the kinetic energies $\epsilon_\Delta - \epsilon_N$ is small for small momentum $\vec{q}$ and can be neglected. Equation (21) contains a first order piece in the density that would go into the real c_0 parameter of the optical potential, actually quite well described by (21) as shown in Ref. 13). However, we are now interested in the higher order pieces of the optical potential and primarily in the imaginary part. Therefore, we will subtract this first order piece and call the rest the absorptive potential. We can now use

$$\begin{aligned} \sum_{M_t} \sum_{m_t} \langle m_t | T^{\lambda'} | M_t \rangle \langle M_t | T^{+\lambda} | m_t \rangle &= \\ = \sum_{m_t} \langle m_t | \frac{2}{3} \delta_{\lambda\lambda'} - \frac{i}{3} \epsilon_{\lambda\lambda'\rho} \vec{\sigma}^\rho | m_t \rangle &= \frac{4}{3} \delta_{\lambda\lambda'} \end{aligned} \quad (22)$$

By expanding (21) in power series of $\Sigma_\Delta/(q^0 - \omega_R)$ up to first order in Σ_Δ and subtracting the first order contribution, we get

$$\begin{aligned} \Pi^{abs}(q) &= \delta_{\lambda\lambda'} \int \frac{d^3k}{(2\pi)^3} \frac{f^{*2}(q)}{\mu^2} \frac{4}{3} n(\vec{k}) \langle M_s | \vec{S}^{+\vec{q}} \vec{S}^{\vec{q}} | M'_s \rangle \\ &\cdot \frac{1}{(q^0 - \omega_R)^2} \left(\sum_{\Delta}^S \delta_{M_s M'_s} + \sum_{\Delta}^T \langle M'_s | \vec{S}^+(\vec{k} + \vec{q}) \vec{S}(\vec{k} + \vec{q}) - \frac{1}{3} \vec{S}^+ \vec{S}(\vec{k} + \vec{q})^2 | M_s \rangle \right) \end{aligned} \quad (23)$$

where we have included the tensor part of the Δ self-energy. Note that the second rank tensor in (23) gives a vanishing contribution in the limit $\vec{q} \rightarrow 0$, (leaving aside the $\vec{S}^{+\vec{q}} \vec{S}^{\vec{q}}$ factor) after performing the $\vec{k}$ integration. $\Sigma_\Delta^S, \Sigma_\Delta^T$ are functions of $\epsilon_N(\vec{k}) + q^0, |\vec{k} + \vec{q}|$, hence the tensor part gives a contribution of $O(\vec{q}^2)$ with respect to the scalar part and can be neglected in pionic atoms.

By using similar arguments as in (22) for the spin sum of $\vec{S}^{+\vec{q}} \vec{S}^{\vec{q}}$ we now get

$$\Pi^{abs}(q) = \delta_{\lambda\lambda'} \left(\frac{4}{3} \right)^2 \vec{q}^2 \frac{f^{*2}}{\mu^2} \frac{\rho}{4} \frac{\sum_{\Delta}^S}{(q^0 - \omega_R)^2} \quad (24)$$

where ρ is the nuclear density.

The optical potential has a factor $(2q^0)^{-1}$ with respect to the pion self-energy^{13),18)}. Thus, for pionic atoms

$$V^{abs}(q) = \frac{1}{2\mu} \frac{4}{9} \frac{f^{*2}}{\mu^2} \vec{q}^2 \int \frac{\sum_{\Delta}^S}{(q^0 - \omega_R)^2} \quad (25)$$

Since Σ_{Δ}^S is not strongly dependent on $|\vec{k}|$, we have taken an average momentum in the Fermi sea to evaluate (24) and (25).

4. - RESULTS FOR Σ_{Δ}

We will now show results for the Δ self-energy (19) both in the lowest order and the full calculation.

In Fig. 3 we can see the results for the imaginary part of the Δ self-energy calculated for $k^0 = \mu$ and $|\vec{k}| = 150$ MeV, an average nucleon momentum which will appear in the Δ 's of Fig. 1c. As a function of the density the lowest order of Σ_{Δ}^S is a function nearly linear in the nuclear density. The small divergence from linearity at $\rho \approx \rho_0$ is a consequence of the Pauli blocking in the p-h excitation in Fig. 1b.

However, the remarkable results is that when one carries out the calculation to all orders of ph or Δ h excitation, as in Fig. 2c, the imaginary part of Σ_{Δ} , as a function of the nuclear density, shows signs of saturation around $\rho = 0.7\rho_0$ and shows appreciable differences with respect to the lowest order calculation.

These results are very similar to those found in the self-energy of a Σ particle inside a nuclear medium⁵⁾. In the latter case this saturation for the imaginary part of the self-energy also appeared and was the key to the understanding of the apparent contradiction between the Σ atoms and Σ hypernuclei data when one analyzed them in terms of an optical potential linear in the density.

As in the case of the Σ particle, the saturation is a peculiar feature of the imaginary part. In Fig. 4 we show the real part of Σ_{Δ}^S (the Pauli piece excluded) for the lowest order and the full calculation. It is remarkable how close $\text{Re } \Sigma_{\Delta}^S$ is to a linear function in the density and how little difference there is between the lowest order result and the full calculation. Thus, in the case of the Δ , the saturation is also a peculiar feature of the imaginary part. It is not difficult to see how higher order terms in $U(q^0, q)$ can affect the real and imaginary part in different ways. Indeed, in order to have an imaginary

part of Σ_{Δ}^S we need an imaginary part of $U(q^0, q)$, which comes from the excitation of physical particle-hole components. This puts severe restrictions on q^0, q , and the particular structure of $U(q^0, q)$ for these values of q^0, q is ultimately responsible for the saturation. However, the real part comes from the whole integration regime and is affected differently.

It is quite illustrative to look in more detail at the reason for this reduction of the imaginary part of Σ_{Δ}^S . In order to do so, let us go back to the sum in Eq. (13). The lowest order contribution comes from the term proportional to U and we will compare it to the contribution coming from the next term in the series, proportional to U^2 . In order to show this effect in a more qualitative manner, we will write our expression (15) in an approximate form

$$\tilde{V}_{ij}(q) \simeq g' \delta_{ij} + D_0 q_i q_j \quad (26)$$

which is quite commonly used in nuclear pion physics. The parameter g' is the spin-isospin correlation parameter, and generally accepted values range from $g' \approx 0.5 - 0.7$.

Due to the spin sum over intermediate nucleon states, which provided the factor δ_{ij} in (6), the term with g' in (26) gets an extra weight with respect to $D_0 q_i q_j$, which is ultimately responsible for the reduction. Indeed

$$\begin{aligned} \delta_{ij} \tilde{V}_{ie} U \tilde{V}_{ej} &= \delta_{ij} (g'^2 \delta_{ij} + 2g' D_0 q_i q_j + D_0^2 \vec{q}^2 q_i q_j) U \\ &= (3g'^2 + 2g' D_0 \vec{q}^2 + D_0^2 \vec{q}^4) U \\ &= \{ 2g'^2 + (g' + D_0 \vec{q}^2)^2 \} U \end{aligned} \quad (27)$$

In an analogous way we can also write

$$\begin{aligned} \delta_{ij} \tilde{V}_{ie} U \tilde{V}_{em} U \tilde{V}_{mj} &= \\ &= \{ 2g'^3 + (g' + D_0 \vec{q}^2)^3 \} U^2 \end{aligned} \quad (28)$$

In the range of $q^0, \vec{q}$ which appreciably contributes to $\text{Im } \Sigma$, one has $q^0 \approx \mu/2$; $|\vec{q}| \approx 2\mu$ and U has both the real and imaginary parts negative (showing the attractive character of the p -h and Δ -h excitation in the pion self-energy). Thus we have

$$\begin{aligned}
 g' + D_0 \vec{q}^2 &\approx -0.2 \\
 g' &\approx 0.6 \\
 \text{Re } U &< 0 \\
 \text{Im } U &< 0 \\
 \text{Im } U^2 &> 0
 \end{aligned}
 \tag{29}$$

The coefficient of U in (27) is positive and the imaginary part of this term is thus negative. On the other hand, both the coefficients of U^2 in (28) and $\text{Im } U^2$ are now positive and as a consequence the imaginary part of the second order term is also positive and will lead to a reduction of the first order contribution.

This quenching observed here contrasts with the enhancement of other quantities, like the pion propagator. The difference lies essentially in the fact that in the latter case we have two external pion legs, while in our case, these lines would be attached to the Δ -N vertex and would also contain the corresponding correlation pieces. In the case of the pion propagator one would obtain a geometrical series with $g' + D_0 \vec{q}^2$ (< 0) as ratio, while here the different coefficients of U^n are positive. The repulsive character of this g' piece, and the special way in which it is weighed after the spin sum, is ultimately responsible for the reduction of $\text{Im } \Sigma$ observed here.

5. - RESULTS FOR THE PION OPTICAL POTENTIAL

If we look at Figs. 3 and 4 in lowest order we get a self-energy that looks very close to the linear extrapolation in the density

$$\Sigma_{\Delta}(\rho) = (-56 - i 38) \rho / \rho_0 \quad [\text{MeV}]
 \tag{30}$$

This expression substituted in (25) would give us the absorptive pion optical potential which should be compared to the standard one used in the analysis of pionic atoms in terms of the C_0 parameter¹⁹⁾. Leaving aside the Lorentz-Lorenz-Ericson-Ericson correction²⁰⁾, which should be considered apart, we have

$$V^{\text{opt}}(q) = - \frac{4\pi}{2\mu} \vec{q}^2 C_0 \rho^2 \frac{1}{1 + \mu/2M}
 \tag{31}$$

(use of the local density approximation and $\vec{q} \rightarrow -i\vec{\nabla}$ is understood in order to construct the potential in $\vec{r}$ space). Hence the parameter C_0 is given by

$$C_0 = - \frac{1}{4\pi} (1 + \mu/2M) \frac{4}{9} \left(\frac{f^*}{\mu} \right)^2 \frac{1}{f} \frac{\sum_{\Delta}^s}{(q^0 - \omega_R)^2} \quad (32)$$

which with the value $f^{*2}/4\pi = 0.36$ gives

$$C_0 = (0.12 + i 0.08) \mu^{-6} \quad (33)$$

in agreement with similar calculations performed in finite nuclei for the ^{16}O case⁶⁾.

Actually, when calculating the optical potential, we assumed $\Sigma_{\Delta}/(\mu - \omega_R)$ to be a small parameter and expanded in terms of it. We can try to refine a little more by including these higher order terms. For this we will use a fact, discussed later, that the effective nuclear density that the pion sees in all measured pionic atoms is, as a good approximation, around $\rho_{\text{eff}} \approx 0.65\rho_0$. Hence we substitute $\Sigma_{\Delta}/(q^0 - \omega_R)^2$ by $(q^0 - \omega_R - \Sigma_{\Delta})^{-1} - (q^0 - \omega_R)^{-1}$ at $\rho = 0.65\rho_0$ and, by neglecting the square of the tensor piece, which is actually small, we get the new value of C_0

$$C_0 = (0.13 + i 0.13) \mu^{-6} \quad (34)$$

One of the fits for the pionic atoms data¹⁹⁾ gives $C_0^{\text{emp}} = (0.20 + i 0.09) \mu^{-6}$.

It is also interesting to compare this with former calculations. As was stated before, these results compare very well with a similar calculation carried out in finite nuclei⁶⁾, but give much larger results than those from Refs. 7) and 8). These latter calculations omitted the complexity of the q^0 integration, which was carried out with some care here by means of a Wick rotation. The absence of these retardation effects should be blamed for the differences, as mentioned in Ref. 13).

We have presented the results in lowest order, but as we have seen, consideration of the meson propagation in the medium had some consequences in the imaginary part of the self-energy which showed appreciable differences with the lowest order calculations as the density increased.

The results in Fig. 3 for the imaginary part can be cast in an easy analytical formula with great accuracy. Hence we find for the full calculation

$$I_m \Sigma^s(\rho) = -38 \frac{1}{2.78} \arctg(2.78 \rho/\rho_0) [\text{MeV}] \quad (35)$$

As a consequence, by means of (25), we now get an optical potential which instead of being quadratic in the density looks like

$$I_m V^{opt}(q) = -\frac{4\pi}{2\mu} \vec{q}^2 C_0 \left(1 + \frac{\mu}{2M}\right)^{-1} \rho \frac{\rho_0}{2.78} \arctg(2.78 \rho/\rho_0) \quad (36)$$

which is normalized as (31) for small values of ρ . Because of the coincidence of both calculations at small densities and the special normalization in (36), $\text{Im } C_0$ has again the same value as in (33). Once more we can use the same arguments which lead to (34) to include approximately higher order terms in this potential. The results which we obtain then are

$$\text{Re } V^{opt}(q) = -\frac{4\pi}{2\mu} \vec{q}^2 \text{Re } C_0 \rho^2 \frac{1}{1 + \mu/2M} \quad (37)$$

$$I_m V^{opt}(q) = -\frac{4\pi}{2\mu} \vec{q}^2 I_m C_0 \rho \frac{\rho_0}{2.78} \arctg(2.78 \rho/\rho_0) \frac{1}{1 + \mu/2M}$$

with

$$\begin{aligned} \text{Re } C_0 &= 0.15 \mu^{-6} \\ I_m C_0 &= 0.14 \mu^{-6} \end{aligned} \quad (38)$$

The functional that appears in (37) for the imaginary part of the optical potential differs appreciably from the ρ^2 functional universally accepted up to now. In view of the good fits obtained for pionic atoms with the ρ^2 functional, it seems a real challenge to try to fit them with the new functional that (37) provides.

As we have seen in Sections 2 and 4, consideration of higher orders in the induced interaction led to the non-linearity of the Δ self-energy as a function of the density. One can also see that our calculated values of C_0 range around the empirically determined ones. However, there are other pieces that also contribute to the p wave pion self-energy like that in Fig. 5a. We may expect similar modifications in the pionic self-energy like those already seen, when we go from diagram 5a to 5b. Indeed, the tensor piece of one pion exchange plays an important role in these pieces, as mentioned in Ref. 8).

This indicates that the p wave pion self-energy as a whole will be well parametrized in the form of (37). The s wave pionic self-energy depicted diagrammatically in Fig. 5c could undergo a similar fate. Once again going from Fig. 5c to Fig. 5d will introduce modifications, however, the s wave character of the coupling of the external pion to the first p -h excitation will stress other types of correlations in the pionic line than the spin-isospin correlations which we found relevant in the p wave pionic self-energy [work on these lines is under way²²⁾].

In view of the theoretical support for the functional of the type (37) for $\text{Im } U^{\text{opt}}$, one may wonder why such good fits to the data have been obtained up to now by using the ρ^2 functional. One may also wonder whether one can fit these data equally well with the new functional that we have obtained. The answer to this last question is offered in the next section.

6. - FIT TO THE PIONIC ATOMS DATA. DISCUSSION

We have carried out a best fit calculation to all the available data, shifts and widths of the pionic states, in the standard procedure, by using the same parameters as in Ref. 19) but keeping $\text{Im } B_0$ and $\text{Im } C_0$ free.

The optical potential is taken as in Ref. 19) but with substitutions

$$\begin{aligned} B_0 p^2 &\rightarrow \text{Re } B_0 p^2 + i \text{Im } B_0 p \frac{p_0}{2.78} \arctg(2.78 p/p_0) \\ C_0 p^2 &\rightarrow \text{Re } C_0 p^2 + i \text{Im } C_0 p \frac{p_0}{2.78} \arctg(2.78 p/p_0) \end{aligned} \quad (39)$$

and a best fit is done to determine $\text{Im } B_0$, $\text{Im } C_0$, by keeping all the other parameters fixed to the values of Ref. 19).

Although there is not much support for such a renormalization in the s wave part (B_0 term), we have chosen to allow such modifications for this piece also and a proper discussion of the consequences will be made later on.

The result of this fit is somewhat surprising. We obtain a fit to all data which is as good (in fact, slightly better when readjusting $\text{Re } B_0$ and $\text{Re } C_0$ slightly) than the one obtained by the ρ^2 functional.

The numerical results for $\text{Im } B_0$ and $\text{Im } C_0$ with the ρ^2 functional are

$$\begin{aligned} \text{Im } B_0 &= 0.049 \mu^{-4} \\ \text{Im } C_0 &= 0.080 \mu^{-6} \end{aligned} \quad (40)$$

while, with the potential of Eq. (39), they are

$$\begin{aligned} \text{Im } B_0 &= 0.083 \mu^{-4} \\ \text{Im } C_0 &= 0.136 \mu^{-6} \end{aligned} \quad (41)$$

It is very hard to understand how two so different functional of the density can give rise to equally good fits, if the different pion atomic states have a large range of overlapping densities with the nucleus. These two good fits with the different functionals seem to suggest that the range of this effective nuclear density is very narrow. Comparison of (41) to (40) considering (39) would give $\rho_{\text{eff}} \approx 0.65\rho_0$. These results obtained from the analysis of the absorptive potential, without changing the first order parameters, are in qualitative agreement with similar results obtained by looking at the interference between the ρ and ρ^2 terms in a conventional optical potential²³⁾.

This narrow band of effective nuclear densities is quite a surprising result which we have confirmed by looking at the overlap between the distorted atomic wave function and the nucleus:

$$\chi(\vec{r}) = |\varphi_{\text{at}}(\vec{r})|^2 \rho_{\text{nuc}}(\vec{r}) \quad (42)$$

This constancy of the effective density in the measured pionic atoms contrasts with the situation of Σ atoms and Σ hypernuclei, which provide two physical situations where the effective nuclear densities felt by the Σ particles are quite different ($\rho \approx 0.1 - 0.2\rho_0$ for Σ atoms and $\rho \approx \rho_0$ for the hypernuclear case). In the latter case the saturation properties of the imaginary part of the self-energy were essential in providing a simultaneous understanding of the pionic and hypernuclear data⁵⁾.

The real part of C_0 in this best fit is as in Ref. 19) $\text{Re } C_0 = 0.20\mu^{-6}$, hence the best fit values $C_0 = (0.20 + i0.14)\mu^{-6}$ compare well with our calculated results, Eq. (38), $C_0 = (0.15 + i14)\mu^{-6}$. However, we should note here that in order to keep the same parameters as in Ref. 19) for purpose of comparison, we have used a Lorenz-Lorentz parameter $\xi = 1$. Should we have used larger values of ξ ($\xi \sim 2$), as suggested by a variety of experiments¹³⁾, we would have obtained larger values of $\text{Re } C_0$ and $\text{Im } C_0$ ²⁴⁾ in the fit. Hence we think that although the Δ mechanism studied in Sections 2 and 3 provides an appreciable amount of the absorptive p wave optical potential, there is still ground for other mechanisms as discussed in Ref. 8) and 21). However, it is also clear that because of the mechanism of virtual meson renormalization studied here, together with the

relevance of the effects of retardation [q^0 is an integration variable in (18), instead of being a fixed quantity, and the integration requires consistency with the special analytical structure of the integrand] a re-analysis of these pieces would be most advisable.

The study of the s wave absorptive potential has only been marginal here. The good fit to the data obtained by changing the density functional for this piece stresses the interesting fact of the narrow band of nuclear densities felt by the pions. On the other hand, the q^0 integration of the loop, together with the inclusion of higher orders of the induced interaction, which proved quite relevant in the present paper in connection with the p wave absorptive potential, are likely to play a similar role in the s wave part²⁵⁾. This calls for a re-analysis of these pieces.

With respect to the fit itself, the bulk of the data are equally well reproduced as in other fits. The results are thus like those in Ref. 19) with small differences. We have checked the region of the $3d$ states for heavy atoms where the widths are abnormally small compared to the theoretical predictions^{26),27)}. There we found 10-20% decrease in Γ with respect to the fit with the ρ^2 functional, but insufficient to provide good agreement with the experimental data.

The $4f$ states in heavy nuclei are also increased by 10-20% with respect to the old fit making the agreement with experiment slightly better.

As stated before the effective density felt by the pions is appreciably constant for the different pionic states known. In order to have smaller densities we should go to high angular momentum. However, even such a state as a $3d$ in ^{16}O , which still does not have too large an angular momentum, would have such a small width that it would make observation extremely tough with present experimental facilities.

We might also wonder whether the optical potential proposed here, smoothly extrapolated to low pion energies, would be equally successful in explaining the pion nucleus scattering data. The answer to this question can be found in Ref. 23). The authors there, after showing the interference between the ρ and ρ^2 terms of the optical potential in pionic atoms, go to low energy pion nucleus scattering and find the same interference and the same effective density. Thus, any potential that fits the pionic atoms fits the scattering data equally well. The existence of this effective density is then the key to understanding the numerous excellent fits to the data with very different optical potentials. Hence it is not a special merit of this potential that it would also explain the low energy pion nucleus scattering data. However, there are some pionic processes like pion nucleus single or double charge exchange which take place preferentially

in the surface and hence could provide the low density regime where one could find complementary information to the one provided by pionic atoms or low energy elastic scattering, and check the new pion optical potential versus the conventional ones.

7. - SUMMARY

We have calculated the Δ self-energy in nuclear matter and found the imaginary part highly non-linear in the nuclear density, an effect which could be traced back to the properties of virtual meson propagation in the medium, essentially pion propagation. This non-linearity had repercussions in the p wave absorptive optical potential which now had a different shape to the functional normally used in pionic atoms' analysis. With the new functional in density provided by the calculation, we carried out a best fit analysis to all existing pionic atoms data and found an even slightly better fit than with the ρ^2 functional. The widths of $3d$ states in heavy atoms were reduced slightly and the $4f$ states slightly increased, in better agreement with the experiment. We attributed the simultaneous good fit with the ρ^2 or the new functional to the fact that pionic atoms have a very narrow range of effective densities felt by the pions, a feature which was numerically checked by looking at the overlap of the pion wave function and the nuclear density, and which agrees with results of another different analysis²³⁾. Other pionic atoms, where the overlap with the nucleus would be much smaller, have such small width that their observation would be extremely difficult. This situation is contrasted with the analogous one in Σ atoms, where the overlapping densities are very low. However, one has access to another regime of large nuclear densities felt by the Σ , except that these states are no longer called atomic states but hypernuclei, since here the strong interaction dominates over the Coulomb attraction. A possible reaction which forces pions in a low density regime would be pion nucleus single or double charge exchange reaction, where the actual charge exchange process takes place in the nuclear surface. Here we could find the information of pions in a low density regime, which would be complementary to the information from pionic atoms and would offer an extra check for the new density functional found here.

Our calculated results for the p wave absorptive potential provided a fair part of the empirically estimated one, although there was room for extra pieces. However, the results of our study showed that these extra pieces, as well as the s wave part of the absorption called for a re-analysis that would account for the meson renormalization in the medium, as well as the retardation effects, by means of a proper energy variable integration in the internal loops, a work which is in progress at the moment.

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FIGURE CAPTIONS

- Fig. 1 : a) The ΔN -NN transition potential mediated by meson exchange;
b) piece of Δ self-energy;
c) second order contribution to the pion nucleus optical potential.
- Fig. 2 : a) Δ self-energy piece, zeroth order;
b) Pauli correction to the Δ self-energy as explained in the text;
c-d) Medium corrections to the Δ self-energy through particle hole and Δ hole excitation in first order;
e) higher orders contribution.
- Fig. 3 : Imaginary part of Σ_{Δ} in lowest order and with the full calculation of Fig. 2. The dashed line is a linear extrapolation from the low density results.
- Fig. 4 : Real part of Σ_{Δ} . Dashed line, first order calculation, no Pauli piece included. Full line, full calculation [Figs. 2c)-e)], no Pauli piece included. Dashed dotted line, full calculation with Pauli piece included Fig. 2b.
- Fig. 5 : a) Pion absorption mechanism from correlated pairs⁸⁾;
b) higher order terms from renormalization of meson propagators;
c) s wave pion absorption piece⁷⁾;
d) higher order terms in s wave absorption from pion renormalization in the medium. The full dots in the s wave pieces stand for the $\pi N \rightarrow \pi N$ s wave scattering amplitude.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5