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CONTINUUM MIXING AND COUPLED CHANNEL EFFECTS

IN $c\bar{c}$ AND $b\bar{b}$ QUARKONIUM

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A B S T R A C T

We apply the unitarized quark model to heavy $c\bar{c}$ and $b\bar{b}$ quarkonium and study mass shifts and mixings induced by hadronic coupled channel effects. In particular, we study the two-meson continuum mixing in the quarkonium wave functions. It is found that the continuum component is 2-20% in the $c\bar{c}$ and $b\bar{b}$ states, measured as the squared sum of two-meson amplitudes ($D\bar{D}$, $D\bar{D}^* + cc$, $D^*\bar{D}^*$, $F\bar{F}$, $F\bar{F}^* + cc$, $F^*\bar{F}^*$ for $c\bar{c}$). It is the largest for states near or above the first threshold. These continuum mixings reduce the predicted radiative widths by 5-30%. The mass shifts of the P and F wave $q\bar{q}$ states are similar to those of S and D wave states previously studied. The computed resonance mixing matrices are explicitly given, enabling future calculations of relativistic effects, QCD effects, etc., to include also the coupled channel effects.

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1. - INTRODUCTION

In our recent paper¹⁾ we combined the quark pair creation model (QPC)²⁾ and the unitarized quark model (UQM)³⁾ and studied hadronic mass shifts, mixings and decay rates for S and D wave states of $c\bar{c}$ and $b\bar{b}$ quarkonia. The purpose of this paper is to (i) include also P and F states into our study, (ii) compute the continuum mixing component (composed of $D\bar{D}$, $D\bar{D}^*$ etc.), of the meson wave functions and (iii) study the effects on electromagnetic M1 and E1 transition rates.

We shall discuss the coupled channel problem in quite general terms starting from a multichannel Schrödinger equation, where the Hilbert space is divided into two sectors, the confined states on one hand and the continuum or decay channels on the other hand. This discussion is a generalization of the Weisskopf-Wigner⁴⁾ formalism. Once a model like the QPC model, for the coupling between the confined states and the continuum is given, one can compute exactly (within the model) the hadronic mass shifts, the resonance mixing and the continuum mixing. Especially, the continuum mixing has not been previously properly taken into account.

It is known that⁵⁾ in non-relativistic potential models the theoretical values of the electromagnetic transition rates of a heavy quarkonium depends mainly on the quark mass and only very weakly on the shape of the potential. For E1 transitions this result can be derived^{6),7)} by using the Thomas-Reich-Kuhn sum rule [see, for example, Ref. 8)] which is a consequence of the Heisenberg uncertainty principle. Thus one could determine the charm quark mass m_c by comparing electromagnetic transition rates with data. However, the c-quark mass obtained in this way comes out rather high, $m_c \approx 2.2$ GeV, which indicates that either relativistic or coupled channel effects (or both) are important.

Various authors [see, for example, Refs. 9)-15)] have tried to include relativistic corrections, but coupled channel effects have not been studied systematically

before. As to the coupled channel effects, the Cornell group¹⁶⁾ did not consider the mixing between the P-wave states and did not study in detail the continuum mixing effects.

The outline of the paper is as follows. In Sec. 2 we discuss in very general terms how to unitarize a model like the quark pair creation model and we derive formulas for the continuum mixing. In Sec. 3 we compare with experimental mass spectra and radiative widths and give tables of resonance and continuum mixing. Finally, our conclusions are in Sec. 4.

2. - THE UNITARIZED QUARK MODEL

Our coupled channel model is obtained by using the quark pair creation model (QPC)²⁾ to determine hadronic vertices. These are needed as input to a unitarized model which we describe below.

2.1 The unitarization of the quark pair creation model.

The unitarization can be formulated using a quite general multichannel Schrödinger equation. One divides the Hilbert space into two sectors: the confined channels $|i\rangle$, which are the same as in naïve single channel potential models ($c\bar{c}$, $b\bar{b}$), and (ii) the continuum channels $|k\rangle$, which are composed of two meson states (for example, $D\bar{D}$, $D\bar{D}^*$, etc.). The Hamiltonian can be decomposed in the following way:

$$H = \begin{matrix} \text{confined} \\ \text{channels} \end{matrix} \left\{ \begin{matrix} H_0 & 0 \\ 0 & H_0^c \end{matrix} \right\} + \begin{matrix} \text{continuum} \\ \text{channels} \end{matrix} \left\{ \begin{matrix} 0 & H_{QPC} \\ H_{QPC}^\dagger & 0 \end{matrix} \right\}.$$

(2.1)

Here H_0 is given by the usual $c\bar{c}$ or $b\bar{b}$ single channel model whose mass spectrum (= bare mass M_i^{bare}) and wave functions $|i\rangle$ are assumed to be known from solving the Schrödinger equation, $H_0|i\rangle = M_i^{\text{bare}}|i\rangle$, in the usual way. The part H_0^c is assumed to contain only the kinetic energy of the two meson channels; i.e., were it not for H_{QPC} no meson-meson interaction would occur, and the states $|i\rangle$ would be stable (zero width) states.

The quark pair creation Hamiltonian H_{QPC} can be pictured by the quark line diagram of Fig. 1, where A represents the confined state $|i\rangle$ and B and C form together the continuum states $|k\rangle$. Our model can be looked upon as a generalization of the Weisskopf-Wigner formalism⁴⁾ for states coupled to a continuum. Consequently, in analogy with the Weisskopf-Wigner case, our full Hamiltonian Eq. (2.1) generates finite widths to the resonances and shifts the masses by an amount ΔE which depends on H_{QPC} . In addition, one finds, at the same time, resonance mixing induced by H_{QPC} such that the physical resonance states are mixtures between the bare confined states $|i\rangle$. Finally, the physical resonance states will mix with the continuum states $|k\rangle$, i.e., they contain a continuum component in their wave function.

The actual form of H_{QPC} is not very important for the present discussion. The crucial feature that we need is that H_{QPC} will give energy dependent vertex functions which, due to the finite size of the hadrons, give an automatic cut-off in the integrals below. In Sec. 2.3 we recapitulate the most important properties of H_{QPC} . Below we suppress for simplicity many indices (L, J^{PC} , flavour, channels B C, etc.), which can be included in an obvious way.

Starting from the full Hamiltonian of Eq. 2.1, one can derive an effective Hamiltonian H^{eff} (or a mass matrix), which describes the finite width physical resonances. For this purpose one uses as reference frame the eigenstates $|i\rangle$ of H_0 and the free eigenstates of H_0^c

$$H_0 |i\rangle = M_i^{\text{bare}} |i\rangle, \quad (2.2)$$

$$H_0^c |k\rangle = E_k |k\rangle, \quad (2.3)$$

and looks for eigenstates of the full Hamiltonian of the form

$$|\Psi_n\rangle = \sum_{ij} a_{ij} |n\rangle + \int dk c_n(k) |k\rangle dk. \quad (2.4)$$

Here the symbolic integral sign also stands for sums over the discrete variables (for example, different channels BC) which we have suppressed. Inserting this in the full problem:

$$H |\Psi_n\rangle = M_n |\Psi_n\rangle, \quad (2.5)$$

one easily finds the exact relations:

$$c_n(k) = \sum_i a_{ni} \frac{\langle k | H_{QPC} | i \rangle}{M_n - E_k}, \quad (2.6)$$

and

$$(H_{ni}^{\text{eff}} - M_{ni}) a_{ni} = 0, \quad (2.7)$$

where

$$H^{\text{eff}} = M_n^{\text{bare}} \delta_{ni} - \int \frac{\langle n | H_{QPC} | k \rangle \langle k | H_{QPC} | i \rangle}{M_n - E_k} dk. \quad (2.8)$$

In deriving these relations one needs only the orthogonality relations $\langle i | j \rangle = \delta_{ij}$, $\langle i | k \rangle = 0$ and $\langle i | H_{QPC} | j \rangle = 0$ and $\langle k | H_{QPC} | k' \rangle = 0$ which follow from the basic form of H assumed in Eq. (2.1).

When the resonance is below all thresholds Eq. (2.8) can be used as such. Above the first threshold the resonance states are not stationary, and the integral

must be interpreted as a principal value integral. It is then appropriate to add an imaginary part [Eq. (2.11) below], which describes the finite widths of decaying states.

Note that Eqs. (2.6)-(2.8) are exact, although Eq. (2.8) looks like a second order perturbation term. The integrals are all finite, since H_{QPC} gives a natural cut-off due to the finite size of the hadrons, described by the $q\bar{q}$ wave functions. The second term in Eq. (2.8) is the mass shift term and can be written:

$$\text{Re } \Pi_{ij}(E) = - \int \frac{\langle i | H_{QPC} | k \rangle \langle k | H_{QPC} | j \rangle}{E - E_k} dk \quad (2.9)$$

$$= - \frac{1}{\pi} \int \frac{\langle i | H_{QPC} | k \rangle \langle k | H_{QPC} | j \rangle \rho(k)}{E - E_k} dE_k \quad (2.10)$$

where the latter form is obtained when the k variable is replaced by E_k , whereby the phase space factor $\rho(E)$ appears.

For the case when a continuum channel is open, one introduces an absorptive part to the effective Hamiltonian by adding an imaginary part:

$$\text{Im } \Pi_{ij}(E) = - \langle i | H_{QPC} | k \rangle \langle k | H_{QPC} | j \rangle \rho(E) \cdot \theta(E - E_{th}) \quad (2.11)$$

whereby $\Pi_{ij}(E)$ has the proper analytic threshold behaviour containing the threshold cuts. The $\text{Im } \Pi_{ij}$ is simply related to the widths. When one can neglect mixing between the resonances: $\text{Im } \Pi_{ii} = -\Gamma_i/2$. The complex mass matrix is then:

$$M_{ij}(E) = M_i^{\text{bare}} \delta_{ij} + \Pi_{ij}(E) \quad (2.12)$$

which is non-diagonal, in general, but which can be diagonalized by an orthogonal matrix α ($\alpha^T = \alpha^{-1}$).

$$M_i^{diag}(E) = \alpha^{-1}(E) M(E) \alpha(E) \quad (2.13)$$

Above the first threshold α becomes complex, but is not Hermitian. We shall, for convenience, normalize α in such a way that

$$\sum_n |\alpha_{ni}|^2 = 1 \quad (2.13')$$

although above the first threshold the resonance states are not exactly orthogonal in the usual sense [c.f. Ref. 7)]. The eigenvalues $M_i^{diag}(E)$ give the true physical masses and widths:

$$M_i^{Phys} = \text{Re } M_{ii}^{diag}(E) \Big|_{E=M_i^{Phys}} \quad (2.14)$$

$$\Gamma_i^{Phys} = -2 \text{Im } M_{ii}^{diag}(E) \Big|_{E=M_i^{Phys}} \quad (2.15)$$

With relativistic kinematics one obtains similar formulae as above:

$$(M_i^{Phys})^2 = \text{Re } (M_{ii}^{diag}(s))^2 \Big|_{s=(M_i^{Phys})^2} \quad (2.14')$$

$$\Gamma_i^{Phys} = - (M_i^{Phys})^{-1} \text{Im } (M_{ii}^{diag}(s))^2 \Big|_{s=(M_i^{Phys})^2} \quad (2.15')$$

These definitions differ somewhat from a definition obtained from the S -matrix pole positions, i.e., the zeros of $\det(M_{ij}^2 - s)$. For narrow resonances which do not mix strongly the pole is, however, very close to $s = (M_i^{Phys})^2 - i M_i^{Phys} \Gamma_j^{Phys}$. The definitions (2.14)-(2.15) have the advantage that they do not depend on analytic continuation and can be determined from data directly.

2.2 The continuum mixing

The mixing matrix α_{ij} determines the coefficients a_{ni} in Eq. (2.4) up to a normalization coefficient which depends on both a_{ni} and the continuum mixing $C_n(k)$

$$a_{ni} = \alpha_{ni} N_n \quad (2.16)$$

For the case when the resonance n is below all thresholds, no imaginary part appears and the coefficients a_{ni} and α_{ni} are real. From Eqs. (2.6) and (2.16) one gets

$$c_n(k) = N_n \sum_i \alpha_{ni} \frac{\langle k | H_{QPC} | i \rangle}{M_n - E_k} \quad (2.17)$$

The part in the normalization, which comes from the continuum states is

$$\int |c_n(k)|^2 dk = N_n^2 \int \left| \sum_i \alpha_{ni} \frac{\langle k | H_{QPC} | i \rangle}{M_n - E_k} \right|^2 dk \approx \quad (2.18)$$

$$N_n^2 \int \left| \frac{\langle k | H_{QPC} | i \rangle}{M_n - E_k} \right|^2 dk,$$

where the approximation is justified when α_{ni} is near diagonal. Replacing the integration variable k by the energy one finds that the numerator of the integrand is again $-\frac{1}{\pi} \text{Im } \Pi_{ii}$:

$$\int |c_n(k)|^2 dk \approx N_n^2 \frac{1}{\pi} \int_{E_{th}}^{\infty} \frac{-\text{Im } \Pi_{ii}(E)}{(M_n - E)^2} dE \quad (2.19)$$

The normalization can then be determined from

$$\sum_i |a_{ni}|^2 + \int |c_n(k)|^2 dk = 1 \quad (2.20)$$

One finds if α_{ni} is normalized as in Eq. (2.13'):

$$N_n = \left[1 + \frac{1}{\pi} \int_{E_{th}}^{\infty} \frac{-\text{Im } \Pi_{ii}(E)}{|M_n - E|^2} dE \right]^{-1} \quad (2.21)$$

Eqs. (2.19) and (2.21) determine the continuum component in the hadron wave function. With relativistic kinematics we have the result:

$$\int |C_n(k)|^2 dk = \frac{1}{\pi} \int \frac{-\text{Im} \bar{\Pi}_{ii}(s)}{|M_n^2 - s|^2} ds \left[1 + \frac{1}{\pi} \int \frac{-\text{Im} \bar{\Pi}_{ii}(s)}{|M_n^2 - s|^2} ds \right]^{-\frac{1}{2}} \quad (2.22)$$

When the resonance is below all thresholds, Eq. (2.21) determines uniquely the continuum mixing. When one or more decay channels are open, the integral diverges for real M_n , i.e., if one evaluates the mixing in the physical region. This is, in fact, what is to be expected since unstable states eventually decay 100% to open channels.

In order to define a more useful continuum mixing for states above threshold it is more appropriate to define the mixing at a complex value of M_n , most naturally at the S-matrix pole position, or up to the approximation discussed above: $M_n^2 = (M_n^{\text{Phys}})^2 - i M_i^{\text{Phys}} M_i^{\text{Phys}}$. Then Eq. (2.22) is finite and defines the continuum mixing at the position of the S-matrix pole.

In Ref. 18), a multichannel problem similar to our Eq. (2.1) is discussed with special emphasis on the generalization of the Levinson theorem and other formal problems related to this Hamiltonian. Another more phenomenological approach to a similar problem is discussed by the Nijmegen group¹⁹⁾.

2.3 The quark pair creation model (QPC).

To complete the discussion of the previous section we need the matrix elements of H_{QPC} which couple the confined states to the continuum states. We use the QPC model²⁾ which gives a clear physical picture of the decay process, and which has been quite successful in describing many experimentally known resonance widths^{20,21)}.

In this model the driving mechanism is the creation of a quark pair of

vacuum quantum numbers, therefore it is also called the 3P_0 model. After the pair creation follows a recombination of the initial quarks in meson A and the 3P_0 pair to the meson state BC. The matrix elements H_{QPC} are then obtained as an overlap of the three wave functions ψ_A, ψ_B, ψ_C (cf. Fig. 1) and the pair creation P or symbolically:

$$\langle k | H_{QPC} | i \rangle = \langle \psi_B \psi_C | \psi_C P \rangle. \quad (2.23)$$

More explicitly, with flavour, sign, channel, etc., indices, one can write Eq. (2.23) in a factorized form containing a flavour symmetry factor $\mathcal{F}$ a spin-angular momentum overlap $\mathcal{S}$ and a space overlap function $\mathcal{L}$ (the colour factor is constant and therefore here absorbed in γ_{QPC})

$$\langle k | H_{QPC} | i \rangle = \gamma_{QPC}^2 \frac{1}{\sqrt{3}} \mathcal{F}_A^{BC} \sum_{S_{BC}, L} \mathcal{S}_{A, S_{BC}, L}^{BC} \mathcal{L}_{A, L, L_A}^{BC}(s) \quad (2.24)$$

Here γ_{QPC} is a universal constant determining the strength of the pair creation [which we found to be 3.03, see Ref. 1)]. The flavour factor is

$$\mathcal{F}_A^{BC} = \text{Tr} (\Delta_A \Delta_B^\dagger \Delta_C^\dagger)_{\pm}, \quad (2.25)$$

where Δ_A one $N \times N$ flavour matrices and the sign is that of $C_A C_B C_C$. This form assures flavour symmetry and the OZI rule for the bare vertices.

The spin-angular momentum overlap is composed of two 9j symbols

$$\mathcal{S}_{A, S_{BC}, L}^{BC} = \left\{ \begin{matrix} \frac{1}{2} & \frac{1}{2} & S_B \\ \frac{1}{2} & \frac{1}{2} & S_C \\ S_A & 1 & S_{BC} \end{matrix} \right\} \left\{ \begin{matrix} L_A & S_A & J_A \\ 1 & 1 & 0 \\ L & S_{BC} & J_A \end{matrix} \right\}. \quad (2.26)$$

$$\cdot (2S_A+1)3(2S_B+1)(2S_C+1)(2L+1)(2S_{BC}+1)(2J_A+1).$$

and the spatial overlap is

$$\mathcal{L}_{A,L,L_A}^{BC}(s) = \sum_{m_p, m_A} \begin{pmatrix} L_A & 1 & L \\ m_A & m_p & 0 \end{pmatrix} \sqrt{4\pi} \int d\vec{q} Y_{1, m_p}(\vec{q}) Y_{1, m_A}(\vec{q} + h_Q \vec{k}) \psi_A^{m_A}(\vec{q} - \frac{1}{2} h_q \vec{k}) \psi_B^*(\vec{q}) \psi_C^*(\vec{q}) , \quad (2.27)$$

where

$$h_q = \frac{2m_Q}{m_q + m_Q} , \quad h_Q = \frac{2m_q}{m_q + m_Q} .$$

Once the potential is fixed the ψ 's are known and one can compute the overlaps. We used a linear plus Coulomb-like potential [Eq. (1.4) of Ref. 1)] which was studied in more detail in Ref. 22).

3. - COMPARISON WITH DATA

The unitarity effects which are of interest can be classified in three different groups: (i) the mass shifts ΔM , (ii) the resonance mixing α_{ij} Eqs. (2.13, 16) and (iii) the continuum mixing Eq. (2.22). The two latter effects are important, in particular, in the calculation of electromagnetic transition widths.

3.1 The mass shifts

We summarize in Fig. 2 the mass shifts for the $c\bar{c}$ and $b\bar{b}$ n^3S_1 , n^3D_1 , n^3P_0 , n^3P_1 , and n^3P_2 states. In Table I we give the numerical values of the P and F wave mass shifts (i.e., including also n^1P_1 and n^3F_2). The corresponding table for the n^1S_0 and n^3S_1 and n^3D_1 mass shifts was already given in Table IV of Ref. 1).

Taking into account the mass dependence (Fig. 2) the P wave mass shifts are rather similar to those of the S wave shifts, as could be expected from a model where $SU(6)_W$ is built in. The main breaking of $SU(6)$ is due to the $D-D^*$

(and $F-F^*$ etc.,) mass splittings. The mass shifts do, however, depend on the resonance mass (Fig. 2) in a different way for $c\bar{c}$ than for $b\bar{b}$. For $c\bar{c}$ we have a decreasing ΔM with mass, whereas for $b\bar{b}$ the opposite is true.

This behaviour can be understood by noting that the relative positions of the $D\bar{D}$ etc., thresholds on one hand and the $B\bar{B}$ etc., on the other hand are quite different (Fig. 2). Since the maximal mass shift is expected near the threshold position [cf. Fig. 2 of Ref. 1)], the increase in ΔM for $b\bar{b}$ with resonance mass follows because one approaches the thresholds from below. For resonances above the $B\bar{B}$ threshold $T(5S)$ etc., the mass shift should turn to a decrease as is the behaviour in the case of the $c\bar{c}$ resonances, most of which lie well above the $D\bar{D}$ threshold.

As was discussed in Ref. 1), these bare spectra can be fitted with a potential whose shape is slightly broader than the conventional (plus a larger constant term). For example, for the Lichtenberg-Wills QCD motivated potential the Λ parameter decreases from 676 to 350 MeV when instead of the physical masses one fits our bare masses.

As to the splittings of the P-wave states, one could have hoped for a bigger effect which could have explained the anomalously high value of the 1^3P_1 mass or equivalently the smallness of the ratio $[m(^3P_2) - m(^3P_1)]/[m(^3P_2) - m(^3P_1)]$. However, our mass shifts turn out to be nearly $SU(6)$ symmetric and no significant effect in modifying this ratio is obtained. From the spin-angular momentum overlaps $\int_A^{BC}$ Eq. (2.25), one can easily derive the relative shifts which the D^*-D and F^*-F mass splittings generate. They are of opposite sign for the meson-meson S and D wave contribution:

$$\frac{\Delta M(^3P_2 - ^1P_1)}{\Delta M(^1P_1 - ^3P_1)} = \begin{array}{cc} \text{S-wave} & \text{D-wave} \\ \frac{1}{1} & \frac{-1}{-1} \end{array} \quad (3.1)$$

i.e., the S wave and D wave both give equal splitting of the levels, but of opposite sign. The relative contribution of the S and D waves depends on details of the model. In our case they are nearly equal and cancel especially for $b\bar{b}$. For $c\bar{c}$ our D wave is slightly stronger. We believe this result is very sensitive to the details of the QPC model. A slightly different model for the hadronic vertex function could give a different result, possibly explaining part of the anomalously high mass of the 1^3P_1 state.

3.2 The resonance and the continuum mixing

In Tables 2 and 3 we give the resonance mixing matrices α_{ij} for the 3P_0 , 3P_1 , 3P_2 , 1P_1 , 3F_2 , and 1S_0 states normalized as in Eq. (2.13'). The corresponding matrices for the 3S_1 and 3D_1 states were already given in Ref. 1). (Tables VII and VIII of Ref. 1). One notices that the matrices are rather similar for all the P wave states as should be expected from a model with near $SU(6)_W$ symmetry.

These matrices are needed in the calculation of electromagnetic transitions discussed in the following subsection. In order to make possible future calculations of relativistic corrections and QCD gluonic effects, etc., we give these matrices explicitly. In Table 4 we give the normalization constant N_n which depends on the continuum mixing [Eqs. (2.20)-(2.22)]. These numbers show that the continuum mixing [Eq. (2.21)] is 2-20% measured as the sum of squared amplitudes. The mixing is the smallest for a deeply bound state like the $T(1S)$ (2%), but increases, as one approaches the first OZI allowed threshold, to slightly more than 20%. These numbers reduce the radiative widths, since as a reasonable first approximation the continuum components do not contribute in the radiative decay.

3.3 Electromagnetic transitions

From the preceding section, Eqs. (2.4), (2.16) and (2.22), it follows that in $\chi(n_i P) \rightarrow \psi(n_f S)$ decay, the initial state is a mixture of n^3S_1 and n^3D_1 states and the continuum, while the final state is a mixture of P, F and the continuum. We can thus write:

$$|\Psi_i\rangle = N_i \left[\sum_m \alpha_{i,ns} |nS\rangle + \sum_m \alpha_{i,mD} |mD\rangle \right] + \int c_i(k) |k\rangle dk, \quad (3.2)$$

$$|\Psi_f\rangle = N_f \left[\sum_m \alpha_{f,nP} |nP\rangle + \sum_m \alpha_{f,mF} |mF\rangle \right] + \int c_f(k) |k\rangle dk. \quad (3.3)$$

The continuum states can in principle couple to the photon (giving rise to part of, e.g., the anomalous magnetic moment) but such contributions should be small, since the cross terms vanish. As an example let us consider a case when both initial and final states contain our maximal 20% of the continuum states. Since a γ -transition from a $q\bar{q}$ to a $(q\bar{q})(q\bar{q})$ state (two meson state) or from $(q\bar{q})(q\bar{q})$ to a $q\bar{q}$ state vanish, only the $(q\bar{q})(q\bar{q}) \rightarrow (q\bar{q})(q\bar{q}) + \gamma$ remains. But the latter is suppressed in amplitude by the square of 20%, i.e., 4%. Furthermore, since many of the $(q\bar{q})(q\bar{q}) \rightarrow (q\bar{q})(q\bar{q}) + \gamma$ are forbidden or small ($D^0\bar{D}^0 \nrightarrow D^0\bar{D}^0 \gamma$, $D^+D^- \nrightarrow F^+F^- \gamma$; $D^+D^- \nrightarrow D^{*+}D^- \gamma$, etc.,) the contribution should be even smaller than 4%. Thus we can safely neglect them. The continuum thus affects the γ transitions only by the renormalization of the mixing coefficients a_{ij} by N_n (c.f., Table 4) as expressed by Eq. (2.16). Thus we can write for the E1 transition rates the formula:

$$\Gamma_{i \rightarrow f} = (2J+1) \frac{16}{243} \alpha k^3 N_i^2 N_f^2 \left[\sum_{n,n'} \alpha_{i,ns} \alpha_{f,n'p} \langle R_{ns} | r | R_{n'p} \rangle + c_f \sum_{m',m} \alpha_{i,mD} \alpha_{f,m'p} \langle R_{mD} | r | R_{n'p} \rangle + d_f \sum_{m,m'} \alpha_{i,mD} \alpha_{f,m'F} \langle R_{mD} | r | R_{m'F} \rangle \right]^2, \quad (3.4)$$

where

$$\begin{aligned} c_J &= -\sqrt{2} \quad , \quad d_J = 0 \quad \text{for } J=0 \quad , \\ c_J &= 1/\sqrt{2} \quad , \quad d_J = 0 \quad \text{for } J=0 \quad , \\ c_J &= -\sqrt{2}/10 \quad , \quad d_J = 2 \quad \text{for } J=2 \quad . \end{aligned} \quad (3.5)$$

The decay rate for the inverse transition, i.e., $\chi \rightarrow \psi \gamma$ is obtained by replacing $2J+1$ by 3 in Eq. (3.4). The transition between a vector state and a pseudoscalar state is a magnetic dipole (M1) transition. The wave functions are

$$|\Psi_i\rangle = N_i \left[\sum_n \alpha_{i,ns} |n^3S_1\rangle + \sum_m \alpha_{i,m0} |m^3D_1\rangle \right] + \int c_i(k) |k\rangle dk, \quad (3.6)$$

$$|\Psi_f\rangle = N_f \sum_m \alpha_{f,ms} |n^1S_0\rangle + \int c_f(k) |k\rangle dk, \quad (3.7)$$

and the decay rate is given by

$$\begin{aligned} \Gamma_{i \rightarrow f} &= \frac{4\alpha e_a^2 \hbar^3}{m_c^2} N_i^2 N_f^2 C_s \\ &\quad \left| \sum_{m,m'} \alpha_{i,ns} \alpha_{f,m's} \langle ms | j_0(kr/2) | n's \rangle \right|^2 \\ &\approx \frac{4\alpha e_a^2 \hbar^3}{3m_c^2} N_i N_f C_s \left| \sum_{n,n'} \alpha_{i,ns} \alpha_{f,n's} \left(\delta_{nn'} - \frac{\hbar^2}{24} \langle ns | r^2 | n's \rangle \right) \right|^2, \end{aligned} \quad (3.8)$$

where

$$\begin{aligned} C_s &= 1 \quad \text{for } V \rightarrow P \gamma, \\ C_s &= 3 \quad \text{for } P \rightarrow V \gamma. \end{aligned}$$

Our mixing matrices α are given in Tables 2 and 3 [and Tables VII and VIII of Ref. 1)] and the normalizations N_i and N_f due to the continuum mixing in Table 4.

The results of applying Eqs. (3.4) and (3.8) are shown in Tables 5 and 6. The matrix elements of r and r^2 were calculated using the potential (4.2) of Ref. 1), although as mentioned in the Introduction the result is insensitive to this choice. The value 1.9 GeV was used for the charmed quark mass. As seen from this Table, the resonance mixings α_{ij} do not affect the allowed M1 transition much, but increase substantially the forbidden M1 transitions - in some cases by more than a factor 10! This is expected, since after mixing, the orthogonality between the initial and final states is lost. Thus the forbidden transitions become partially allowed.

As for the E1 transitions the situation is more complicated due to the change of sign of C_j in Eq. (3.5). Thus in some processes we have constructive interference and in others destructive interference.

The effect of the continuum mixing is always a suppression of the width through the normalization factor $N_i N_f$ in Eqs. (3.4) and (3.8). This varies between 5 and 30%. In comparison with the data our predictions generally go in the right direction by reducing the allowed theoretical widths. Most of these become consistent with the data. An exception is the $\psi' \rightarrow \chi_1 \gamma$ width which comes out too large. Inclusion of relativistic and QCD gluonic effects further modify the widths [cf. Refs. 9)-15)], although at present different calculations give different results.

5. - CONCLUSIONS

This paper concludes our analysis, which we initiated in Ref. 1), of the $c\bar{c}$ and $b\bar{b}$ states using the UQM with hadronic vertices given by the QPC model. In general, we find unitarity effects to be far from negligible.

Some of these effects, in particular most of the mass shifts, can be absorbed

into the parameters of conventional single channel potential models by renormalization. The potential for the bare states is slightly broader than usual, and the constant term larger, or the quark masses are slightly larger than usual. In detail, there are some important exceptions, where such a renormalization does not work, in particular for states near the most important thresholds, such as the $T(4S)$ mass, which is much better fitted in the unitarized model.

The other important effects are the mixings induced by the hadronic coupled channel effects. These modify considerably the electromagnetic couplings. The most dramatic example is the e^+e^- width of the $1D$ state, which naively would vanish or be very small, but which through $1D - 2S$ mixing becomes substantial in agreement with data. Other couplings are the $E1$ and $M1$ γ - transition rates which are all modified by the resonance mixing as studied in Sec. 4. Many otherwise suppressed decays get highly increased predicted widths through the resonance mixing. The non-suppressed widths get modified by up to 20%, usually downwards (except $\psi' \rightarrow \chi_1 \gamma$), through the resonance mixing.

Finally the continuum mixing of $D\bar{D}$, $D\bar{D}^*$ etc., states in the $c\bar{c}$ (and $B\bar{B}$ etc. , in $b\bar{b}$) turns out to be up to 20%. This mixing always reduces the radiative widths by amounts which vary between 5 and 30% improving the comparison with experiment. We have included tables of the mixing matrices and the wave function normalizations in such a way that it is easy to evaluate the coupled channel effects for future more elaborate models with relativistic effects, QCD gluonic effects etc.

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Table 1

The mass shifts of the $c\bar{c}$ and $b\bar{b}$ states and the bare and physical masses as given by the potential model.

$c\bar{c}$ states	Physical mass	Bare mass	Mass shifts
$\chi_c(1^3P_0)$	3415	3588	173
$\chi_c(2^3P_0)$	3893	3990	97
$\chi_c(3^3P_0)$	4231	4259	28
$\chi_c(1^1P_1)$	3520	3702	182
$\chi_c(2^1P_1)$	3934	4061	127
$\chi_c(3^1P_1)$	4264	4294	30
$\chi_c(1^3P_1)$	3510	3690	180
$\chi_c(2^3P_1)$	3930	4038	108
$\chi_c(3^3P_1)$	4261	4285	23
$\chi_c(1^3P_2)$	3550	3735	185
$\chi_c(2^3P_2)$	3957	4104	147
$\chi_c(1^3F_2)$	4034	4161	127
$\chi_c(3^3P_2)$	4287	4324	37
$\chi_c(2^3F_2)$	4336	4390	54
$b\bar{b}$ states			
$\chi_b(1^3P_0)$	9870	9913	43
$\chi_b(2^3P_0)$	10231	10286	55
$\chi_b(3^3P_0)$	10487	10551	64
$\chi_b(1^1P_1)$	9896	9940	44
$\chi_b(2^1P_1)$	10252	10309	57
$\chi_b(3^1P_1)$	10500	10564	64
$\chi_b(1^3P_1)$	9892	9936	44
$\chi_b(2^3P_1)$	10249	10305	56

Table 1 (continued)

$b\bar{b}$ states	Physical mass	Bare mass	Mass shifts
$\chi_b(3^3P_1)$	10498	10562	64
$\chi_b(1^3P_2)$	9912	9957	45
$\chi_b(2^3P_2)$	10264	10322	58
$\chi_b(1^3F_2)$	10325	10387	62
$\chi_b(3^3P_2)$	10506	10571	65
$\chi_b(2^3F_2)$	10563	10626	63

Table 2

Resonance mixings α_{ij} for $c\bar{c}$ states

	1P		2P		3P	
	Re	Im	Re	Im	Re	Im
$\chi_c(1^3P_0)$	0.9992	-	0.039	-	-0.008	-
$\chi_c(2^3P_0)$	- 0.064	0.011	0.9967	0.000	0.045	0.007
$\chi_c(3^3P_0)$	0.052	- 0.030	- 0.147	0.115	0.9804	0.189
$\chi_c(1^1P_1)$	0.9992	-	0.039	-	- 0.010	-
$\chi_c(2^1P_1)$	0.012	0.068	0.9961	0.001	0.038	- 0.041
$\chi_c(3^1P_1)$	0.078	- 0.015	- 0.196	0.090	0.9731	0.019
$\chi_c(1^3P_1)$	0.9991	-	0.041	-	- 0.010	-
$\chi_c(2^3P_1)$	- 0.021	0.124	0.9870	0.008	0.074	- 0.066
$\chi_c(3^3P_1)$	0.075	- 0.040	- 0.204	0.118	0.9678	0.028

Table 2 (continued)

	1P		2P		1F	
	Re	Im	Re	Im	Re	Im
$\chi_c(1^3P_2)$	0.9993	-	0.0363	-	0.001	-
$\chi_c(2^3P_2)$	0.056	0.042	0.9851	0.006	0.136	-0.055
$\chi_c(1^3F_2)$	0.005	-0.015	-0.090	-0.004	0.9885	0.003
$\chi_c(3^3P_2)$	0.058	0.005	-0.153	0.082	-0.013	-0.010
$\chi_c(2^3F_2)$	-0.001	-0.001	0.022	-0.003	-0.117	0.078

	3P		2F	
	Re	Im	Re	Im
$\chi_c(1^3P_2)$	-0.011	-	0.000	-
$\chi_c(2^3P_2)$	0.021	-0.047	-0.014	-0.016
$\chi_c(1^3F_2)$	-0.010	-0.021	-0.115	-0.029
$\chi_c(3^3P_2)$	0.9765	0.013	0.112	-0.007
$\chi_c(2^3F_2)$	-0.065	-0.063	0.9856	0.005

	1S		2S		3S		4S	
	Re	Im	Re	Im	Re	Im	Re	Im
$\eta_c(1S)$	0.9997	-	0.015	-	-0.012	-	0.013	-
$\eta_c(2S)$	0.012	-	0.9993	-	0.030	-	-0.017	-
$\eta_c(3S)$	0.013	-0.015	0.064	0.102	0.9914	-0.006	0.014	-0.045
$\eta_c(4S)$	0.037	-0.028	0.036	0.045	-0.080	0.035	0.9934	0.002

Table 3

Resonance mixings α_{ij} for $b\bar{b}$ states

	1P	2P	3P
$\chi_b(1^3P_0)$	0.9981	-0.059	-0.018
$\chi_b(2^3P_0)$	0.092	0.9939	-0.061
$\chi_b(3^3P_0)$	0.034	0.138	0.9900
$\chi_b(1^1P_1)$	0.9980	-0.061	-0.018
$\chi_b(2^1P_1)$	0.096	0.9932	-0.065
$\chi_b(3^1P_1)$	0.037	0.140	0.9895
$\chi_b(1^3P_1)$	0.9980	-0.061	-0.018
$\chi_b(2^3P_1)$	0.095	0.9933	-0.065
$\chi_b(3^3P_1)$	0.036	0.139	0.9896

	1P		2P		1F	
	Re	Im	Re	Im	Re	Im
$\chi_b(1^3P_2)$	0.9978	-	-0.063	-	0.000	-
$\chi_b(2^3P_2)$	0.099	-	0.9927	-	-0.001	-
$\chi_b(1^3F_2)$	0.000	-	0.002	-	0.9972	-
$\chi_b(3^3P_2)$	0.038	-	0.146	-	-0.004	-
$\chi_b(2^3F_2)$	0.002	0.001	-0.001	0.001	-0.169	0.006

	3P		2F	
	Re	Im	Re	Im
$\chi_b(1^3P_2)$	-0.019	-	0.000	-
$\chi_b(2^3P_2)$	-0.069	-	0.000	-
$\chi_b(1^3F_2)$	0.000	-	-0.075	-
$\chi_b(3^3P_2)$	0.9884	-	-0.008	-
$\chi_b(2^3F_2)$	0.050	0.026	0.9839	-0.002

Table 3 (continued)

	1S	2S	3S	4S
$\eta_b(1^1s_0)$	0.9995	-0.030	-0.010	-0.002
$\eta_b(2^1s_0)$	0.048	0.9973	-0.052	-0.015
$\eta_b(3^1s_0)$	0.021	0.095	0.9942	-0.044
$\eta_b(4^1s_0)$	0.008	0.019	0.175	-0.9844

Table 4

The continuum mixing contribution to the wave function
normalization N_n .

state	$N_n = (1 + \int c_n(k)^2 dk)^{-\frac{1}{2}}$	
	$c\bar{c}$	$b\bar{b}$
1^3s_1	0.95	0.99
2^3s_1	0.90	0.97
1^3d_1	0.91	0.97
3^3s_1	0.92	0.93
2^3d_1	0.91	0.92
4^3s_1	0.93	0.92
3^3d_1		0.91
1^1s_0	0.96	0.99
2^1s_0	0.93	0.97
3^1s_0	0.93	0.94
4^1s_0	0.95	0.91
1^3p_0	0.94	0.98
2^3p_0	0.93	0.95
3^3p_0	0.93	0.92

Table 4 (continued)

state	$N_n = (1 + c_n(k)^2 dk)^{-\frac{1}{2}}$	
	$c\bar{c}$	$b\bar{b}$
1^1P_1	0.93	0.98
2^1P_1	0.88	0.95
3^1P_1	0.92	0.89
1^3P_1	0.93	0.98
2^3P_1	0.90	0.95
3^3P_1	0.92	0.89
1^3P_2	0.93	0.98
2^3P_2	0.87	0.95
1^3F_2	0.86	0.96
3^3P_2	0.92	0.89
2^3F_2	0.92	0.91

Table 5

The magnetic dipole transition rate for the following cases.

(a) Potential model prediction, (b) Resonance mixing is included, (c) Resonance and continuum mixing are included.

(in KeV)

$c\bar{c}$, M1	without mixing (a)	resonance mixing (b)	resonance continuum mixing (c)	exp
$\psi(1S) \rightarrow \eta_c(1S)$	1.76	1.76	1.46	0.4 - 1.0
$\psi(2S) \rightarrow \eta_c(1S)$	1.15	2.76	2.10	0.69 ± 0.34
$\psi(2S) \rightarrow \eta_c(2S)$	0.754	0.737	0.515	0.6 - 2.6

Table 5 (continued)

$b\bar{b}$, M1	without mixing (a)	resonance mixing (b)	resonance continuum mixing (c)	exp
$T(1S) \rightarrow \eta_b(1S)$	5.27×10^{-4}	5.27×10^{-4}	5.11×10^{-4}	
$T(2S) \rightarrow \eta_b(1S)$	0.00157	0.00866	0.00803	
$T(2S) \rightarrow \eta_b(2S)$	7.75×10^{-5}	7.74×10^{-5}	7.31×10^{-5}	
$T(3S) \rightarrow \eta_b(1S)$	0.00323	0.0132	0.0114	
$T(3S) \rightarrow \eta_b(2S)$	0.00056	0.00584	0.00481	
$T(3S) \rightarrow \eta_b(3S)$	2.89×10^{-5}	2.87×10^{-5}	2.19×10^{-5}	

Table 6

Same as Table 5 but for electric dipole transition. (in KeV)

$c\bar{c}$, E1	without mixing (a)	resonance mixing (b)	resonance continuum mixing (c)	exp
$\psi' \rightarrow \chi_2 \gamma$	38.5	31.4	22.0	16.3 ± 4.1
$\psi' \rightarrow \chi_1 \gamma$	49.2	53.9	38.2	18.1 ± 4.7
$\psi' \rightarrow \chi_0 \gamma$	57.5	26.2	18.9	21.7 ± 5.3
$\chi_2 \rightarrow J/\psi \gamma$	387	378	298	490 ± 330
$\chi_1 \rightarrow J/\psi \gamma$	298	292	232	< 700
$\chi_0 \rightarrow J/\psi \gamma$	141	139	112	97 ± 38

Table 6 (continued)

$b\bar{b}$, E1	without mixing (a)	resonance mixing (b)	resonance continuum mixing (c)	exp
$1^3P_0 \rightarrow 1^3S_1$	22.4	23.5	22.3	
$1^3P_1 \rightarrow 1^3S_1$	26.1	27.9	26.5	
$1^3P_2 \rightarrow 1^3S_1$	29.8	31.6	29.9	
$2^3P_0 \rightarrow 1^3S_1$	11.9	18.0	16.1	
$2^3P_1 \rightarrow 1^3S_1$	12.7	19.2	17.2	
$2^3P_2 \rightarrow 1^3S_1$	13.5	20.8	18.4	
$2^3S_1 \rightarrow 1^3P_0$	1.17	1.11	1.01	1.05 ± 0.5
$2^3S_1 \rightarrow 1^3P_1$	2.20	2.53	2.31	1.77 ± 0.7
$2^3S_1 \rightarrow 1^3P_2$	2.21	2.38	2.16	1.87 ± 0.7
$2^3P_0 \rightarrow 2^3S_1$	10.1	8.46	7.25	
$2^3P_1 \rightarrow 2^3S_1$	12.9	13.3	11.4	
$2^3P_2 \rightarrow 2^3S_1$	15.6	14.6	12.4	
$3^3S_1 \rightarrow 1^3P_0$	0.069	0.212	0.178	
$3^3S_1 \rightarrow 1^3P_1$	0.180	0.0002	0.0001	
$3^3S_1 \rightarrow 1^3P_2$	0.263	0.163	0.136	
$3^3S_1 \rightarrow 2^3P_0$	1.60	1.26	0.97	
$3^3S_1 \rightarrow 2^3P_1$	2.94	3.17	2.25	
$3^3S_1 \rightarrow 2^3P_2$	3.03	2.93	2.29	

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FIGURE CAPTIONS

Fig. 1. Quark line diagram for the QPC model vertex function describing the decay of meson A to two mesons B and C.

Fig. 2. The mass shifts of the (a) $c\bar{c}$ states and (b) $b\bar{b}$ states. The NS and ND shifts are connected by the solid lines. Note the different relative positions of the thresholds in (a) compared to (b), which explain the different behaviour of ΔM with energy.

FIG. 1

Fig. 2