TWO-PARTICLE RAPIDITY CORRELATIONS IN $\alpha\alpha$, αp , AND pp INTERACTIONS AT
THE CERN INTERSECTING STORAGE RINGS

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ABSTRACT

Two-particle rapidity correlations have been studied for $\alpha\alpha$, αp , and pp interactions at the CERN ISR using the Split-Field Magnet (SFM) detector. In order to isolate the true two-particle correlations, the analysis was performed at fixed charged multiplicity. In the framework of a simple cluster model, it is found that cluster widths as well as cluster multiplicities are the same for $\alpha\alpha$, αp , and pp interactions, and both decrease with increasing charged multiplicity.

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1. INTRODUCTION

In a recent paper we have studied the multiplicity distributions for charged particles produced in high-energy $\alpha\alpha$ and αp interactions [1]. In this paper we investigate two-particle correlations, in order to take a further step towards understanding the underlying mechanisms responsible for particle production in nuclear interactions.

Particle correlations have been measured in various hadronic interactions during the last ten years [2,3], and lately also in e^+e^- interactions [4]. The observed correlations were interpreted in the framework of various theoretical models; the most popular ones were the cluster models [5] and, more recently, the string models [6].

When α -particles were accelerated and stored in the CERN Intersecting Storage Rings (ISR), a unique possibility of studying two-particle correlations for interactions of multinucleon systems was provided. Speculations regarding a phase transition when high enough energy densities are reached suggest that the size or mass of the clusters might be larger in $\alpha\alpha$ interactions than in interactions between elementary particles [7]. Also, other collective phenomena could lead to an increase in the cluster size.

The present experiment was performed at the ISR using the Split-Field Magnet (SFM) detector. A description of the SFM detector and of the experimental set-up can be found in ref. 1. The data were taken with a "minimum bias" trigger, requiring at least one track candidate anywhere in the detector. The data sets used for this analysis are listed in table 1, together with the c.m. energies.

This paper is organized as follows. First we study the inclusive correlations, and then turn to the semi-inclusive correlations by fixing the observed multiplicity. By doing the analysis at fixed multiplicity in terms of a cluster model, we extract cluster-widths in rapidity and cluster-strengths or decay multiplicities as a function of the total multiplicity, and compare these quantities for our different data sets. We believe this to be a sensitive procedure for recognizing differences of

real dynamical correlations. The main aim of this investigation has been to measure and compare correlations in $\alpha\alpha$, αp , and pp interactions.

2. INCLUSIVE CORRELATIONS

In high-energy interactions, most of the produced particles have low transverse momenta; consequently, one expects dynamical correlations to show up primarily in the longitudinal phase space. It is thus common to study two-particle rapidity correlations. The normalized correlation function

$$R(y_1, y_2) = \frac{\rho^{(2)}(y_1, y_2)}{\rho^{(1)}(y_1) \cdot \rho^{(1)}(y_2)} - 1, \quad (1)$$

where $\rho^{(1)}(y) = \sigma_{in}^{-1} d\sigma/dy$ and $\rho^{(2)}(y_1, y_2) = \sigma_{in}^{-1} d^2\sigma/dy_1 dy_2$ are the charged particle densities, has the advantage that most of the experimental efficiencies cancel out; $d\sigma/dy$ and $d^2\sigma/dy_1 dy_2$ are the single and double inclusive cross-sections, respectively; and σ_{in} is the inelastic cross-section. One problem in comparing $\alpha\alpha$, αp , and pp interactions is to find corresponding energies at which comparisons can be made. In fig. 1 we have plotted $R(y_1, y_2)$ at fixed y_1 as a function of y_2 , for all charged particles produced in pp interactions at three different energies. Here and elsewhere we have symmetrized the R -function for the cases of symmetric beams, since there were no significant differences, induced by the detector, between the data observed in the two hemispheres. The main features of this plot are the prominent peaks around the fixed rapidity y_1 . However, an energy dependence is clearly seen, i.e. at large absolute rapidities y_1 and y_2 the value of R decreases with decreasing energy. If, instead, the "normalized" rapidity, defined as $\psi = y/y_{inc}$ [8], is used (y_{inc} is the beam rapidity in the c.m. system), the behaviour changes drastically. In fig. 2, $R(\psi_1, \psi_2)$ is plotted versus ψ_2 at fixed ψ_1 . An almost perfect energy independence is observed. This suggests that by using the "normalized" rapidity, we can compare correlations from the three reactions even when the incident energies are different. However, for αp interactions we have to transform the data into a system where the proton and the α -particle have the same incident rapidities, i.e. the nucleon-nucleon c.m. system. In fig. 3 we display the results of such a

comparison. Except for the peaks around $y_1 = y_2$, the most striking effect is the difference between correlations in $\alpha\alpha$ interactions and those in pp interactions. At high absolute rapidities the $\alpha\alpha$ data flatten out, giving $R \approx 0$, in contrast with pp data, where large negative correlations are seen in this region.

The data from αp interactions have the following features. When the "triggering" rapidity ψ_1 is close to 0, the correlations on the α -side (positive rapidities) are close to the correlations observed in $\alpha\alpha$ interactions, whilst on the p -side (negative rapidities) the function behaves like the one obtained for pp interactions. When the "triggering" rapidity is shifted to the α -side, the R -function becomes similar to the one observed for $\alpha\alpha$ interactions, and the corresponding behaviour is observed for ψ_1 on the p -side, i.e. the function follows closely the one obtained from pp interactions.

It has long been known [3] that the main structure of such fully inclusive two-particle correlations results from mixing events with different structure in momentum space (here y) and has nothing to do with the true two-particle correlations. In the following we derive a quantitative form for R assuming that all true two-particle correlations are absent, and show that the change in shape of the particle densities in rapidity space with increasing multiplicity leads to a large apparent correlation. The total densities used in the definition of R are summed over the contributions from different charged multiplicities, i.e.

$$\rho^{(1)}(y) = \sum_n \pi(n) \cdot \rho_n^{(1)}(y) \quad (2a)$$

and

$$\rho^{(2)}(y_1, y_2) = \sum_n \pi(n) \cdot \rho_n^{(2)}(y_1, y_2) , \quad (2b)$$

where $\pi(n)$ is the probability of having n charged particles. The assumption that two-particle correlations are absent can be formulated:

$$\frac{\rho_n^{(2)}(y_1, y_2)}{\int \rho_n^{(2)} dy_1 dy_2} = \left[\frac{\rho_n^{(1)}(y_1)}{\int \rho_n^{(1)} dy} \right] \left[\frac{\rho_n^{(1)}(y_2)}{\int \rho_n^{(1)} dy} \right],$$

that is,

$$\rho_n^{(2)}(y_1, y_2) = \frac{n-1}{n} \rho_n^{(1)}(y_1) \cdot \rho_n^{(1)}(y_2) \quad (3)$$

for all n , since

$$\int \rho_n^{(2)} dy_1 dy_2 = n(n-1) \quad \text{and} \quad \int \rho_n^{(1)} dy = n.$$

Let us now assume that the density at any rapidity y depends linearly on n [9], i.e.

$$\rho_n^{(1)}(y) = n \cdot \alpha(y) + \beta(y). \quad (4)$$

Defining the average multiplicity associated with a particle at rapidity y (including this particle) as

$$\langle n \rangle_y = \sum_n \pi(n) \cdot n \cdot \rho_n^{(1)}(y) / \sum_n \pi(n) \rho_n^{(1)}(y), \quad (5)$$

we obtain (Appendix A) the following expression for R :

$$R(y_1, y_2) \approx \frac{\langle n \rangle^2}{D^2} \left(\frac{\langle n \rangle y_1}{\langle n \rangle} - 1 \right) \left(\frac{\langle n \rangle y_2}{\langle n \rangle} - 1 \right) - \frac{1}{\langle n \rangle}, \quad (6)$$

where $D^2 = \langle n^2 \rangle - \langle n \rangle^2$. In fig. 4 we have plotted the normalized associated multiplicity excess $(\langle n \rangle_y / \langle n \rangle) - 1$ for pp interactions at $\sqrt{s} = 62.2$ GeV and for $\alpha\alpha$ interactions at $\sqrt{s}_{nn} = 30.4$ GeV. We can immediately see a resemblance with the corresponding R -functions (see fig. 2). Figures 5 show comparisons between eq. (6) and the experimental R -functions, the overall behaviour of the latter being very well described by this equation, apart from the maximum at $y_1 \approx y_2$ which is essentially absent from it.

Even if the assumption of linearity in the density [eq. (4)] is not fulfilled, this example shows that the R -function would have a structure similar to the one observed, even if all two-particle correlations were absent. Here we have shown

quantitatively that this structure has its origin in correlations between the phase-space distribution of particles and total multiplicity; the existence of such correlations follows from very general principles such as energy and momentum conservation. Thus we arrive at the well-known conclusion that in order to get rid of these so-called long-range correlations, we have to study R (or some other function) at fixed total multiplicity.

As an additional remark, we would like to point out that studies of local multiplicity fluctuations [10], i.e. the fluctuation of the number of particles in a given rapidity interval, are identical to studies of two-particle correlations with $y_1 = y_2$. We can easily derive the relation

$$f_2(y) = R(y, y) \left[\rho^{(1)}(y) \cdot \Delta y \right]^2, \quad (7)$$

where Δy is the chosen bin-width. Thus the local second cumulant $f_2(y)$ reflects both the density distribution and the chosen bin-width; and by choosing a small enough bin-width we obtain $f_2(y) \rightarrow 0$ for any y , thus showing "Poisson likeness" all over the longitudinal phase space.

3. SEMI-INCLUSIVE CORRELATIONS

Figure 6 shows the R -function for all charged particles at a fixed negative multiplicity $n_- = 5$ for $\alpha\alpha$, αp , and pp interactions as a function of y_2 for two different values of y_1 . The negative multiplicity has been fixed (instead of the total), since this measures the number of produced particles -- thus avoiding problems due to the break-up of the α -particles -- and since the detector has a more homogeneous acceptance for negative particles at large x . The different interactions give similar features with a peak around y_1 of about the same height. The strengths of the correlations are much smaller than those of figures 3, and it should be noted that the distributions shown in fig. 6 are very similar to those obtained from fig. 5 if eq. (6) is subtracted from the experimental data. In fig. 7 the overall behaviour of the R -function for $\alpha\alpha$ interactions at $n_- = 5$, is given as a contour plot. The main feature of this plot is a saddle-shaped ridge stretching diagonally along $y_1 = y_2$ surrounded by a relatively flat region. Contour plots for pp interactions show the same ridge, and the only significant difference

in shape is seen at large absolute rapidities ($|y| > 3$) along the diagonal $y_1 = y_2$, where the R-function for pp data drops rapidly. Figure 7 can be compared with contour plots for inclusive data given in ref. 3, where the shape and size of the ridge is quite different.

In order to study systematically whether differences of the width and height of the correlation peak occur for $\alpha\alpha$, αp , and pp interactions, we need a parametrization of the peak and the background level. Therefore we analysed the data in the context of a simple cluster model, where the description "cluster" should be understood in a very broad sense, i.e. a group of particles with some correlated properties. In Appendix B we have derived a relevant formula which can be written:

$$R_n(y_1, y_2) = \frac{F_n \cdot \rho_n^{(1)}(y_a)}{\rho_n^{(1)}(y_1) \cdot \rho_n^{(1)}(y_2)} \frac{1}{2\sqrt{\pi} \delta_n} \exp \left[-(\Delta y)^2 / 4\delta_n^2 \right] - \frac{1 + F_n}{n}, \quad (8)$$

where δ_n is the width in rapidity of clusters produced in events with multiplicity n , and F_n is the corresponding cluster-strength defined as $F_n = [\langle K(K-1) \rangle / \langle K \rangle]_n$, where K is the charged multiplicity of the cluster. Furthermore, $y_a = (y_1 + y_2)/2$ and $\Delta_y = y_1 - y_2$. This is an extension of the formula for $R(0, y_2)$ given in reference 11.

When the experimental data are used to fix the parameters F_n and δ_n of eq. (8), two problems have to be considered: The one-dimensional densities have to be corrected for losses due to the acceptance; and the statistical errors of $R(y_1, y_2)$ have to be calculated correctly. The first problem is handled by using acceptance tables calculated by the use of reconstructed, Monte Carlo generated events. This method is described in ref. 1. The formulae used for the statistical errors are given in Appendix C.

We are now ready to use eq. (8) to calculate F_n and δ_n by χ^2 minimization for different values of the multiplicity n . Here we have used the R-function in

the central region of rapidity -- more specifically $|y_1|, |y_2| < 2$ -- where the prediction of the simple cluster model, described by eq. (8), seems to be valid (see fig. 7). Both the height of the correlation peak and the background level are determined by the cluster-strength F_n . We obtain the same values of F_n within $\approx 3\%$ from both terms if they are fitted independently. In earlier investigations only the strengths given by the height of the correlation peak were used.

In fig. 8 the values of δ_n are displayed as a function of the negative multiplicity n_- . In fig. 8a we have compared δ_n from pp data at three different energies, and the energy independence is clearly seen. In fig. 8b we compare δ_n from $\alpha\alpha$ and αp data with the δ_n from pp data, and no significant deviations can be detected. Thus, for $\alpha\alpha$, αp , and pp interactions, the cluster widths seem to decrease with increasing n_- at low values of n_- and flatten off for higher values of n_- .

Figure 9a shows F_n from αp data at $\sqrt{s}_{nn} = 44$ GeV and from pp data at $\sqrt{s} = 44$ GeV (i.e. at the same nucleon-nucleon c.m. energy) as a function of n_- ; in fig. 9b we show F_n from $\alpha\alpha$ data at $\sqrt{s}_{nn} = 31.2$ and from pp data at $\sqrt{s} = 30.4$ GeV (again at a similar nucleon-nucleon c.m. energy). Figure 9 shows that also F_n decreases with increasing multiplicity and that the incident particles seem to be without importance. On the other hand, in contrast to the case for δ_n , there seems to be an energy dependence of F_n .

From the simple cluster model (Appendix B) it is also possible to obtain the quantities $F_n^+ = [\langle K_+(K_+ - 1) \rangle / \langle K_+ \rangle]_n$, $F_n^- = [\langle K_-(K_- - 1) \rangle / \langle K_- \rangle]_n$, and $M_n = [\langle K_+ K_- \rangle / \langle K \rangle]_n$. If we assume an average charge symmetry of the clusters, we obtain, apart from $\langle K_+ \rangle = \langle K_- \rangle = \langle K \rangle / 2$,

$$F_n^+ = F_n^- \quad (9a)$$

and

$$F_n = \frac{1}{2}F_n^+ + \frac{1}{2}F_n^- + 2M_n. \quad (9b)$$

Thus all information is contained in two of the strengths entering eq. (9b). Indeed, the two relations given by eqs. (9) are well fulfilled by the experimental data within the experimental errors. Furthermore, the cluster widths obtained in the determinations of F_n^+ , F_n^- , and M_n are almost the same and are very close to the ones obtained in connection with F_n . In Appendix D we describe a method of calculating $\langle K \rangle_n$ from the knowledge of F_n and M_n . Figure 10 shows the results of this calculation.

4. SUMMARY AND CONCLUSIONS

When two-particle correlations from $\alpha\alpha$, αp , and pp interactions are compared, they show a similar behaviour. The only differences are seen at large absolute rapidities, where the distributions from $\alpha\alpha$ interactions and from the α -side of the αp interactions give correlations close to zero. This is, however, a trivial effect, since most of the particles observed in this region are spectator protons (or heavier fragments) left over from the interacting α -particles. When the correlations are analysed in the framework of a simple cluster model, the widths as well as the multiplicities of the clusters turn out to be the same for $\alpha\alpha$, αp , and pp interactions. The conclusion must be that the mechanisms responsible for the short-range correlations are the same for the different kinds of interactions. We find no trace of collective phenomena with this kind of analysis; but if it were possible to observe such phenomena, then either the energy or the masses of the interacting nuclei would have to be increased.

When the results from this work are compared with earlier correlation studies on pp interactions, a few interesting differences are seen. We have clear indications that the cluster widths and multiplicities K decrease with increasing total multiplicity n , contradicting earlier measurements where an increase of $\langle K \rangle$ was seen for n larger than the average multiplicity [3]. *A priori* the cluster models predict an increase as long as the mechanism is n independent, since an increase of n allows higher K to be produced. Except for low n the cluster widths we observe are smaller than those seen in earlier works [3,12].

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APPENDIX A

CORRELATIONS INDUCED BY MIXING MULTIPLICITIES

In this appendix we will show that the assumptions made in section 2 [a) two-particle correlations are absent, b) the density of produced particles grows linearly with n] introduce a rapidity dependence of the R-function.

From the definition of $\langle n \rangle_y$ [eq. (5)] it follows that

$$\begin{aligned} \langle n \rangle_y &= \sum_n \pi(n) [n^2 \alpha(y) + n \beta(y)] / \sum_n \pi(n) \rho_n^{(1)}(y) \\ &= [\langle n^2 \rangle \alpha(y) - \langle n \rangle \beta(y)] / \rho^{(1)}(y) \\ &= [(\langle n^2 \rangle - \langle n \rangle^2) \alpha(y) + \langle n \rangle \rho^{(1)}(y)] / \rho^{(1)}(y) \\ &= \frac{D^2 \alpha(y)}{\rho^{(1)}(y)} - \langle n \rangle, \end{aligned}$$

where $D^2 = \langle n^2 \rangle - \langle n \rangle^2$, and we obtain

$$\alpha(y) = \frac{\rho^{(1)}(y) \cdot \langle n \rangle}{D^2} \left(\frac{\langle n \rangle}{\langle n \rangle} - 1 \right). \quad (\text{A.1})$$

If, for a moment, we neglect the factor $(n-1)/n$ in eq. (3), the R-function can be rewritten in the following way:

$$\begin{aligned} R(y_1, y_2) &= \frac{\sum_n \pi(n) \rho_n^{(2)}(y_1, y_2)}{\rho^{(1)}(y_1) \cdot \rho^{(1)}(y_2)} - 1 \\ &= \frac{\sum_n \pi(n) [n \cdot \alpha(y_1) + \beta(y_1)] [n \cdot \alpha(y_2) + \beta(y_2)]}{\rho^{(1)}(y_1) \cdot \rho^{(1)}(y_2)} - 1 \\ &= \frac{\langle n^2 \rangle \alpha(y_1) \alpha(y_2) + \langle n \rangle [\alpha(y_1) \beta(y_2) + \alpha(y_2) \beta(y_1)] + \beta(y_1) \beta(y_2)}{\rho^{(1)}(y_1) \cdot \rho^{(1)}(y_2)} - 1 \\ &= \frac{D^2 \alpha(y_1) \alpha(y_2) + \rho^{(1)}(y_1) \cdot \rho^{(1)}(y_2)}{\rho^{(1)}(y_1) \cdot \rho^{(1)}(y_2)} - 1 \end{aligned}$$

and

$$R(y_1, y_2) = \frac{D^2 \alpha(y_1) \alpha(y_2)}{\rho^{(1)}(y_1) \cdot \rho^{(1)}(y_2)} .$$

Inserting $\alpha(y)$ from eq. (A.1) finally gives

$$R(y_1, y_2) = \frac{\langle n^2 \rangle}{D^2} \left(\frac{\langle n \rangle y_1}{\langle n \rangle} - 1 \right) \left(\frac{\langle n \rangle y_2}{\langle n \rangle} - 1 \right) . \quad (A.2)$$

If the factor $(n-1)/n$ is taken into account, eq. (A.2) becomes

$$R_n(y_1, y_2) = \frac{\langle n^2 \rangle}{D^2} \left(\frac{\langle n \rangle y_1}{\langle n \rangle} - 1 \right) \left(\frac{\langle n \rangle y_2}{\langle n \rangle} - 1 \right) - \frac{1}{\langle n \rangle} + o\left(\frac{1}{\langle n \rangle} - \frac{1}{\langle n \rangle}\right) ,$$

where the last term can be neglected.

APPENDIX B

A SIMPLE CLUSTER MODEL

Here we derive a formula for the normalized correlation function R , using a few assumptions that are common to most models.

Let us assume that a cluster at rapidity Y decays into K charged particles at rapidities y_i ($i = 1, 2, \dots, K$), so that y_i are uncorrelated and Gaussian distributed with mean Y and width δ . If we now select events with n particles, these n particles will come from m independent clusters with multiplicities K_j , at rapidities Y_j ($j = 1, 2, \dots, m$). We can now write the one-dimensional particle density

$$\rho_n^{(1)}(y) = n \int_{-\infty}^{+\infty} C(Y) \frac{1}{\sqrt{2\pi}\delta} \exp \left[-(y - Y)^2 / 2\delta^2 \right] dY ,$$

where $C(Y)$ is the probability of having a cluster at rapidity Y . The transformation $Y = y + \epsilon$ gives

$$\rho_n^{(1)}(y) = n \int_{-\infty}^{+\infty} C(y + \epsilon) \frac{1}{\sqrt{2\pi}\delta} \exp (-\epsilon^2 / 2\delta^2) d\epsilon . \quad (B.1)$$

If it is assumed that the width of the cluster decay δ is much smaller than the width of the rapidity distribution $C(Y)$, we can use a δ -function approximation, and

$$\rho_n^{(1)}(y) = n \cdot C(y)$$

or

$$C(y) = \frac{1}{n} \rho_n^{(1)}(y) . \quad (B.2)$$

The two-dimensional density will be the sum of two terms, one (A_n) taking care of particles coming from different clusters, and the other (B_n) of particles coming from the same cluster. The two terms can be written

$$A_n(y_1, y_2) = \sum_m P(m) \sum_{i=1}^m \frac{K_i(n - K_i)}{n^2} \rho_n^{(1)}(y_1) \cdot \rho_n^{(1)}(y_2) \quad (B.3a)$$

$$B_n(y_1, y_2) = \sum_m P(m) \sum_{i=1}^m K_i(K_i - 1) \int_{-\infty}^{+\infty} C(Y) \frac{1}{\sqrt{2\pi}\delta} \exp \left[- (y_1 - Y)^2 / 2\delta^2 \right] \frac{1}{\sqrt{2\pi}\delta} \exp \left[- (y_2 - Y)^2 / 2\delta^2 \right] dY, \quad (B.3b)$$

where $P(m)$ is the probability that m clusters are produced. The integral of eq. (B.3b) can be rewritten in the following way:

$$\begin{aligned} & \int_{-\infty}^{+\infty} C(Y) \frac{1}{\sqrt{2\pi}\delta} \exp \left[- (y_1 - Y)^2 / 2\delta^2 \right] \frac{1}{\sqrt{2\pi}\delta} \exp \left[- (y_2 - Y)^2 / 2\delta^2 \right] dY \\ &= \int_{-\infty}^{+\infty} C(Y) \frac{1}{2\pi\delta^2} \exp \left\{ - (y_1^2 + y_2^2 - 2y_1y_2) / 4\delta^2 - \left[(y_1 + y_2)^2 - 4(y_1 + y_2)Y + 4Y^2 / 4\delta^2 \right] \right\} dY \\ &= \frac{1}{\sqrt{2\pi}\delta} \exp \left[- (\Delta y)^2 / 4\delta^2 \right] \int_{-\infty}^{+\infty} C(Y) \frac{1}{\sqrt{\pi}\delta} \exp \left[- (y_a - Y)^2 / \delta^2 \right] dY, \end{aligned}$$

where $\Delta y = y_1 - y_2$ and $y_a = (y_1 + y_2)/2$. Using a δ -function approximation and eq. (B.2), one finds

$$B_n(y_1, y_2) = \sum_m P(m) \sum_{i=1}^m K_i(K_i - 1) \frac{\rho_n^{(1)}(y_a)}{n} \frac{1}{2\sqrt{\pi}\delta} \exp \left[- (\Delta y)^2 / 4\delta^2 \right]. \quad (B.3b')$$

After summing over m and i we finally get

$$\rho_n^{(2)}(y_1, y_2) = \left[1 - \frac{1}{n} (1 + F_n) \right] \rho_n^{(1)}(y_1) \cdot \rho_n^{(1)}(y_2) + \frac{F_n}{2\sqrt{\pi}\delta} \exp \left[- (\Delta y)^2 / 4\delta^2 \right] \rho_n^{(1)}(y_a), \quad (B.4)$$

where $F_n = [\langle K(K - 1) \rangle / \langle K \rangle]_n$, and

$$R_n(y_1, y_2) = \frac{F_n \rho_n^{(1)}(y_a)}{\rho_n^{(1)}(y_1) \cdot \rho_n^{(1)}(y_2)} \frac{1}{2\sqrt{\pi}\delta} \exp \left[- (\Delta y)^2 / 4\delta^2 \right] - \frac{1 + F_n}{n} . \quad (B.5)$$

It should be noted that if several multiplicities around some average n_a are used together in order to gain statistics, eq. (B.5) will become

$$R_{n_a}(y_1, y_2) = \frac{F_{n_a} \rho_{n_a}^{(1)}(y_a)}{\rho_{n_a}^{(1)}(y_1) \cdot \rho_{n_a}^{(1)}(y_2)} \frac{1}{2\sqrt{\pi}\delta} \exp \left[- (\Delta y)^2 / 4\delta^2 \right] - \frac{1 + F_{n_a}}{n_a} + \frac{\sigma^2(n)}{n_a^2} ,$$

where $\sigma(n)$ is the dispersion in the n -interval used, i.e. the background level changes neither the height nor the width of the correlation peak.

APPENDIX C

STATISTICAL ERRORS ON R

Experimentally $R(y_1, y_2)$ is calculated as

$$R(y_1, y_2) = \begin{aligned} & \frac{\langle n_1 n_2 \rangle}{\langle n_1 \rangle \langle n_2 \rangle} - 1 \quad \text{for } y_1 \neq y_2, \\ & \frac{\langle n_1(n_1 - 1) \rangle}{\langle n_1 \rangle^2} - 1 \quad \text{for } y_1 = y_2, \end{aligned} \quad (C.1)$$

where n_1 and n_2 are the multiplicities in a small interval Δy around y_1 and y_2 , respectively. Statistical calculations give the estimate of the variance of R as

$$\begin{aligned} \widetilde{V[R]} = & \frac{1}{N \langle n_1 \rangle^4 \langle n_2 \rangle^4} \left[\langle n_1^2 n_2^2 \rangle \langle n_1 \rangle^2 \langle n_2 \rangle^2 - 2 \langle n_1^2 n_2 \rangle \langle n_1 n_2 \rangle \langle n_1 \rangle \langle n_2 \rangle^2 - 2 \langle n_1 n_2^2 \rangle \langle n_1 n_2 \rangle \langle n_1 \rangle^2 \langle n_2 \rangle \right. \\ & \left. + \langle n_1^2 \rangle \langle n_1 n_2 \rangle^2 \langle n_2 \rangle^2 + \langle n_2^2 \rangle \langle n_1 n_2 \rangle^2 \langle n_1 \rangle^2 + 2 \langle n_1 n_2 \rangle^3 \langle n_1 \rangle \langle n_2 \rangle - \langle n_1 n_2 \rangle^2 \langle n_1 \rangle^2 \langle n_2 \rangle^2 \right] \\ & + O\left(\frac{1}{N^2}\right) \quad \text{for } y_1 \neq y_2, \end{aligned} \quad (C.2a)$$

and

$$\begin{aligned} \widetilde{V[R]} = & \frac{1}{N \langle n_1 \rangle^6} \left[\langle n_1^4 \rangle \langle n_1 \rangle^2 - 4 \langle n_1^3 \rangle \langle n_1^2 \rangle \langle n_1 \rangle + 4 \langle n_1^2 \rangle^3 - \langle n_1^2 \rangle^2 \langle n_1 \rangle^2 + 2 \langle n_1^3 \rangle \langle n_1 \rangle^2 \right. \\ & \left. - 4 \langle n_1^2 \rangle^2 \langle n_1 \rangle + 2 \langle n_1^2 \rangle \langle n_1 \rangle^3 + \langle n_1^2 \rangle \langle n_1 \rangle^2 - \langle n_1 \rangle^4 \right] + O\left(\frac{1}{N^2}\right) \quad \text{for } y_1 = y_2. \end{aligned} \quad (C.2b)$$

N is the total number of events.

APPENDIX D

CLUSTER MULTIPLICITIES

In the following we estimate the average number of charged particles produced in a cluster decay, using the known strengths F_n and M_n defined in section 3.

When calculating the cluster multiplicities, our approach is similar to that of Drijard et al. [12], i.e. we use a model that considers only neutral and singly charged clusters, and furthermore we neglect clusters decaying into more than four charged particles. The probability of having a cluster with no charged particles has been estimated to be $\approx 10\%$ of the total probability of having a neutral cluster [12,13].

The total strength F_n can be decomposed into two contributions: i) the neutral strength $F_n^{(0)}$, defined as F_n , but for neutral clusters only; ii) the singly charged strength $F_n^{(ch)}$, similarly defined. Drijard et al. made the assumption $F_n^{(0)} \approx F_n^{(ch)}$. There is, however, a basic difference between $F_n^{(0)}$ and $F_n^{(ch)}$, since $F_n^{(0)}$ has to be larger than unity, whilst $F_n^{(ch)}$ can be smaller. If we assume a Poisson distribution, we find $F_n^{(0)} = F_n / \tanh F_n$, and for a geometric distribution we find $F_n^{(0)} = F_n + 1/(F_n + 1)$. We have used these two relations as limits of $F_n^{(0)}$, i.e. a variation of about 10%.

Using these assumptions and solving the subsequent system of equations gives

$$\langle K \rangle_n = \frac{27 - 9F_n^{(0)}}{(21M_n - 9 - 4F_n)F_n^{(0)} - (33M_n - 27 - 2F_n)} . \quad (D.1)$$

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Table 1

Data sets used in this work

Interaction	$\sqrt{s}$ (GeV)	$\sqrt{s}_{nn}$ (GeV)	No. events
$\alpha\alpha$	124.6	31.2	104214
αp	88.1	44.0	39360
pp	62.2		62565
	44.0		43981
	30.4		38359

Figure captions

- Fig. 1 : The normalized correlation function for pp interactions at three different energies. a) $y_1 = 0$, b) $y_1 = -2$.
- Fig. 2 : The normalized correlation function for pp interactions at three different energies, with the normalized rapidity ψ as argument. a) $\psi_1 = 0$, b) $\psi_1 = -0.5$.
- Fig. 3 : Comparison of the normalized correlation function from $\alpha\alpha$, αp , and pp interactions. The pp data from fig. 2 are shown as curves. a) $\psi_1 = 0$, b) $\psi_1 = -0.5$.
- Fig. 4 : The normalized associated multiplicity excess for pp interactions at $\sqrt{s} = 62.2$ GeV and for $\alpha\alpha$ interactions at $\sqrt{s} = 124.6$ GeV.
- Fig. 5 : Comparison between the normalized correlation function and calculations from eq. (6). a) pp $\sqrt{s} = 62.2$ GeV, $y_1 = 0$; b) pp $\sqrt{s} = 62.2$ GeV, $y_1 = -2$; c) $\alpha\alpha$ $\sqrt{s} = 124.6$ GeV, $y_1 = 0$; d) $\alpha\alpha$ $\sqrt{s} = 124.6$ GeV, $y_1 = -2$.
- Fig. 6 : The normalized correlation function at fixed multiplicity ($n_- = 5$) for $\alpha\alpha$, αp , and pp interactions. a) $y_1 = 0$, b) $y_1 = -1.5$.
- Fig. 7 : Two-dimensional representation of the normalized correlation function for $\alpha\alpha$ interactions at $\sqrt{s} = 124.6$ GeV and $n_- = 5$. The central square indicates the region used in the analysis.
- Fig. 8 : The cluster width δ_n as a function of the corrected negative multiplicity (i.e. the points are plotted at the estimated average true multiplicity, for a given observed multiplicity): a) pp interactions, b) $\alpha\alpha$ interactions. The shaded area corresponds to the pp data from (a).
- Fig. 9 : The cluster strength F_n as a function of the corrected negative multiplicity. a) αp $\sqrt{s}_{nn} = 44.0$ GeV and pp $\sqrt{s} = 44.0$ GeV, b) $\alpha\alpha$ $\sqrt{s}_{nn} = 31.2$ GeV and pp $\sqrt{s} = 30.4$ GeV.

Fig. 10 : The calculated cluster multiplicity $\langle K \rangle_n$ as a function of the corrected negative multiplicity. a) $\alpha_p \sqrt{s}_{nn} = 44.0$ GeV and $pp \sqrt{s} = 44.0$ GeV, b) $\alpha\alpha \sqrt{s}_{nn} = 31.2$ GeV and $pp \sqrt{s} = 30.4$ GeV.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10