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SONIC SEARCH FOR MONOPOLES, GRAVITATIONAL WAVES, AND NEWTORITES

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A B S T R A C T

Hopes for the detection of gravitational waves and of very massive magnetic monopoles rely on cosmological sources - the big bang itself in the monopole's case. Some gravitational wave antennas are "acoustic" resonant detectors, that have reached a sensitivity not far from the "quantum limit". We investigate in detail the response of these sensitive detectors to the passage of "relic" monopoles. We compute signal to noise ratios for a variety of target materials, and we find them to be favourable for the very cold, high quality resonators that are presently contemplated. A monopole traversing a metal produces a "thermo-acoustic" pulse, whose amplitude is linear in the monopole's velocity, β . If the metal is superconducting, there is a novel additional "magneto-acoustic" source, whose amplitude is β -independent. Monopole detectors that rely on ionization have a sensitivity threshold in β , and may conceivably be blind to relic monopoles. The response of superconducting loop detectors is β -independent, but their collection areas are limited by the requirement of a sophisticated magnetic shielding. Neither of the above limitations would be shared by an acoustic monopole detector. We sketch a "sonic antenna" that would respond directionally to conventional cosmic rays, gravitational radiation, monopoles, and even to more exotic signals, like newtorites (elementary or composite "meteorites" that interact with ordinary matter only gravitationally).

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1. - FOREWORD

Half a century after Dirac's pioneering discussion of magnetic charges¹⁾, two events have rekindled strong interest in monopoles. Most recent is the monopole candidate found by Cabrera²⁾ in an experiment measuring magnetic flux jumps in a superconducting ring. In addition, there is the discovery by Polyakov and 't Hooft³⁾ that unified gauge theories [in which a simple group breaks to a subgroup containing an unbroken $U(1)$ symmetry] predict the existence of very massive monopoles. In the vacuum (minimal energy solution) of these theories a scalar ("Higgs") field has a non-zero vacuum expectation value that spontaneously breaks the gauge symmetry. The monopoles of unified gauge theories are topologically stable classical solutions for which the direction of the scalar field (in group space) is inextricably wound into the different directions at spatial infinity. In a "grand" unified scheme, let M be the mass scale at which weak, strong and electromagnetic interactions are unified into a single simple gauge group, and let α_M be the electromagnetic fine structure constant renormalized at mass M . The predicted mass of the monopoles is $m \sim M\alpha_M^{-1}$. In $SU(5)$, the simplest grand unified model⁴⁾, the estimate is $m \sim 10^{16} \text{ GeV}/c^2 \sim 20 \text{ nanograms}$ ⁵⁾, a considerable mass for a single particle, even if it is not fully "elementary".

When grand unification dynamics is incorporated into the standard big bang model of the Universe, monopoles immediately create significant difficulties. Large numbers of primordial monopoles are predicted to have been created at the very early stages of the bang, and to have survived to the present⁶⁾. Even their passive contribution to the average energy density of the Universe is large enough to rule out the scenario, exceeding observation by many orders of magnitude. "Inflationary" and "new inflationary" modifications of the standard cosmology appear to solve the monopole and other puzzles⁷⁾. But the pendulum swings all the way back: the predicted abundance of relic monopoles is nil. Surely the pendulum will keep on moving⁸⁾, unless relic monopoles are either observed or the upper limits on their flux are enormously improved. In view of the mobile character of our prejudices on the very early Universe, it may be a reasonable attitude to discuss the natural monopole abundance at a more down to earth, or at least down to galaxy, level.

Several arguments conservatively based on the properties of the Universe as we see it today have been used to limit the cosmic monopole flux on earth. The most stringent limit on monopoles with masses within a few orders of magnitude around the "official" 20 nanograms is the Parker bound, or refinements thereof⁹⁾. This bound is based on the observed existence of magnetic fields in our own

galaxy, with field strengths of order $B \sim 3 \cdot 10^{-6}$ gauss, coherently extending over regions of a few kpc size. These magnetic fields would be consumed as they accelerate monopoles and lose energy to them. The Parker bound results from the requirement that the time scale of consumption of the magnetic field not be greater than the time scale, $\tau \sim 10^8$ years, of regeneration¹⁰⁾ (τ is the revolution period of the galaxy, which is assumed to generate its B field as a "dynamo" effect). Numerically, the "Parker" constraint on the flux of monopole is

$$F \lesssim 10^{-4} \frac{\text{monopoles}}{\text{m}^2 \cdot \text{yr} \cdot \text{sr}} \cdot \left[\frac{B}{3 \cdot 10^{-6} \text{ Gauss}} \right] \cdot \left[\frac{10^8 \text{ yr}}{\tau} \right] \cdot \begin{cases} 1 & (m \lesssim 10^{17} \text{ GeV}) \\ \frac{m}{10^{17} \text{ GeV}} & (m \gtrsim 10^{17} \text{ GeV}) \end{cases} \quad (1.1)$$

This bound would point to the necessity of extremely large detector arrays.

Cabrera and collaborators have now announced results for 382 days of observation with the original one-loop detector of $\sim 10 \text{ cm}^2$ effective surface and 131 days with a $\sim 70 \text{ cm}^2$ three-loop detector¹¹⁾. Should the original event be real, the present flux "measurement" in these detectors is

$$F \sim 20 \text{ monopoles} / \text{m}^2 \cdot \text{yr} \cdot \text{sr} \quad (1.2)$$

The conflict between Eq. (1.1) and Eq. (1.2) is striking. The negative reaction would be to give up the belief in either the monopole candidate or in our understanding of galactic magnetic fields. A more positive suggestion is that monopoles, like meteorites, may be gravitationally bound to the sun, resulting in a local density enhancement relative to the average galactic population¹²⁾.

The typical velocity of relic monopoles has not played an explicit role in the foregoing discussion. The Parker limit only implicitly depends on velocity, in that it deals with galactically distributed monopoles whose speed, as we shall see, is expected to be in the neighbourhood of $v/c \sim 10^{-3}$. The quantum jumps of flux in superconducting ring detectors are independent of monopole speed. But the question of velocity is crucial to most other methods of monopole detection implemented or discussed so far. We proceed to review order of magnitude expectations for relic monopole velocities.

A heavy monopole falling onto earth from afar would have its velocity increased by $\Delta v \sim 3 \cdot 10^{-5}c$, as a result of acceleration by the earth's gravitational field. The earth's orbital velocity is $v \sim 10^{-4}c$, of the same magnitude as the speed of a "meteorite" monopole bound to the sun (meteorites fall with velocities in the narrow range of 10 to 70 km/sec). It is difficult to imagine heavy monopoles reaching the earth's surface with velocities much smaller than the above $10^{-5}c$ to $10^{-4}c$, since their interactions with matter are not expected to slow them down very significantly^{*}). A monopole traversing a galactic coherent magnetic field region with the properties previously described, would have its speed changed by $\Delta v \sim 3 \cdot 10^{-3}c$ $(10^{16} \text{ GeV/m})^{1/2}$. This is of the same order of magnitude as the galactic escape velocity or the orbital velocity of the solar system. Thus monopoles distributed more or less uniformly over the galaxy are expected to cross the earth with $v \lesssim 3 \cdot 10^{-3}c$. Heavy monopoles uniformly distributed over the Universe would be red shifted to very small velocities. A terrestrial observer would see them with $v \sim 10^{-3}c$, our velocity relative to the background 3°K radiation. To summarize, the heavy monopoles predicted by grand unified theories are unlikely to hit a detector with velocities outside the generous range $\beta = v/c = 10^{-5}$ to 10^{-2} .

In table 1.1¹³⁾ we list monopole flux limits (in units of monopoles per square metre year and steradian) from ionization-sensitive detectors. The table lists the threshold level of ionization I_{thr} set by the different experiments, in units of minimum ionization, I_{min} . Also given are the effective area ($m^2 \times sr$) and the bracket in monopole velocity (β_{min}, β_{max}) set by the electronic readout. The last column, labelled β_{thr} , is an estimate of the monopole velocity necessary to produce the threshold ionization, I_{thr} of each experiment. The values of β_{thr} are taken from calculations by Ritson¹⁴⁾ of the ionizing power of monopoles in scintillators and proportional chambers. For some experiments $\beta_{thr} > \beta_{min}$ implying that the lowest velocity monopoles in the velocity window defined by the electronics may not produce sufficient ionization to be observable. This is not to be taken too literally: calculations are very delicate for slow monopole velocities, $\beta < 10^{-3}$, below which the ionizing power decreases very sharply, reflecting the minimum energy transfer necessary to produce ionization^{**)}.

^{*}) A slow monopole is estimated to lose $\sim 10^3(v/c) \text{ GeV}$ per centimeter in traversing a metal, see Eq. (1.4) below. The corresponding range in kilometers is $10^8(v_0/c)(m/10^{16} \text{ GeV})$. For a dielectric target, the range is even larger.

^{**)} The maximum energy transfer to a free electron at rest in a collision with a heavy non-relativistic particles is $2m_e\beta^2c^2$. When $\beta \leq \alpha$ this is comparable or smaller than atomic binding energies $\sim m_e c^2 \alpha^2$ and ionization is suppressed.

Table 1.1

Experiment	Detector	I_{thr}/I_{min}	$(\beta_{min}, \beta_{max})$	Flux limit	Surface $m^2 \times sr$	β_{thr}
BNL	Prop.	2	$3 \times 10^{-4}, 1.2 \times 10^{-3}$	<11	4	1.5×10^{-3}
Bologna	Scint.	20	$2 \times 10^{-3}, 0.5$	$<7 \times 10^{-2}$	38	10^{-3}
Kolar mine	Prop.	2.5	$1 \times 10^{-3}, 1$	$<10^{-2}$	--	2×10^{-3}
Berkeley	CR 39	*	*	$<5 \times 10^{-2}$	90	~ 0.02
Baksan	Scint.	0.25	$4 \times 10^{-3}, 5 \times 10^{-2}$	$<5 \times 10^{-3}$	--	2.5×10^{-4}
Soudan I	Prop.	0.5	$3 \times 10^{-4}, 3 \times 10^{-2}$	$<7 \times 10^{-2}$	71	3×10^{-4}
Tokyo I	Scint.	1.2	0.01, 0.1	<5	1.1	3×10^{-4}
Tokyo II	Scint.	1/40	$2 \times 10^{-4}, 5 \times 10^{-3}$	<9	1.4	10^{-4}
Tokyo III	Scint.	1/16	$3 \times 10^{-4}, ---$	<1.3	2.2	10^{-4}
Utah Stanford	Scint.	0.05	$9 \times 10^{-5}, 10^{-2}$	<4	2.7	1.5×10^{-4}
Michigan	Scint.	~ 0.01	$3 \times 10^{-4}, 10^{-2}$	<37	3.16	10^{-4}

Monopole flux limits from ionization-sensitive detectors (monopoles/ m^2 year sr), as gathered by E. Loh and D.M. Ritson¹¹⁾. "Prop." refers to proportional chambers, "Scint." to scintillation counters. The different entries are explained in text. The Berkeley experiment is a search for "fast" monopoles, its value of β_{thr} is based on calculations by Ahlen¹⁵⁾ of the minimum visible ionization in the CR 39 plastic target.

By the same token, the actual threshold of ionization experiments for $\beta < 10^{-3}$ is hard to pin down with confidence. The limits of Table 1.1, as well as Cabrera's "flux" Eq. (1.2), and the Parker bound Eq. (1.1), are displayed in Fig. 1.1. The apparent conflict of most of the limits of Table 1.1 with the "flux measurement" Eq. (1.2) can be resolved by assuming that the monopole velocity distribution is very sharply cut off for $\beta \geq 10^{-4} - 10^{-3}$ and/or by remembering that the low velocity ($\beta < 10^{-3}$) monopole ionizing power is hard to estimate with precision. (Notice in particular the β_{thr} values, shown in the figure as triangle marks whenever they may be relevant.) The possibility of an enhanced flux of low velocity monopoles is the rationale behind the sun-orbiting monopole scenario¹²⁾.

Fig. 1.1: Monopole flux limits as a function of monopole velocity. We have not included in this graph limits related to the controversial subject of monopole-induced baryon decay¹⁶⁾.

To summarize, Fig. 1.1 indicates that monopole searches could be improved not only in the obvious direction of larger detectors, but also in the direction of improved sensitivity to small velocities. The necessary range of velocities to be covered with greater confidence in the detector's sensitivity may be rather narrow, $\beta \sim 10^{-4}$ to 10^{-3} .

In the next section we shall outline a detector technique that, like Cabrera's, should give a non-zero signal down to $\beta = 0$. Though the detector is not of the football field size apparently required by the Parker bound, the hope is that it can cover much larger areas than superconducting rings, without running into insuperable technical problems. The minimum aim is to lower Cabrera's β independent bound without reference to grand and cosmic prejudices on monopole velocities, masses and abundances.

The signal in a superconducting coil detector is β -independent and that of ionization detectors has a threshold^{*)} in $\beta \sim O(\alpha)$. A "sonic" detection technique has been discussed^{18),19)}, whose sensitivity to slow monopoles is somewhat in between that of the above detectors: the signal amplitude is linear in β . The method we shall propose is related to the sonic approach, which we proceed to outline.

Consider a heavy monopole sailing through a bulk metal. Let it be slow enough for its ionizing power to be suppressed or nil ($\beta \ll 10^{-2}$). Electrons in the metal feel an electric field $\vec{v} \times \vec{B}$, and eddy currents are generated around the monopole's straight trajectory. The currents dissipate due to ohmic losses. Thus the monopole loses a (tiny) fraction of its energy E per unit length of travel x . This loss has been estimated by Ford, and Allen and Kinoshita²⁰⁾, to be:

$$\frac{dE}{dx} \sim 0.3 \, n_D^2 \, n_c^{2/3} \left[\frac{\beta}{10^{-3}} \right] \frac{\text{GeV}}{\text{cm}} \quad (1.3)$$

where n_D is the monopole's charge in Dirac magnetic units and n_c is the number of conducting electrons per atom. Even for a slow monopole ($\beta = 10^{-4}$ to 10^{-3}) and for the minimal case ($n_D = n_c = 1$), this energy loss is comparable to or larger than that of a minimum ionizing particle ($dE/dx \sim 2.6\rho \text{ MeV/cm}$, with ρ the target's density relative to water). This energy dissipates in the material

^{*)} Drell et al.¹⁷⁾ have analyzed the energy loss of monopoles in the form of excitation, rather than ionization, in H and He targets. This loss is found to be greater than that due to ionization and to lower the low-velocity cut-off of ionization-related detectors (the excitation can be transferred as observable ionization to a quenching gas additive: the Penning effect).

as ohmic losses: it results in a local heating along the monopole's track. As we analyze in detail in Chapter 3, this heating changes the local pressure in the medium, thereby acting as a source of an (ultra-)sonic wave. The obstacle to the detection of this signal is thermal noise. Detailed analysis by Akerlov¹⁸⁾ has shown that the expected signal would be lost in thermal noise even for milli-Kelvin temperatures. In the rest of this paper, we will attempt to reverse this pessimistic conclusion by considering resonant gravitational antenna-like detectors ("improving" the noise properties) and superconducting targets (for which a β -independent source of local pressure should also exist).

2. - INTRODUCTION

Consider a monopole which, as in Fig. 2.1a, is about to fall on a bulk piece of superconducting material. A superconductor does not allow magnetic fields below a certain critical value H_c to permeate it, except for surface effects confined to a layer of depth λ , the London penetration depth ($\lambda \sim 10^{-5}$ cm is a typical value). As the monopole approaches, eddy supercurrents are generated; they are shown as dashed lines in Fig. 2.1a. The magnetic field lines, that for an isolated monopole would be "Coulombic" and spherically symmetric, are modified by the supercurrents so as to avoid the material. As the unstoppable monopole hits, and crosses the superconductor (Figs. 2.1b,c), the magnetic field has no choice but to penetrate it. For a monopole with Dirac's minimal charge ($g = e/2\alpha$) the value of the magnetic flux is $2\phi_0$, with ϕ_0 the quantum flux unit ($\phi_0 = hc/2e = 2.067 \times 10^{-7}$ Gauss cm²). In the superconductor this flux is constrained to a tubular vortex maintained by circling supercurrents (strictly speaking, as we shall see, this is only true for type-I superconductor; in Type II the single vortex is unstable and two vortices will form). These vortices are alternatively referred to in the literature as flux tubes, fluxons or fluxoids. As the monopole moves away from the superconductor the magnetic field vanishes at points like the one shown by an asterisk in Fig. 2.1d, and the field lines may "cross". Eventually, Fig. 2.1e, the monopole leaves with its kinetic energy negligibly decreased and its magnetic field back to its spherical shape. A scar is left behind in the superconductor in the form of one or two "vortices". We proceed to discuss these vortices and how they may be detected. A hypothetical reader who is an expert on superconductivity will be bored (or worse) by the next two paragraphs²¹⁾.

Fig. 2.1: Avatars of the magnetic field of a monopole that pierces a bulk superconductor.

There are two types of superconductors, whose differences are well described (semiquantitatively) by the Ginzburg-Landau model^{22),23)}. With amazing intuition, these authors introduced a complex order parameter ψ , subsequently interpreted as the Cooper pair wave function, and entirely analogous to a singlet doubly charged Higgs field in non-relativistic spontaneously broken electrodynamics. The behaviour of flux vortices and of interfaces between normal and superconducting regions depends on the ratio $\kappa \equiv \lambda/\xi$ between two parameters with dimensions of length. The parameter λ is the characteristic magnetic field penetration depth (the inverse of the "photon mass"). The parameter ξ is a measure of the "rigidity" of the wave function ψ ; the distance over which it may vary significantly. The sign of the surface energy between a superconducting region and a region rendered normal by the presence of a magnetic field is a function of κ , positive for $\kappa < 1/\sqrt{2}$ (Type I superconductors) and negative for $\kappa > 1/\sqrt{2}$ (Type II). In Type I superconductors the area of the normal-to-superconducting interphase interface tends to be minimized. When a Type I superconductor is placed in an increasing external magnetic field H , the field is completely expelled from the material as long as H is smaller than the critical field

$H_c(T)$. At $H = H_c(T)$ the field abruptly penetrates, and for a conveniently chosen geometry, the whole specimen is rendered normal. The behaviour of a Type II superconductor in an external magnetic field is richer. At a critical field H_{c1} the magnetical field starts to penetrate the material in the form of "Abrikosov²³⁾ vortices", that contain a single quantum unit of flux and are identical in structure to the monopole-induced vortices we previously described. As H increases above H_{c1} , it is energetically favourable for vortices to stream in from the outside: this increases the surface between the superconducting background and the vortices, where the magnetic field suppresses superconductivity. At a higher field $H_{c2} > H_{c1}$ the vortices eventually coalesce and the whole material is rendered normal. The $H < H_{c1}(T)$ phase of a Type II superconductor is called the "Meissner" phase, as in Type I. The intermediate phase, $H_{c1}(T) < H < H_{c2}(T)$ is often called the "mixed state". Much is known about Abrikosov vortices in Type II superconductors, where they occur naturally. In a Type I superconductor one of the few ways to produce a vortex is to thread it in with a monopole^{*)}. This is the reason why the theory of Type I vortices has not been developed, and we will be forced to do it to the extent of our needs.

In a subcritical external magnetic field a vortex like that of Fig. 2.1e is in unstable equilibrium: it would be energetically favourable for it to move to the surface and be radiated away. But in realistic materials vortices are "pinned" by bulk and surface imperfections. Cabrera and collaborators¹¹⁾ and independently Wipf²⁶⁾ contemplate an experiment in which a moving superconducting loop would detect fresh monopole-induced vortices in a cylindrical plate of Nb, a Type II superconductor, against a background of pre-existing pinned vortices.

At a constant subcritical magnetic field H , the volume of a superconductor changes as its temperature is increased towards the critical temperature, $T_c(H)$. A similar behaviour occurs for bulk superconductors under the influence of a magnetic field: a magnetostrictive volume change for $H < H_c(T)$ and a volume jump at the phase transition in the Type I case. The volume of a superconductor also changes locally, as a magnetic flux vortex is injected into the material

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At the time when ideas about vortices were in their infancy, Cabibbo and Doniach²⁴⁾ discussed vortex formation in Type I superconductors in connection with the peculiar results of an experiment by De Feo and Sacerdoti²⁵⁾. In the experiment, a foil of superconducting Pb was exposed to α particles. The experimental results could be explained by the α particles heating the material along their track and boring a "normal" hole through which the magnetic field could sneak in, ending up after heat diffusion as an ensemble of artificially-made Type I trapped vortices. Another way to produce Type I vortices suggests itself: force them to migrate from a Type II to a Type I superconductor in contact with it, with the help of an electrical current.

and a local stress is developed. When a monopole pierces a superconductor leaving behind a flux vortex, the sudden change of stress of magnetic origin gives rise to a sonic wave. It is on this velocity-independent signal, in addition to the velocity-dependent thermal effect mentioned in the previous section, that we shall capitalize.

3. - THE THERMO-ACOUSTIC PRESSURE SOURCE

Ford and Allen and Kinoshita²⁰⁾ have computed the energy loss of a monopole as it traverses a normal conductor and induces dissipative eddy currents. The energy lost by the monopole ends up via the ohmic losses as local heating along its trajectory. For a monopole traversing a superconductor one would at first sight not expect such an energy loss, since supercurrents are non-dissipative. However, superconductors act as such only for low frequency electromagnetic signals. For signals above a certain frequency, the energy in the quanta is sufficient to overcome the energy gap and to break up the Cooper pairs responsible for superconductivity. In this section we briefly review a heuristic presentation of a monopole's energy loss in a conductor, due to Allen and Kinoshita²⁰⁾, proceed to generalize the results to a bulk superconductor target, discuss a diversity of mechanisms of energy loss by monopoles, and compute the monopole-induced thermo-acoustic source of sound waves.

Consider a monopole travelling within a metal, as in Fig. 3.1, where several geometric quantities are defined. Electrons at a distance R from the monopole feel a magnetic field g/R^2 and an electric field $\vec{E} = \vec{\beta} \times \vec{B}$ whose magnitude is $\beta gb/R^3$, with b the "impact parameter". The eddy currents which are created around the monopole's trajectory, in a torus as that in Fig. 3.1, lose energy to heat at a differential rate $dE = \sigma e^2 2\pi b db dx dt$, with σ the conductivity. The energy lost by the monopole per unit length of voyage is, after integration over time:

$$\frac{dE}{dx} = 3\pi^2 g^2 \frac{\beta}{4c} \int_0^\infty \sigma \frac{db}{b^2} \quad (3.1)$$

This integral would diverge if it were not for the fact that σ is a function of wave number and frequency (implicitly, a function of b). Further thought and effort yield the approximate result²⁰⁾:

$$dE/dx \sim \beta N_c^{2/3} \ln [2k_F \Lambda] \hbar c \quad (3.2a)$$

$$N_c = n_c N_A P/A \quad (3.2b)$$

where $N_C(n_C)$ is the number of conduction electrons per unit volume (per atom), $k_F = (3\pi^2 N_C)^{1/3}$ is the Fermi wave-number, Λ is the electron's mean free path, N_A is Avogadro's number, ρ is density and A is atomic number. The explicit functional form of σ has turned the apparent linear divergence of Eq. (3.1) into a logarithmic one, regulated by an upper cut-off $b_{\max} \sim \Lambda$, above which the conductivity is approximately constant and the integral in Eq. (3.1) converges rapidly, and a lower cut-off, $b_{\min} \sim k_F^{-1}$, below which a loop like that of Fig. 3.1 would carry less than one quantum of angular momentum.

Fig. 3.1: The induction of eddy currents by a monopole moving inside a conductor.

The electric and magnetic fields at a point at distance b from the monopole's trajectory vary with a characteristic frequency $\nu(b) \sim \beta c / 2\pi b$. As previously mentioned, a superconductor acts as a normal conductor for frequencies $\nu(b)$ high enough to bridge the energy gap 2Δ , $\nu(b) > \nu_{\max} = 2\Delta/h$. Thus, even in a superconductor, there is a cylinder of radius $b_{\max} \sim \beta c / 2\pi \nu_{\max}$ around the monopole's trajectory, inside which the metal acts as a conductor, with a "normal" conductivity, no screening of the time-varying magnetic field, and the consequent ohmic loss of the induced currents. To estimate $b_{\max}$, use the approximate BCS expression $\Delta \sim 1.76 k_B T_c$, with k_B Boltzman's constant and T_c the critical temperature²¹⁾. The result

$$b_{\max} \sim \frac{\hbar \beta c}{3.52 k_B T_c} = 6 \cdot 10^{-5} \text{ cm} \left[\frac{\beta}{10^{-3}} \right] \left[\frac{10^4 \text{ K}}{T_c} \right] \quad (3.3)$$

is numerically much smaller for "slow" monopoles than the typical values of $\Lambda \sim 10^{-2}$ to 10^{-3} cm, the low temperature electron mean free path, governed by electron-phonon interactions. To estimate the ohmic energy loss in a superconductor, substitute $b_{\max}$ for Λ in the log of Eq. (3.2)

$$\begin{aligned} \ln(2k_F \Lambda) &\rightarrow \ln(2k_F b_{\max}) \\ &\sim 9.4 + \ln \left[\frac{\beta}{10^3} n_c^{1/3} (1^\circ K_c) \right] \\ &\simeq 9.4 \end{aligned} \quad (3.4)$$

to obtain

$$\left. \frac{dE}{dx} \right|_e \sim 0.5 \frac{\text{GeV}}{\text{cm}} \left(\frac{n_c}{3} \right)^{2/3} \left(\frac{\beta}{10^{-3}} \right) \quad (3.5)$$

where we have normalized to "typical" values of n_c and β , and have used the fact that ρ/Λ is approximately constant for all solid elements.

The preceding calculations of the energy loss of magnetic charges are not unlike the corresponding calculations for electric charges. But the magnetic charge of a monopole is large enough to affect significantly the electronic orbitals of the atoms through which it passes. Aware of this fact, Drell et al.¹⁷⁾ have investigated a mechanism of energy loss by monopoles that, they argue, is very relevant for gaseous targets. So far, their published results apply only to atomic hydrogen and helium targets. In these materials, the effect of the magnetic charge of the slowly passing monopole on the atomic energy levels is sufficient to promote electrons to higher atomic orbitals. The cross-section for this process is higher and its effective β threshold ($\beta \sim 10^{-4}$) lower, than for ionization. Presumably this effect is also very important in noble gases other than He. The electronic structure of the atoms in a metal is not unlike that of a noble gas, with the valence electrons gone to join the conduction band. Monopoles may have a considerable cross-section for the excitation of atoms in a metal, with a corresponding dE/dx that may be comparable to that of Eq. (3.5), for $\beta \gtrsim 10^{-4}$. Calculations of this effect in $Z > 2$ atoms are likely to be incredibly complicated. To estimate it, recall the fact that the binding energy of outer electrons, the geometric cross-section of atoms, and the values of ρ/Λ and Z/Λ are comparable within an order of magnitude for different atoms. This leads us to expect the values of $dE/\rho dx$ in $Z > 2$ materials to be

comparable or smaller than the corresponding value in helium. They will be comparable if most electrons play a similar role, and smaller if only the outermost ones contribute significantly to the Drell et al. effect.

In Fig. 3.2 we have compiled values of dE/pdx as a function of β , calculated by a variety of authors for a variety of materials^{15),17),20)}. They are compared with the same quantity as computed by us for aluminium below T_C , using Eq. (3.5) with $n_C = 3$, $T_C = 1.176$ °K, $\rho = 2.7$ gr/cm³. The curve for supercold Al is somewhat above the Drell et al. curve for helium, that we have argued to be an upper bound for their mechanism in a metal target. Thus, we shall conservatively use Eqs. (3.5) as an estimate of the monopole's energy loss in a superconductor. This neglect of a possibly large effect à la Drell et al.¹⁷⁾ may be particularly conservative for high density metals. The reason is that, as we shall see, the figure of merit in the thermo-acoustic excitation of the vibrational modes of a detector is dE/dx , and not dE/pdx . The neglected effect may therefore be a multiplicative factor of order $\rho/\rho(\text{Al})$ in the assumed dE/dx for a heavy metal of density ρ . Also given in Fig. 3.2 are values of dE/pdx for a dielectric target: Si. The curve labelled "Ritson" is a calculation of that part of the energy loss that results in ionization¹⁴⁾.

Fig. 3.2: Thermal energy loss of monopoles in different materials, as a function of monopole velocity.

So far we have discussed the mechanisms for which the energy lost by a monopole in a superconductor ends up as a local heating along its trajectory. This heat deposition produces a sudden pressure change $\delta p(\vec{x}, t)$, that acts as the source of a sonic wave. In Chapter 5 we shall compute the projection of this sonic wave onto the vibrational eigenmodes of a resonant detector. The typical wavelengths of the eigenmodes of interest are comparable to the dimensions of the detector, many orders of magnitude bigger than the region suddenly heated by the monopole. We shall see that this implies that we do not need to consider the precise $\vec{x}$ dependence of the pressure source $\delta p(\vec{x}, t)$ but only its integral over the two dimensions, $\vec{x}_\perp$, transverse to the monopole trajectory. We proceed to compute this integral of the local pressure.

Let δE be the amount of energy deposited by the monopole in the form of heat, in an elementary volume V around the point $\vec{x}$. The deposited heat raises the temperature in the elementary volume by an amount

$$\delta T = \frac{1}{\rho C_V} \frac{\delta E}{V} \quad (3.6)$$

where, by definition, C_V is the specific heat at constant volume (expressed here in units of energy per degree per unit mass). The temperature rise induces a local overpressure

$$\delta p(\vec{x}) = \left. \frac{\partial p}{\partial T} \right|_V \delta T \quad (3.7a)$$

$$= - \left. \frac{\partial V}{\partial T} \right|_p \left(\left. \frac{\partial V}{\partial p} \right|_T \right)^{-1} \delta T \quad (3.7b)$$

where in the second of the equalities we have used the equation of state $V = V(p, T)$. To express the derivatives in the last of the above equalities in terms of conventional thermodynamic quantities, recall the definitions of the coefficient of thermal expansion, β_p ; and of the bulk modulus at constant temperature, K_T (the inverse of the compressibility χ_T):

$$\beta_p \equiv \left. \frac{\partial \ln V}{\partial T} \right|_p \quad (3.8a)$$

$$\chi_T \equiv \frac{1}{K_T} \equiv - \left. \frac{\partial \ln V}{\partial p} \right|_T \quad (3.8b)$$

Combine Eqs. (3.6), (3.7) and (3.8) to obtain

$$\delta p(\vec{x}) = \gamma \frac{\delta E}{V} \quad (3.9)$$

$$\gamma \equiv \frac{\beta_p K_T}{\rho C_V} \quad (3.10)$$

where γ is Grüneisen's dimensionless parameter. The relevant integral of the local overpressure over the dimensions transverse to the monopole trajectory is

$$\Sigma_{th} \equiv \int \delta p(\vec{x}) d^2 \vec{x}_\perp = \gamma \int \frac{dE}{V} d^2 \vec{x}_\perp = \gamma \frac{dE}{dx} \quad (3.11)$$

where dE/dx is the thermal energy loss per unit length that we previously discussed. The figure of merit in the conversion of thermal energy deposition into sound is therefore Grüneisen's²⁷⁾ parameter γ . This dimensionless quantity has the practical advantage of being a very slowly varying function of temperature even at low absolute temperatures, in contrast with the specific heat and thermal expansion coefficients that enter into its definition.

So far we have not considered the time-dependence of the pressure source Eq. (3.9), nor have we mentioned the related problem of thermal diffusion of the heat deposited by the monopole. We proceed to argue that for the problem at hand: the excitation of the eigenmodes of a resonator; it suffices to treat the time dependence of Eq. (3.11) as a step function $\theta(t)$ at the time of passage of the monopole. The energy that a monopole transfers to the electrons of broken Cooper pairs can be interpreted as heat only after the electrons have travelled for one or a few interactions lengths, Λ , and thermally excited the lattice, in a cylinder of radius $O(\Lambda) \sim 10^{-2}$ to 10^{-3} cm. This raises the temperature within the cylinder by an amount δT , to which the local pressure change δp is proportional:

$$\delta T = \frac{1}{C_p \rho \pi \Lambda^2} \frac{dE}{dx} \quad (3.12)$$

$$\delta p = K_s \beta_p \delta T \quad (3.13)$$

Here, C_p is the specific heat of constant pressure and K_S is the adiabatic bulk modulus, whose definition is analogous to Eq. (3.8b). After a time t has elapsed, long enough for heat to have diffused out to distances $x_{\perp} \gg \Lambda$, the temperature distribution acquires the Gaussian form

$$\delta T = \frac{1}{4\pi\alpha t g C_p} \frac{dE}{dx} \exp\left\{-\frac{x_{\perp}^2}{4\alpha t}\right\} \quad (3.14)$$

where α is the coefficient of thermal diffusivity, assumed to be temperature-independent for the sake of simplicity in presentation (the actual form of the temperature distribution is given by the solution of the heat diffusion equation $\partial T/\partial t = \alpha(T)\nabla^2 T$, with the initial monopole-induced temperature rise as the $t = 0$ boundary condition). In the excitation of the eigenmodes of a resonator, the relevant Fourier components of the pressure pulse Eq. (3.11) have periods of order $\tau = \ell/c_s$, with ℓ a linear dimension of the resonator and c_s the speed of sound. For these long periods the Fourier transform of Eq. (3.11) with a $\theta(t)$ time-dependence coincides with the Fourier transform and $\vec{x}_{\perp}^2$ integral of a pressure source with the space and time dependence given by Eq. (3.14), provided $\ell c \gg 4\alpha$, a condition that is amply met in practice. Loosely speaking, we are saying that thermal diffusion is "slower" than sound propagation, and that the explicit space and time-dependence of the former are irrelevant, provided one is "listening" to long enough wavelengths and periods. The structure of the short transients induced by the fast passage of the monopole need not be studied in detail.

4. - THE MAGNETO-ACOUSTIC PRESSURE SOURCE

The passage of a monopole through a metal creates stresses around its trajectory, part of which are due to the ohmic losses, discussed in the previous chapter, which locally raise the temperature. In a superconductor there is a relatively large additional magneto-acoustic source of stress, due to the flux tube created by the monopole and to the special response of a superconductor to the magnetic field of the fluxon. In a bulk superconductor, a change of internal pressure occurs when an increasing magnetic field drives the specimen out of the Meissner phase. This pressure change is generally discussed in terms of the volume change that takes place after the local stresses are released and have become strains. Since we are interested in the mechanisms by which the local stresses in a newly born flux tube generate sound, the language of local stresses at constant volume is more appropriate to our analysis. In a single flux tube

the density of Cooper pairs is lowered, relative to the value in a bulk superconductor, but the density vanishes only at the tube's axis. Therefore, the relation between the stresses in a newly-born flux tube and the parameters describing the behaviour of bulk superconductors is not entirely straightforward: we shall have to discuss flux tubes in some detail, in order to relate the magneto-acoustic pressure source to measured quantities.

The monopole-induced injection of a flux tube into a superconductor, being a sudden process, changes not only the local pressure^{*)}, but also the local temperature, even when the additional ohmic losses discussed in the previous chapter are not included. The creation of the flux tube is neither isothermal; nor locally adiabatic, since heat transfer will take place through the initially fast process of temperature re-equilibration. However, we have learned in Chapter III that the details of thermal diffusion are not relevant to the excitation of the low eigenmodes of vibration of a resonator. When heat diffusion is similarly disregarded in the analysis of the purely magneto-acoustic pressure change, the stresses produced in the superconductor by the injection of a monopole can be computed adiabatically: everything is as if they occurred reversibly and with no heat exchange or deposition^{**)}. In spite of this lengthy preamble, we shall see that the fine distinctions between isothermal or adiabatic magneto-acoustic effects play essentially no role in the numerics.

We proceed to compute the magneto-acoustic pressure source $\delta p(\vec{x})$ and its integral along the directions transverse to the monopole trajectory, relevant to the excitation of the eigenmodes of a resonator (see Chapter 5). The sudden creation of the flux tube occurs at constant local volume. Let δE be the extra internal energy added to a small volume V around $\vec{x}$ by the creation of the flux tube. The corresponding pressure change is found by differentiation of δE with respect to V :

$$\delta p(\vec{x}) = - \left. \frac{\partial \delta E}{\partial V} \right|_s \quad (4.1)$$

*) Strictly speaking the superconductor also experiences shear stresses. But we shall see in Chapter 5 that in an isotropic material the source of sound is indeed a local pressure.

**) For conceptual simplicity, one may imagine that, within the medium, the flux tube is enclosed initially in a container with fixed, thermally insulating walls. The container should be small compared to the wavelength of the vibrational modes of interest, but large compared to the size of the flux tube. When thermal equilibrium is reached inside the container, the walls are removed and the region containing the flux tube then acts as a source of sound for the medium as a whole. From the discussion below, it will be clear that the final result is independent of the size of the imaginary container employed, as long as it is within the limits stated.

where, as we have argued, we may use the adiabatic expression at constant entropy S . Re-expressing the above equality in terms of pressure and temperature derivatives of δE , we obtain

$$\delta p = - \left. \frac{\partial p}{\partial V} \right|_S \left. \frac{\partial \delta E}{\partial p} \right|_T - \left. \frac{\partial T}{\partial V} \right|_S \left. \frac{\partial \delta E}{\partial T} \right|_p \quad (4.2)$$

The first coefficient may be related to the adiabatic bulk modulus

$$K_S = - \left. \frac{\partial p}{\partial \ln V} \right|_S \quad (4.3)$$

which for metals differs negligibly from the isothermal bulk modulus K_T , defined in Eq. (3.8b). To re-express the second coefficient in the right-hand side of Eq. (4.2), we first use the equation of state $T = T(V, S)$, to obtain

$$\left. \frac{\partial T}{\partial V} \right|_S = - \left. \frac{\partial T}{\partial S} \right|_V \left. \frac{\partial S}{\partial V} \right|_T \quad (4.4a)$$

Next, since $S = -(\partial F / \partial T)_V$ and $p = -(\partial F / \partial V)_T$, with F the Helmholtz free energy:

$$\left. \frac{\partial S}{\partial V} \right|_T = \left. \frac{\partial p}{\partial T} \right|_V = - \left. \frac{\partial p}{\partial V} \right|_T \left. \frac{\partial V}{\partial T} \right|_p \quad (4.4b)$$

where the last equality makes use of the equation of state $p = p(T, V)$. Combining Eqs. (4.4) with Eqs. (3.8) we conclude

$$\left. \frac{\partial T}{\partial V} \right|_S = -\gamma \frac{T}{V} \quad (4.5)$$

where γ is the Grüneisen parameter, defined in Eq. (3.10). Finally, integrating Eq. (4.2) over the volume of the superconductor, and making use of the longitudinal translational invariance of the flux tube, we can express the magneto-acoustic pressure source as:

$$\sum_{mag} \equiv \int \delta p(\vec{x}) d^2 \vec{x}_\perp = K_S \left. \frac{\partial \mathcal{E}}{\partial p} \right|_T + \gamma T \left. \frac{\partial \mathcal{E}}{\partial T} \right|_p \quad (4.6)$$

where $\mathcal{E}$ is the total energy of the flux tube per unit length. The quantity ϵ is the string tension of the flux tube.

The conclusion we have reached is that in order to calculate the source of low frequency sound from the creation of a flux tube, we only need to know its string tension $\epsilon(p,T)$. This is entirely analogous to the result of the previous chapter where the source of sound due to ohmic losses was shown to depend only on the total energy lost as heat per unit length, not on the details of the process. The magneto- and thermo-acoustic sources should of course be added in amplitude in order to get the total sonic source. The remarks made in the previous chapter concerning the diffusion of heat apply to the last term in Eq. (4.6), which reflects the local temperature charges in the adiabatic creation process: the transients due to the details of heat diffusion do not influence the source of low frequency sound. If we had calculated the isothermal source of sound only the first term in Eq. (4.6) would be present, with K_S replaced by K_T . The last term in Eq. (4.6) is generally much smaller than the first. In particular it vanishes for $T = 0$.

To relate the string tension ϵ to measurable quantities, and for the sake of a hypothetical reader not familiar with superconductivity, we briefly recall certain well known aspects of the thermodynamics of bulk superconductors, and then proceed to study single flux tubes²¹⁾. Consider a long cylindrical piece of superconducting material with volume V and length L , immersed in a longitudinal magnetic field H (such as could be created by a co-axial solenoid) and held at constant pressure and temperature. The superconductor's free energy, $F_S(T,V,H)$, satisfies the differential relation

$$\delta F_S = -S \delta T - p \delta V - \frac{1}{4\pi} \int (\vec{B} - \vec{H}) \cdot \delta \vec{H} d\vec{x} \quad (4.7)$$

where $\vec{B}$ is the magnetic induction and the last term represents the magnetic work due to the presence of the material. (In our cylindrical geometry the superconducting material does not affect $\vec{H}$ itself.) The material reacts as a quasi-perfect diamagnet (Meissner effect); supercurrents flow at the cylinder's surface and reduce $\vec{B}$ to zero everywhere but in a small surface layer of depth λ (the magnetic penetration depth). We neglect this surface effect to obtain

$$F_S(T, V, H) = F_S(T, V) + \frac{V H^2}{8\pi} \quad (4.8)$$

where $F_S(T, V)$ is the Helmholtz free energy in the absence of magnetic fields. Since temperature and pressure are the quantities held externally fixed, we perform a standard Legendre transformation to obtain the Gibbs potential²⁸⁾

$$G_S(T, p, H) = F_S(T, V, H) + pV = G_S(T, p + \frac{H^2}{8\pi}) \quad (4.9)$$

where $G(T,p)$ in the second equality is the potential for $\vec{H} = 0$. In a sense the only effect of the magnetic field on a bulk superconductor is the magnetostrictive increase of pressure at the surface of the body. Let $G_n(T,p)$ be the Gibbs potential of the normal state of the specimen, essentially independent of H , due to the small susceptibility of normal metals. In a Type I superconductor, the critical magnetic field H_c is given by the solution of the phase equilibrium equation

$$G_s(T,p,H_c) = G_n(T,p) \quad (4.10)$$

The volume change in the superconducting to normal transition of our bulk specimen can be found by differentiation with respect to p of Eq. (4.10) and use of Eq. (4.9), to be²⁹⁾

$$\frac{V_n - V_s}{V_s} = \frac{1}{4\pi} H_c \left. \frac{\partial H_c}{\partial p} \right|_T \quad (4.11)$$

[In a Type II superconductor, a "thermodynamic" critical field H_c , intermediate between H_{c1} and H_{c2} , can also be defined via the above procedure, and a somewhat more formal expression identical to Eq. (4.11) is still valid³⁰⁾.] The volume change of Eq. (4.11) results from the release of the stresses induced in the material as the critical magnetic field permeates it. We are interested in the local version of these stresses within a single freshly made flux tube.

We proceed to estimate the string tension $\epsilon_{2m}(p,T)$ of the monopole-induced flux tube, that contains $2m$ units of quantum flux, with m the monopole's magnetic charge in Dirac units. We discuss in great detail only the case of a minimally charged monopole, $m = 1$. As we saw in Chapter 2, vortices with one unit of quantum flux naturally occur in the mixed phase of Type II superconductors [$H_{c1}(T) < H < H_{c2}(T)$]. In Type I superconductors vortices do not naturally form. In terms of the string tension, ϵ_n , the distinction between the two types of superconductors is the following. For a Type II superconductor $\epsilon_n > n\epsilon_1$ and the equilibrium magnetic flux permeating the material in its mixed state tends to be distributed in an array of singly quantized vortices. For a Type I superconductor $\epsilon_n < n\epsilon_1$ and a would-be mixed state is thermodynamically unstable. In a Type II superconductor, as we discuss in detail in the next paragraph, the quantity $\epsilon_1(p,T)$ can be directly related to other quantities accessible to experiment, via a general thermodynamic argument very close to the one we have given for a bulk superconductor. But even for a minimally charged monopole,

the generated vortex has two units of quantum flux. It can be argued on general grounds that $\epsilon_2 \approx 2\epsilon_1$, and therefore very little detailed theory is actually necessary to estimate the magneto-acoustic effect in Type II targets. In Type I, the calculation of the string tension requires a detailed model of superconductivity. For the sake of a unified presentation (and to estimate the deviations from the equality $\epsilon_2 = 2\epsilon_1$ in Type II targets), we shall eventually discuss both types of superconductors within the same detailed model.

Consider a Type II material in the presence of a magnetic field $\vec{H}$ and containing a single minimal flux tube parallel to $\vec{H}$, of length L , and with string tension ϵ_1 . Under a small change δH , the first term in the integral in Eq. (4.7) receives a contribution $-L\phi_0 \delta H/4\pi$ from the $\vec{B}$ field of the fluxon, with ϕ_0 the quantum of flux. The Gibbs potential of the material is then

$$G_2^{(u)}(p, T, H) = G_2(p, T, H) + L(\epsilon_1 - \phi_0 H/4\pi) \quad (4.12)$$

where we have used the fact that, to first order, a small addition to F is the same as a small addition to G , $\delta G = \delta F + O(\delta F^2)$. When an increasing external field H reaches the lower critical field, $H = H_{c1}$, with

$$H_{c1} = \frac{4\pi\epsilon_1}{\phi_0} \quad (4.13)$$

the last term in Eq. (4.11) vanishes and it becomes energetically possible for flux tubes to creep from the surface into the material²¹⁾. In a Type II superconductor, Eq. (4.13) expresses $\epsilon_1(p, T)$, or its derivatives, as functions of the measurable parameters H_{c1} , $\partial H_{c1}/\partial p$, $\partial H_{c1}/\partial T$.

In a Type II superconductor, we could proceed to estimate the magneto-acoustic source without further ado. For a monopole with a minimal magnetic charge, we use in Eq. (4.6) the string tension ϵ_2 , that corresponds to two quantum units of flux. Assume $\epsilon_2 = 2\epsilon_1$, a relation that is correct for $\kappa = 1/\sqrt{2}$ and valid to within 20% for other realistic values of κ , as we shall see below. Use of Eq. (4.13) leads to the approximate result

$$\sum_{mag}^{(type II)} \approx \frac{\phi_0}{2\pi} \left[\kappa_s \frac{\partial H_{c1}}{\partial p} \Big|_T + \gamma T \frac{\partial H_{c1}}{\partial T} \Big|_p \right] \quad (4.14)$$

which directly relates the magneto-acoustic source to measurable quantities.

In order to explicitly compute the string tension $\epsilon_2(p,T)$ in both types of superconductors, we turn to a specific theoretical model: that of Ginzburg and Landau^{22),23)}. In this model the internal state of the superconductor is locally described by the wave function $\Psi(\vec{x})$ of the Cooper pairs. In the presence of an electromagnetic field $\vec{B} = \text{rot } \vec{A}$ the free energy of the superconductor is

$$F_0(T, V, \psi, \vec{A}) = F_n(T, V) + \int \frac{\vec{B}^2}{8\pi} d^3x + \int_V \left[\frac{1}{2m_c} \left| \left(\frac{\hbar}{i} \vec{\nabla} - \frac{e_c}{c} \vec{A} \right) \psi \right|^2 - \alpha |\psi|^2 + \frac{1}{2} \beta |\psi|^4 \right] d^3x \quad (4.15)$$

where F_n is the free energy of the normal state. The Cooper pair's mass and charge are $m_c = 2m_e$, $e_c = 2e$. The constants α and β are parameters characterizing the properties of the material. With some simplifying assumptions the Landau-Ginzburg model can be derived from the Bardeen-Cooper Schrieffer-microscopic theory of superconductors²¹⁾, for temperatures close to T_c . Provided the correct phenomenological parameters are used, the "Lagrangian" of Eq. (4.15) is known to yield a semiquantitatively correct description of superconductors in the full temperature range $0 \leq T \leq T_c$ ²¹⁾.

From a particle physics point of view, Eq. (4.15) is nothing but the energy of a non-relativistic charged Higgs field. It is well known that such a system, for positive α and β , undergoes a spontaneous breakdown of gauge invariance: the field ψ has a non-vanishing value in the superconducting state with $|\psi|^2$ the density of Cooper pairs.

Thermodynamic equilibrium is obtained at the minimum of the free energy under variations of ψ and $\vec{A}$ subject to the correct boundary conditions. For $\vec{A} = 0$ and ψ constant, the minimum occurs at

$$|\psi|^2 = \frac{\alpha}{\beta} \quad (4.16)$$

and we have

$$F_0(T, V) = F_n(T, V) - V \frac{\alpha^2}{2\beta} \quad (4.17)$$

From this we derive, via a standard Legendre transform:

$$G_0(T, p) = G_n(T, p - \frac{\alpha^2}{2\beta}) \quad (4.18)$$

Equation (4.18), when compared with Eqs. (4.9) and (4.10), yields

$$\frac{1}{8\pi} H_c^2 = \frac{\alpha^2}{2\beta} \quad (4.19)$$

A characteristic length scale for variations in ψ is given by the ratios of the terms involving $|\nabla\psi|^2$ and $|\psi|^2$ in Eq. (4.15):

$$\xi = \frac{\hbar}{\sqrt{2m_c\alpha}} \quad (4.20)$$

The characteristic length scale for electromagnetic phenomena is set by the "photon mass" term, arising when $|\psi|^2$ takes the value of Eq. (4.15)

$$\lambda = \sqrt{\frac{m_c c^2}{4\pi e_c^2} \frac{\beta}{\alpha}} \quad (4.21)$$

There is a fundamental relation between the λ and ξ , obtained by combining the last three definitions:

$$\phi_0 \equiv hc/2e = 2\pi\sqrt{2} H_c \lambda \xi \quad (4.22)$$

The dimensionless ratio

$$\kappa = \lambda/\xi \quad (4.23)$$

is called the Landau-Ginzburg parameter and characterizes the type of superconductor. For $\kappa < 1/\sqrt{2}$ it is of Type I; for $\kappa > 1/\sqrt{2}$, Type II.

We now turn to the case of a fluxon with n units of flux. In cylindrical co-ordinates (r, θ, z) we put²³⁾

$$\psi = \sqrt{\frac{\alpha}{\beta}} f(r) e^{in\theta} \quad (4.24)$$

$$\vec{A} = \frac{n\phi_0}{2\pi} \frac{g(r)}{r} \vec{e}_\theta \quad (4.25)$$

In this gauge the non-trivial topological winding number of the solution appears in the phase factor in ψ . The dimensionless functions f and g only depend on $r = \sqrt{x^2 + y^2}$ and satisfy the boundary conditions

$$f(0) = g(0) = 0 \quad (4.26a)$$

$$f(\infty) = g(\infty) = 1 \quad (4.26b)$$

The magnetic field is

$$\vec{B} = \frac{n\phi_0}{2\pi} \frac{1}{r} \frac{dg}{dr} \vec{e}_z \quad (4.27)$$

and the energy per unit of length becomes

$$\varepsilon_n = \frac{\phi_0^2}{16\pi^2\lambda^2} \mathcal{V}_n(\kappa) \quad (4.28)$$

where the dimensionless quantity

$$\begin{aligned} \mathcal{V}_n(\kappa) = \int_0^\infty r dr & \left[\frac{\lambda^2 n^2}{r^2} \left(\frac{dg}{dr} \right)^2 + \left(\frac{df}{dr} \right)^2 \right. \\ & \left. + \frac{n^2(1-g)^2 f^2}{r^2} + \frac{1}{2\xi^2} (1-f^2)^2 \right] \end{aligned} \quad (4.29)$$

can only depend on κ . The functions f and g minimize $\mathcal{V}_n(\kappa)$ and must therefore satisfy

$$\frac{d^2 f}{dr^2} + \frac{1}{r} \frac{df}{dr} - \frac{n^2(1-g)^2}{r^2} f + \frac{1}{\xi^2} (1-f^2) f = 0 \quad (4.30)$$

$$\frac{d^2 g}{dr^2} - \frac{1}{r} \frac{dg}{dr} + \frac{1}{\lambda^2} (1-g) f^2 = 0 \quad (4.31)$$

subject to the boundary conditions (4.27). A standard transformation³¹⁾ of the integral in Eq. (4.29) brings $\mathcal{V}_n(\kappa)$ into the form

$$\mathcal{V}_n(\kappa) = \frac{1}{\xi^2} \int_0^\infty r dr (1-f^2) \quad (4.32)$$

Since f for small n varies from 0 to 1 within a region of size ξ , we expect $\mathcal{V}_n(\kappa)$ to be a slowly varying function of κ . For $\kappa = 1/\sqrt{2}$, $\mathcal{V}_n$ is exactly known^{32),33)}

$$\mathcal{V}_n(\kappa) \Big|_{\kappa=1/\sqrt{2}} = n \quad (4.33)$$

whereas for large κ it has the behaviour³³⁾

$$\nu_n(\kappa) \simeq n^2 \ln \kappa \quad (\kappa \gg 1) \quad (4.34)$$

The small κ behaviour of $\nu_n(\kappa)$ has to our knowledge not been investigated before, but our numerical studies discussed below indicate that

$$\nu_n(\kappa) \sim |\ln \kappa|^{-1} \quad (\kappa \ll 1) \quad (4.35)$$

This is in accord with our expectation that $\nu_n(\kappa)$ is a slowly varying function of κ .

In order to compute the derivatives of ϵ_n with respect to pressure and temperature we must calculate the derivative of $\nu_n(\kappa)$ with respect to κ . Differentiate Eq. (4.29) with respect to ξ , and use the fact that the implicit ξ -dependence via f and g vanishes, due to the differential equilibrium conditions Eqs. (4.30) and (4.31), to obtain

$$\frac{d\nu_n(\kappa)}{d\kappa} = \frac{1}{\kappa} \frac{1}{3} \int_0^\infty r dr (1-f^2)^2 \quad (4.36)$$

The logarithmic derivative is

$$\rho_n(\kappa) \equiv \frac{d \ln \nu_n}{d \ln \kappa} = \frac{\int_0^\infty r dr (1-f^2)^2}{\int_0^\infty r dr (1-f^2)} \quad (4.37)$$

a quantity which is positive and less than unity.

We have numerically solved the Landau-Ginzburg thermodynamic stability conditions, Eqs. (4.30) and (4.31), using both a fourth order Runge-Kutta method and a direct finite element (lattice) minimization of the integral Eq. (4.29). We found that for intermediate values of κ both methods worked equally well whereas for extreme values of κ the finite element method was definitely superior. The results for $\nu_1(\kappa)$ and $\nu_2(\kappa)$ are shown in Figs. 4.1 and 4.2.

We now turn to the calculation of the magneto-acoustic source. Here we write the string tension in two different forms, to be discussed in turn below, according to whether the superconductor is of Type I or II.

Fig. 4.1: Plot of $v_n(\kappa)/n^2$ versus κ for $n = 1, 2$. The asymptotic behaviour for large κ is $v_1(\kappa) = \ln \kappa + 0.50$, $v_2(\kappa)/4 = \ln \kappa - 0.45$. The first of these constants is in disagreement with that computed in the original reference by Abrikosov²³).

Fig. 4.2: Plot of $1/\rho_n$, with $\rho_n = d \ln v_n / d \ln \kappa$, versus κ for $n = 1, 2$. The linear asymptotes for large κ are and must be the same as those of Fig. 4.1. For $\kappa \ll 1$ the asymptotes are $1/\rho_1 = -0.81 \ln \kappa + 1.0$ and $1/\rho_2 = -0.81 \ln \kappa + 0.0$.

The relevant experimentally available quantities in a Type I superconductor are the critical temperature T_c and its pressure derivative $\partial T_c / \partial p$, the critical magnetic field at zero temperature H_0 and its pressure derivative $\partial H_0 / \partial p$, and for a small selection of pure metals, the electromagnetic penetration depth at zero temperature λ_0 . The temperature dependence is well described by the phenomenological formula^{34),35)}

$$H_c = H_0 (1 - t^2) \quad (4.38)$$

$$\lambda = \lambda_0 (1 - t^2)^{-1/2} \quad (4.39)$$

where $t = T/T_c$ is the reduced temperature. In our estimates we shall concentrate on Type I pure metals for which λ_0 is experimentally known. To compute the magneto-acoustic source of Eq. (4.6), with ϵ given by Eq. (4.28), we also need the pressure derivative of λ_0 . To estimate this quantity, we use the following somewhat elaborate method. Let λ_L be the Landau penetration depth, related to the density of conduction electrons N_c via

$$\lambda_L = \sqrt{\frac{m_e c^2}{4\pi e^2 N_c}} \quad (4.40)$$

The Landau-Ginzburg penetration depth λ_0 , in turn, is related to λ_L via the Pippard relationship³⁶⁾;

$$\lambda_0 = \lambda_L \left(\frac{\sqrt{3}}{2\pi} \frac{\xi_0}{\lambda_L} \right)^{1/3} \quad (4.41)$$

Here ξ_0 is the Pippard, or BCS, coherence length and v_F is the Fermi velocity:

$$\xi_0 = 0.18 \frac{\hbar v_F}{k_B T_c} \quad (4.42a)$$

$$v_F = \frac{\hbar}{m_e} (3\pi^2 N_c)^{1/3} \quad (4.42b)$$

The Pippard relationship, Eq. (4.41), corrects the oversight made when assuming the Cooper pairs to be point-like in the Landau-Ginzburg treatment. In an extreme Type I superconductor, the Cooper pairs are actually quite large on the scale of λ and give rise to effectively non-local interactions at distances of order ξ_0 . The estimates obtained from the above considerations, via Eqs. (4.35)

to (4.37), are sometimes off by as much as a factor of three, but we only use this method in the calculation of logarithmic derivatives, for which only the scaling behaviour, correctly described by Eqs. (4.41) and (4.42), matters.

The Landau-Ginzburg parameter is calculated from the relationship Eq. (4.22) expressed in the form:

$$K = \frac{2\pi\sqrt{2}}{\phi_0} H_c \lambda^2 = \frac{K_0}{1+t^2} \quad (4.43)$$

The string tension is obtained from Eq. (4.28), which may also be written in form

$$\varepsilon_n = \frac{\phi_0 H_c}{4\pi} \cdot \frac{\nu_n(K)}{\sqrt{2}K} \quad (4.44)$$

Using Eqs. (4.33)-(4.37) we may express the logarithmic derivative of ε_n entirely in terms of $\partial \ln T_c / \partial p$, $\partial \ln H_0 / \partial p$ and $\partial \ln N_e / \partial p$. The last quantity is identical to the compressibility (inverse bulk modulus $1/K$). Collecting everything we obtain for the source of sound Eq. (4.6) in a Type I superconductor

$$\sum_{\alpha\beta} \int \delta p(\vec{x}) d^2 \vec{x}_\perp = \varepsilon_n D_n^I \quad (4.45)$$

with ε_n the string tension and

$$D_n^I = A_n^I K \frac{\partial \ln T_c}{\partial p} + B_n^I K \frac{\partial \ln H_0}{\partial p} + C_n^I \quad (4.46)$$

In a Type I target the coefficients A , B and C are given by

$$A_n^I = \frac{2}{3} (1 - \rho_n) + \frac{2t^2}{1+t^2} \left(\rho_n + \frac{2t^2}{1-t^2} \right) \quad (4.47a)$$

$$B_n^I = \rho_n \quad (4.47b)$$

$$C_n^I = \frac{4}{9} (1 - \rho_n) - \gamma \frac{2t^2}{1+t^2} \left(\rho_n + \frac{2t^2}{1-t^2} \right) \quad (4.47c)$$

where we have neglected the difference between K_S and K_T , the adiabatic and isothermal bulk moduli. Here the functions $\rho_n(K)$ are the ones defined in Eq. (4.37) and plotted in Fig. 4.2.

In superconductors of Type II we can either estimate the magneto-acoustic source via the approximate expression Eq. (4.14), or proceed in a manner that makes use of the Ginzburg-Landau model, as we have done for Type I superconductors. But for most elementary Type II superconductors we have not been able to find data on the pressure derivatives of the lower critical field so that we shall have to appeal to the Ginzburg-Landau model even when use is made of Eq. (4.14), which expresses the magneto-acoustic source in terms of measurable parameters. In practice, we relate the magneto-acoustic pressure source to the actually measured quantities: H_{c1} , H_{c2} , T_c , $\partial T_c / \partial p$ and the pressure derivative of the "thermodynamic" critical field $\partial H_c / \partial p$.

The value of Ginzburg-Landau parameter κ can be obtained from the relation³⁷⁾

$$\frac{H_{c1}}{H_{c2}} = \frac{\nu_1(\kappa)}{2\kappa^2} \quad (4.48)$$

and our explicit calculation of $\nu_1(\kappa)$, reported in Fig. 4.1. For convenience, we have plotted Eq. (4.48) in Fig. 4.3, and blown up parts of Figs. 4.1 and 4.2 in Figs. 4.4 and 4.5, respectively.

Fig. 4.3: Plot of H_{c1}/H_{c2} versus κ .

Fig. 4.4: Blown up version of part of Fig. 4.1.

Fig. 4.5: Plot of $\rho_n = d \ln v_n / d \ln \kappa$ in a useful range of κ values.

The thermodynamic critical field is estimated from either of the relations

$$H_c = (H_{c1} H_{c2} / \nu_1(\kappa))^{1/2} \quad (4.49a)$$

$$H_c = \frac{\sqrt{2} \kappa}{\nu_1(\kappa)} H_{c1} \quad (4.49b)$$

The remaining of the calculation of the magneto-acoustic source in a Type II superconductor proceeds as in Type I. All one needs is to compute explicitly the pressure derivative of the string tension ϵ_n of Eq. (4.44). As before we assume the temperature dependence of H_c , λ and κ to be governed by the approximate phenomenological expressions Eqs. (4.38), (4.39) and (4.43). The only differences with the the Type I results originate from the fact that for a Type II superconductor we do not appeal to the Pippard relationship Eq. (4.41), but assume $\lambda_0 \approx \lambda_L$, an approximation that becomes strictly correct in the large κ , extreme Type II limit. The results for the magneto-acoustic pressure source are

$$\Sigma_{mag} = \epsilon_n D_n^{\text{II}} \quad (4.50a)$$

$$D_n^{\text{II}} = A_n^{\text{II}} K \frac{\partial \ln T_c}{\partial p} + B_n^{\text{II}} K \frac{\partial \ln H_0}{\partial p} + C_n^{\text{II}} \quad (4.50b)$$

$$A_n^{\text{II}} = \rho_n \frac{2t^2}{1+t^2} + \frac{4t^4}{1-t^4} \quad (4.50c)$$

$$B_n^{\text{II}} = \rho_n \quad (4.50d)$$

$$C_n^{\text{II}} = 1 - \rho_n - \gamma A_n^{\text{II}} \quad (4.50e)$$

where $H_0 = H_c (T = 0)$; ρ_n are the quantities defined in Eq. (4.37), $t = T/T_c$, and the difference between K_S and K_T has been ignored. In the above treatment of Type II superconductors we have neglected the effect of the "aftershock", in which the unstable $n > 1$ vortex initially formed by the monopole breaks up into elementary vortices of unit flux.

For some Type II superconductors we have been able to locate data on the pressure derivative of the thermodynamic field $\partial H_c / \partial p$, while for others $\partial H_{c1} / \partial p$ is the quantity that has been measured. In this latter case we may

still use the expression Eq. (4.50b), if we relate the pressure derivatives of H_{c1} and H_c via the following procedure. Let H_1 denote H_{c1} at zero temperature. The pressure derivative of Eq. (4.49b) at $T = 0$ is:

$$\frac{\partial \ln H_1}{\partial p} = \frac{\partial \ln H_0}{\partial p} + (\rho_1^0 - 1) \frac{\partial \ln \kappa_0}{\partial p} \quad (4.51)$$

Here $\rho_1^0 = \rho_1(\kappa_0)$ and κ_0 are defined in Eqs. (4.37) and (4.43), respectively. The value of $\partial \ln \kappa_0 / \partial p$ can be obtained from Eqs. (4.22) and (4.23) to be

$$\frac{\partial \ln \kappa_0}{\partial p} = \frac{\partial \ln H_0}{\partial p} + 2 \frac{\partial \ln \lambda_L}{\partial p} = \frac{\partial \ln H_0}{\partial p} - \frac{1}{K} \quad (4.52)$$

where K is the bulk modulus. In the last equality in Eq. (4.52) we have used $\lambda_0 \approx \lambda_L$, and the scaling behaviour of $\ln \lambda_L$ as given by Eq. (4.40): $\lambda_L \sim N_c^{-1/2} \sim V^{1/2}$. We may now combine Eqs. (4.51) and (4.52) to obtain

$$K \frac{\partial \ln H_1}{\partial p} = \rho_1^0 K \frac{\partial \ln H_0}{\partial p} + 1 - \rho_1^0 \quad (4.53)$$

which is the result we were after.

To summarize this lengthy chapter, we recall its main results. The magneto-acoustic pressure source, Σ_{mag} , is expressed via Eq. (4.6) in terms of pressure and temperature derivatives of the fluxon's string tension. In a Type II superconductor, this result can be approximately re-expressed, via simple thermodynamic arguments, in the form of Eq. (4.14), that relates Σ_{mag} to p and T derivatives of the lower critical field. Type I superconductors lack a lower critical field and require more work: we use the Ginzburg-Landau model to express Σ_{mag} in terms of T_c , $\partial \ln T_c / \partial p$ and $\partial \ln H_0 / \partial p$, Eqs. (4.45) and (4.46). The coefficients in these expressions are given by Eqs. (4.44) and (4.47), and require explicit solutions of the model for fixed κ . In a Type II superconductor one can similarly obtain a result for Σ_{mag} that is more than a mere estimate, provided one is ready to accept the explicit assumptions of a Ginzburg-Landau model. The corresponding results are given by Eqs. (4.50).

5. - THE SONIC MONOPOLE SIGNAL IN A RESONANT DETECTOR

The sudden pressure pulse induced by a monopole traversing a solid target acts as the source of a sonic wave. In an isotropic medium, where the thermal expansion parameters and the coefficients describing the superconducting properties of the material are scalars, the aforementioned stresses can only be of the pressure type: $\sigma_{ij} = -p\delta_{ij}$ ^{*)}. We are interested in the projection of the sonic wave onto the eigenmodes of a finite resonant detector. Consider a monopole that has just traversed a piece of material leaving in its wake a local thermo- and magneto-acoustic pressure change $\delta p(\vec{x}, t)$, confined to a narrow cylinder along the trajectory, depicted in Fig. 5.1. The explicit expressions for the resulting excitations of the vibrational modes of the detector are in general very complicated. This is a consequence of the intricate nature of the motions of a solid body, in which coupling between deformations in different directions, and the boundary conditions, play a non-trivial role ³⁸⁾. We shall first isolate the general features of the excitation process and then specialize to a highly simplified geometry in order to obtain explicit expressions for the signal to noise ratio.

Let Y be Young's module and σ Poisson's ratio for the superconducting material which we for simplicity assume to be isotropic. In an infinite medium of density ρ longitudinal (compressional) waves propagate with the speed

$$c_l = \sqrt{\frac{Y}{\rho} \frac{1-\sigma}{(1-2\sigma)(1+\sigma)}} \quad (5.1)$$

whereas transverse (shear) waves propagate with the speed

$$c_t = \sqrt{\frac{Y}{\rho} \frac{1}{2(1+\sigma)}} \quad (5.2)$$

^{*)} Let the strain tensor be $u_{ij} = (\partial u_i / \partial x_j + \partial u_j / \partial x_i) / 2$ and the stress tensor $\sigma_{ij} = \partial F / \partial u_{ij}$ with F the free energy density of the material. The deformation and temperature dependent term in F must, because of rotational invariance, be of the form $(T - T_0)u_{ii}$, leading to a diagonal stress of the pressure type. Under a virtual deformation the Ginzburg-Landau free energy Eq. (4.15) receives two contributions. The first arises from the intrinsic dependence of the material constants α and β on the strain tensor, which again must be proportional to u_{ii} in an isotropic medium. The second is of the form $u_{ij}T_{ij}$ where T_{ij} is the stress tensor of the fields ψ and $\vec{A}$. To lowest order in u_{ij} this tensor is divergence-free and consequently does not act as a source of sound.

(a)

(b)

Fig. 5.1: Definitions of some geometrical quantities for monopoles traversing two differently shaped targets.

The equations of motion for general vibrations in the presence of a pressure source $\delta p(\vec{x}, t)$ are

$$\ddot{\vec{u}} + \vec{D} \cdot \vec{u} = -\frac{1}{\rho} \vec{\nabla} \delta p \quad (5.3)$$

where $\vec{D}$ is the Hermitian, positive definite dyadic operator

$$D_{ij} = c_t^2 (\nabla_i \nabla_j - \delta_{ij} \Delta) - c_l^2 \nabla_i \nabla_j \quad (5.4)$$

and $\vec{u}(\vec{x}, t)$ is the displacement field. Although a pressure source in an infinite medium may only excite longitudinal waves, the boundary conditions for a finite body impose a coupling to the transverse waves.

Let $\vec{u}_n(\vec{x})$ be the eigenfunctions of $\vec{D}$ with eigenfrequency ω_n :

$$\vec{D} \cdot \vec{u}_n = \omega_n^2 \vec{u}_n \quad (5.5)$$

We take the boundary conditions to be those of a free body with no forces acting at the surface. Normalize the eigenfunctions to the volume of the body

$$\int_V \vec{u}_n \cdot \vec{u}_m d^3\vec{x} = V \delta_{nm} \quad (5.6)$$

such that the displacements $\vec{u}_n$ are of order unity. Resolving the sound wave on the normal modes, we have

$$\vec{u}(\vec{x}, t) = \sum_n b_n(t) \vec{u}_n(\vec{x}) \quad (5.7)$$

where the amplitude $b_n(t)$ satisfies

$$\ddot{b}_n + \omega_n^2 b_n = \Delta_n \quad (5.8)$$

with a source

$$\Delta_n = \frac{1}{\rho V} \int_V (-\vec{\nabla} \delta p) \cdot \vec{u}_n d^3\vec{x} \quad (5.9)$$

In realistic materials a damping term $\omega_n \dot{b}_n / Q_n$ should be added to the left-hand side of Eq. (5.8), where Q_n is the Q value of the mode n. Since we are interested in abrupt, non-periodic signals the (large) values of the quality factor Q_n will only play an explicit role in our considerations about reduction of effective thermal noise, a problem to be discussed in the next chapter.

Let us now concentrate entirely on low frequency eigenmodes for which $\vec{u}_n(\vec{x})$ varies slowly throughout the body. Since δp is only non-zero along the trajectory L of the monopole, we may factor the integral in Eq. (5.9) into a line integral along L and a surface integral orthogonal to L , to obtain

$$\Lambda_n = \frac{\sum}{\rho V} \int_L \text{div}_\perp \vec{u}_n d\ell \quad (5.10)$$

where we have also performed a partial integration along the transverse directions, and where

$$\Sigma = \Sigma_{\text{mag}} + \Sigma_H = \int \delta p(\vec{r}) d^3\vec{x}_\perp = K \left. \frac{\partial \mathcal{E}}{\partial p} \right|_T + \gamma T \left. \frac{\partial \mathcal{E}}{\partial T} \right|_p + \gamma \left. \frac{d\mathcal{E}}{dx} \right|_p \quad (5.11)$$

is what we have called the source of sound. In our normalization of the eigenmodes, the integral in Eq. (5.10) is of order $L\omega_n/c_s$, where c_s is a suitably chosen sound velocity, and where L now also denotes the length of the monopole's path in the target. Let us define a "form factor":

$$G_n = \frac{c_s}{\omega_n} \frac{1}{L} \int_L \text{div}_\perp \vec{u}_n d\ell \quad (5.12)$$

This factor is expected to be order unity for a large number of modes and paths. We shall calculate it for simple geometry below. Thus we arrive at the expression

$$\Lambda_n = \frac{L\omega_n}{\rho V c_s} G_n \cdot \Sigma \quad (5.13)$$

for the source Eq. (5.10).

In order to solve Eq. (5.8) we must know the time dependence of the source $s_n(t)$. As the monopole traverses the material, there may be fairly complicated phenomena whose time-dependence we have not investigated in detail. The characteristic time for these transients is set by the crossing time of the monopole, the time it takes electrons of broken Cooper pairs to thermalize in their collisions with phonons, and the time necessary for supercurrents to settle into a stable vortex. For a target with linear dimensions of a few centimeters all these characteristic times are orders of magnitude shorter than the period of a low frequency eigenmode τ_n^*). Thus we expect the quantity $s_n(t)$ to

*) For monopoles with velocity β at the lower end of the conceivable bracket, the speed of sound may begin to be comparable to β . In this case the sonic pulse is conical rather than cylindrical, implying some simple modifications of our estimates, without significantly affecting their order of magnitude. We shall not dwell upon this matter in detail.

reach, in a time large compared to the characteristic transient times but short compared to τ_n , an asymptotic value $s_n(\infty)$ given by Eqs. (5.11) and (5.13), interpreted as being time-independent.

The general causal solution of Eq. (5.8) is

$$b_n(t) = \int_{-\infty}^{\infty} \frac{e^{-i\omega t}}{\omega_n^2 - (\omega + i\varepsilon)^2} \tilde{s}_n(\omega) \frac{d\omega}{2\pi} \quad (5.14)$$

where $\tilde{s}_n(\omega)$ is the Fourier transform of the source:

$$\tilde{s}_n(\omega) = \int_0^{\infty} s_n(t) e^{i\omega t} dt \quad (5.15)$$

where the monopole has been assumed to arrive at time $t = 0$. The behaviour of $\tilde{s}_n(\omega)$ for low ω is

$$\tilde{s}_n(\omega) = \frac{i}{\omega + i\varepsilon} s_n(\infty) + A_n + O(\omega) \quad (5.16)$$

where

$$A_n = \int_0^{\infty} (s_n(t) - s_n(\infty)) dt \quad (5.17)$$

is a constant that characterizes the transient behaviour of the sonic pulse. Substituting Eq. (5.16) into Eq. (5.14) we obtain (for $t > 0$)

$$b_n(t) = s_n(\infty) (1 - \cos \omega_n t) \omega_n^{-2} + A_n (\sin \omega_n t) \omega_n^{-1} + O(\omega_n^0) \quad (5.18)$$

We expect the transient behaviour of $s_n(t)$ not to be violent; that is we conservatively assume $|A_n| \ll \tau_n |s_n(\infty)|$. Thus we take the first term in Eq. (5.18) to be the dominant one in the excitation of low frequency eigenmodes, and we drop the remainder.

The total kinetic plus potential energy in the monopole-induced sound wave is

$$E = \int_V \rho \langle \dot{\vec{u}}^2 \rangle_t d^3\vec{x} = \sum_n \rho V \langle \dot{b}_n^2 \rangle_t \quad (5.19)$$

where the subindex t indicates time average. Use Eqs. (5.18) and (5.13) to obtain, for the energy in a single mode

$$E_n = \frac{1}{2} \frac{L^2}{V} \frac{G_n^2}{\rho c_n^2} \sum^2 \quad (5.20)$$

For gravitational waves the signal energy per eigenmode is proportional to the volume of the antenna. For monopoles it is inversely proportional to a characteristic linear dimension of the detector. This is a reflection of the fact that the monopole is a line-source of vibrations, not a whole-body source.

We now evaluate the form factor G_n in Eq. (5.20), defined in Eq. (5.12) for a specific geometry. The simplest explicitly solvable case is that of a parallelepiped with highly asymmetric dimensions $\ell_1 \gg \ell_2 \gg \ell_3$, i.e., a resonator in the form of a thin strip, see Fig. 5.1a. Among the vibrational eigenmodes of such a body there are "longitudinal" irrotational ones that can be naively described as sinusoidal compressional waves along the three principal directions. Only the three different sound velocities c_i , $i = 1, 2, 3$ reflect the boundary conditions that couple longitudinal and transverse motions. Their values are:

$$C_1 = \sqrt{\frac{Y}{\rho}} \quad (5.21a)$$

$$C_2 = \sqrt{\frac{Y}{\rho} \frac{1}{1-\epsilon^2}} \quad (5.21b)$$

$$C_3 = \sqrt{\frac{Y}{\rho} \frac{1-\epsilon}{(1+\epsilon)(1-2\epsilon)}} = C_2 \quad (5.21c)$$

With the origin of co-ordinates in a corner of the parallelepiped, the normalized eigenmodes for the "longitudinal" waves in the i^{th} direction $\vec{e}_i$ are approximately given by

$$\vec{u}_n^{(i)} \simeq \vec{e}_i \sqrt{2} \cos(k_n^{(i)} x_i) \quad (5.22)$$

with

$$k_n^{(i)} = \frac{\pi n}{l_i} \quad (5.23a)$$

$$\omega_n^{(i)} = c_i k_n^{(i)} \quad (5.23b)$$

The form factors are

$$G_n^{(1)} = \sqrt{2} \sin(k_n^{(1)} x_1) \quad (5.24a)$$

$$G_n^{(2)} = \sqrt{2} \sin(k_n^{(2)} x_2) \quad (5.24b)$$

$$G_n^{(3)} = -\frac{2\sqrt{2}}{\pi n} \delta_{n, \text{odd}} \cos^2 \theta \quad (5.24c)$$

where x_i are the co-ordinates of the centre of the monopole's track within the detector and θ is the angle of incidence as defined in Fig. 5.1a. In deriving these formulas we have excluded the cases of grazing incidence of the monopole $\theta \lesssim l_{2,3}/l_1$. The value of the square of the form factors in the two longest directions, averaged over the positions of incidence of monopoles, is unity for all n . In the shortest direction only the odd modes are excited, with a n^{-2} law in their energy. For the energies in the modes of the parallelepiped, we find

$$E_n^{(i)} = \frac{l_3}{l_1 l_2} \cdot \frac{\Sigma^2}{\rho c_i^2} \cdot \begin{cases} 1/2 \sin^2 \theta & (i=1, 2) \\ 4 \cos^4 \theta / \pi^2 n^2 \sin^2 \theta & (i=3) \end{cases} \quad (5.25a)$$

where Σ is given by Eq. (5.11) and where we have averaged over positions of monopole incidence. Notice that the factor $l_3/l_1 l_2$ in Eq. (5.25) is the ratio of the small dimension to the large ones. This suppression is only a property of very asymmetric targets, like our parallelepiped, wherein the length of the track of an average monopole is of the order of magnitude of the smallest side. In a cylindrical target like that of Fig. 5.1b, the geometrical factor would be of the order of l^{-1} .

6. - MONOPOLE SIGNALS AND THERMAL NOISE IN SPECIFIC MATERIALS

In this chapter we compare the typical vibrational energies deposited by monopoles in targets made of different metals, and we compare the signals with the total noise (the combined and coupled effect of thermal noise in the detector and the noise of the signal readout system). In the expression for the signal energy per eigenmode, Eq. (5.20), the values of ρc_s^2 , the bulk modulus K , and the thermal energy loss dE/dx vary by less than an order of magnitude from metal to metal. This is not true of Grüneisen's parameter γ and the pressure and temperature derivatives of the "string tension" $\partial\epsilon/\partial p|_T$ and $\partial\epsilon/\partial T|_p$. Targets with large values of these parameters are optimal in the sense of the monopole's signal. But the crucial question is the signal to noise ratio. The effective noise, as we shall see, strongly depends on the detector's Q value. It is not possible to make realistic theoretical predictions for Q values, and we will therefore not be able, at this stage, to choose an optimal material to be used in "listening" to monopoles.

To be very specific, let us consider a detector (or detector-element) of fixed "typical linear dimension" $\ell = \ell_1 \ell_2 / \ell_3$, in the sense of Eq. (5.25). We concentrate on the low order vibrational eigenmodes along the longest of the detector dimensions, in which case we can approximate ρc_1^2 in Eq. (5.25) by Young's modulus, Y . For an order of magnitude estimate, we set $\theta = 45^\circ$ in Eq. (5.25a), and give results for the signal energy per eigenmode at $T \approx 0$:

$$E_s = \sum^2 / \ell Y \quad (6.1a)$$

$$\sum = K \left. \frac{\partial \epsilon}{\partial p} \right|_{T=0} + \gamma \frac{dE}{dx} (\beta) \quad (6.1b)$$

In Table 6.1 we list, for a few pure metals, the parameters that determine, via Eqs. (5.11), (4.54), (4.45), (4.14) (3.2) and (3.5), the monopole signal energy at temperatures close to absolute zero ($T \ll T_c$). The values in parentheses are indicative and are not used in the $T = 0$ estimates. The last metal listed in Table 6.1, Cr, is not a superconductor. Notice that at low temperatures some metals (Al, V, Nb) expand as the temperature increases ($\gamma > 0$) while others (α -U, Cr) contract ($\gamma < 0$). The sign of the volume change induced by the injection of a flux tube, governed by the sign of dH_0/dp [as naively indicated by Eq. (4.11)], is also metal-dependent: negative for Al and Nb and positive for V and α -U. In Table 6.2 we list a series of intermediary parameters used in the calculations and the equations from which they are obtained. Notice, in comparing the values of dE/dx and the string tension ϵ_2 , that a very slow monopole ($\beta \sim 10^{-5}$) loses comparable amounts of energy to

Table 6.1

Metal	Supercond. Type	T _c (°K)	λ ₀ (nm)	H _{c1} (gauss)	H _{c2} (gauss)	H _c (gauss)	∂T _c /∂p 10 ⁻¹¹ cm ² /dyn	dH ₀ /dp 10 ⁻⁹ gauss cm ² /dyn	K ⁽⁴⁸⁾ 10 ¹¹ dyn/cm ²	γ ⁽⁴⁸⁾ 10 ¹¹ dyn/cm ²	γ ⁽⁴⁹⁾	n _c
Al	I	1.196 ³⁹⁾	50 ⁴¹⁾	**	**	104 ⁴⁶⁾	-2.9 ⁴⁶⁾	-2.7 ⁴⁶⁾	7.55	7.03	1.6	3
Nb	II	{9.25} ³⁹⁾	{44} ⁴¹⁾	1850 ^{43),49)}	3900 ^{43),49)}	-	{-0.16 to -0.25} ³⁹⁾	-1.2 ⁴⁶⁾	17.0	10.5	1.5	5
V	II	{5.40} ³⁹⁾	40 ⁴²⁾	1100 ⁴⁴⁾	2200 ⁴⁵⁾	-	{1.1} ⁴⁶⁾ {0.58 to 0.65} ³⁹⁾	1.3 ⁴⁶⁾	15.8	12.7	1.6	5
<α-U>	II	{0.2?} ⁴⁰⁾		120 ⁴⁷⁾	240 ⁴⁷⁾	-	{7.6} ⁴⁶⁾	140 ± 30 ⁴⁶⁾	15.7	16.5	-18	6
Cr	**	**	**	**	**	**	**	**	16.0	27.9	-9.3	6

Parameters used in the calculation of monopole signals at zero temperature for different pure metal targets. The brackets for α-U denote a directional average. Notice that we have taken for n_c its maximum possible value (this parameter only occurs as n_c^{2/3} in the expression for dE/dx).

Table 6.2

Metal	H_c (gauss)	κ	$v_1(\kappa)$ Fig. 5.4	$v_2(\kappa)$ Fig. 5.4	$\rho_1(\kappa)$ Fig. 5.5	$\rho_2(\kappa)$ Fig. 4.5	$\frac{d\ln H_c}{dp}$ $(T = 0)$	ϵ_2 10^{-5} ergs/cm	$\Sigma_{\text{mag}} (T = 0)$ 10^{-5} ergs/cm	$\left(\frac{dE}{dx}\right)$ $10^{-5} \left(\frac{\beta}{10^{-3}}\right) \frac{\text{ergs}}{\text{cm}}$
Al	104 (exp.)	0.112 (4.43)	**	0.8	**	0.41	**	0.86 (4.44)	-13 (4.45)-(4.47)	80 (3.5)
Nb	2432 (4.49)	1.13 (4.48)	1.22	2.58	0.425	0.554	+0.218 (4.53)	6.4 (4.44)	1.33 (4.14) -0.125 (4.50)	113 (3.5)
V	1420 (4.49)	1.09 (4.48)	1.20	2.55	0.425	0.554	1.19 (4.53)	3.8 (4.44)	4.3 (4.14) 4.8 (4.50)	113 (3.5)
$\langle \alpha-U \rangle$	155 (4.49)	1.09 (4.48)	1.20	2.55	0.425	0.554	603 (4.53)	0.42 (4.44)	238 (4.14) 325 (4.50)	127 (3.5)
Cr	**	**	**	**	**	**	**	**	**	189 (3.2)

Intermediary parameters in the calculation of the sonic source. The values in parentheses number the equations from which the result is obtained.

heat and to the creation of a flux tube in a superconductor. For the cases of the Type II metals Nb, V and α -U we have given in Table 6.2 the magneto-acoustic source Σ_{mag} as computed via the naïve estimate Eq. (4.14) and via the more elaborate expression Eq. (4.50). It is satisfactory to notice that the estimates are quite compatible and undoubtedly correct to within a factor of two^{*)}. Unfortunately we do not have the same confidence in the calculation of the thermal contribution to the signal. In estimating this contribution we have used values of dE/dx given by Eq. (3.2) for conductors and Eq. (3.5) for superconductors. As discussed in the conductor case by Ford and Allen and Kinoshita²⁰⁾ their estimates may be uncertain by more than a factor of two. Moreover, as we explained in Chapter II, neglect of the atomic excitation effect discussed by Drell and collaborators¹⁷⁾ may result in an underestimate of the thermal signal by an amplitude factor that may be as large as $\rho/\rho(A\ell)$ in a high ρ metal (Recall that we have chosen to overlook this effect in pessimizing our estimates.) Crystals of Al, V, Nb and Cr are cubic, while the structure of α -U is orthorhombic: we use directionally-averaged values of the parameters in this case. We have chosen chromium, not a superconductor, amongst our examples, because of its anomalously large Grüneisen parameter and predicted thermo-acoustic response; the magneto-acoustic effect in a normal metal is purely magnetorestrictive, and much smaller than the phase-transition-related effect in a superconductor^{**)} .

The magneto-acoustic contribution to the monopole signal amplitude is velocity independent, while the thermo-acoustic contribution, proportional to dE/dx , is approximately linear in β . The signal energy as a function of β , Eq. (6.1), is therefore a parabola. Following tradition in work related to gravitational wave detectors, we shall choose to express the signal energy in units of temperature $T_s = E_s/k_B$. For the sake of definiteness, we have taken $\ell = 10$ cm as the linear dimension of the antenna in Eq. (4.1a). The result of multiple substitutions of Eqs. (5.11), (4.50) (4.45), (4.14), (3.5) and (3.2) into Eqs. (4.1) can be expressed, for the five metals of our choice, as:

$$T_s(Al) = 1700^\circ K (\beta - \beta_0)^2 ; \beta_0 = 1.0 \times 10^{-4} \quad (6.2a)$$

$$T_s(Nb) = 1960^\circ K (\beta - \beta_0)^2 ; \beta_0 < 10^{-6} \quad (6.2b)$$

^{*)} This is not the case for Nb, for which an accidental cancellation in Eq. (4.53) makes $\partial H_1/\partial p$ very close to zero, and hard to estimate. In practice, the thermo-acoustic effect dominates for reasonable values of β , and the uncertainty in the Nb magneto-acoustic effect is irrelevant.

^{**)} The antiferromagnetic properties of Cr, presumably involved in its large value of γ at low temperatures, may play a role (which we have not investigated) in the sonic response to monopoles.

$$T_s(V) = 1850^\circ K (\beta + \beta_0)^2 ; \quad \beta_0 = 2.7 \times 10^{-5} \quad (6.2c)$$

$$T_s(\alpha-U) = 2.3 \times 10^5 {}^\circ K (\beta - \beta_0)^2 ; \quad \beta_0 = 1.4 \times 10^{-4} \quad (6.2d)$$

$$T_s(Cr) = 8.1 \times 10^4 {}^\circ K \beta^2 ; \quad \beta_0 \equiv 0 \quad (6.2e)$$

The $\beta = 0$ signal in the above expressions, absent for Cr, is the purely magneto-acoustic effect. For $\beta \gg \beta_0$, only the thermo-acoustic effect matters. Of the four superconducting elements we have considered only vanadium expands via both the magnetic and thermal effects of the monopole. For Al and α -U, the thermal and magnetic acoustic sources have opposite signs, and there is a material-dependent value $\beta = \beta_0$ at which the two effects cancel. If our estimates are not very incorrect, Nature is perverse enough to have placed this "zero" in the interesting range $\beta = 10^{-5}$ to 10^{-3} of monopole velocities^{*}). The functions $T_s(\beta)$ of Eqs. (6.2) are plotted in Fig. 6.1.

Of the superconducting metals we have considered, α -U appears to be the most sensitive to monopoles, except in a narrow range $\beta \sim \beta_0(U)$. A word of caution should be placed on our considerations for this particular material, quite independently of the technical difficulties that may arise in constructing large-mass uranium antennas. The properties of cold uranium, i.e., its superconducting transition temperature, are debatable. Its peculiar and extreme behaviour may be related to the existence of many different phases at low temperature, that are not fully understood⁵⁰⁾. With the controversial exception of α -U, chromium - not a superconductor - appears to be the most monopole-sensitive material for $\beta \geq 10^{-5}$, presumably encompassing the whole range of conceivable monopole velocities.

The thermal noise energy in a resonator is $k_B T$ per mode. Should one have to fight this noise level with no extra weaponry, sonic detectors of monopoles would have to operate at extremely low temperatures, as indicated by Fig. 6.1. But in a high Q antenna, the random thermal noise is "well organized" or "predictable", in the following sense⁵¹⁾. Take as a measure of the antenna's random thermal motion, in a mode with period τ , the mean square displacement $\langle x^2 \rangle$. Because of the long memory (high Q value) of the resonator, this quantity,

^{*}) Whether magnetic and thermal effects are of opposite signs in the three principal directions of a single α -U crystal, we do not know. We have found "directional" data on γ ⁴⁹⁾ ($\gamma_a = -27.2$, $\gamma_b = -4.6$, $\gamma_c = 3.2$) but not on $\partial H_{c1} / \partial \sigma$, with σ a unidirectional stress.

Fig. 6.1: Signal temperature per eigenmode for a variety of materials at very low temperatures, as a function of monopole velocity. T_{eff} (today) and T_{eff} (tomorrow) indicate effective total noise temperatures that have been, or soon will be, achieved in practice. Also shown in the figure is the signal temperature for a minimum ionizing track in Cr, see discussion in Chapter 7.

if measured at two consecutive times separately by a small interval Δt , varies on average only a fraction $\Delta t/Q\tau$ of its mean value. A signal arriving in the time interval Δt is observable if it competes favourably with this effectively reduced thermal noise. This does not mean that noise can be indefinitely reduced by shortening Δt ; there are other sources of noise in the antenna readout system, among them "wide band noise", that behave as $(\Delta t)^{-1}$. We shall not dwell on the optimization of the filter parameters. Suffice it to say that one may characterize the minimum optimized noise by an effective temperature⁵²⁾:

$$T_{\text{eff}} = \frac{2T}{\beta_c Q} + 2T_n \quad (6.3)$$

Here T is the real temperature of the antenna, β_c is a dimensionless coupling constant: the ratio of the electrical energy in the transducer to the total energy in the antenna, and T_n is the noise temperature of the electronic amplifiers. In large existing gravitational wave antennas, values of the parameters

$T \sim 1^{\circ}\text{K}$, $\beta_c \sim 10^{-2}$, $Q > 10^6$ have been reached. Commercial RF-SQUID amplifiers have $T_n \sim 10^{-4}^{\circ}\text{K}$. The above numbers correspond to $T_{\text{eff}} \sim 4 \cdot 10^{-4}^{\circ}\text{K}$, shown in Fig. 6.1 as T_{eff} (today). Users of the Rome-Frascati 5-Ton aluminium antenna at CERN⁵²⁾ are planning to improve its parameters to $T \leq 0.03^{\circ}\text{K}$, $\beta_c \sim 3 \cdot 10^{-2}$, $Q \geq 10^7$, and to develop a dc SQUID with noise temperature $T_n \sim 10^{-7}^{\circ}\text{K}$. This would correspond to $T_{\text{eff}} \sim 3 \cdot 10^{-7}^{\circ}\text{K}$, shown in Fig. 6.1 as T_{eff} (tomorrow). The figure shows that in between today's and tomorrow's noise levels, there is ample room for compromises that would maintain large monopole signal to noise ratios for targets of different metals and sizes.

We have concentrated in this chapter on pure-metal detectors, the only materials for which we have found sufficient information on the relevant thermal and magnetic properties. Possibly, alloys do exist that are much more sensitive than any pure metal to the passage of monopoles, but we have been unable to find the corresponding values of Grüneisen parameters and $\partial H_{c1}/\partial p$ (alloys are typically Type II superconductors). Moreover, not only famous bells, but also high Q antennas are characteristically made of some well chosen alloy. The making of high Q resonators is more an art than a science, and we must conclude that we have at present no specific suggestion for the optimal material to detect the sound of monopoles.

7. - NON-THERMAL NOISE, COSMIC RAY SIGNALS, AND THE SENSITIVITY TO NEWTORITES

A variety of backgrounds of non-thermal origin may be misinterpreted as a signal in a sonic monopole antenna. Gravitational waves and certain hypothetical particles - other than monopoles - may also trigger the detector. For reasons which should become clear in the rest of this chapter we discuss a detector that may look like the one in Fig. 7.1, conceived as a "telescope" for rare cosmic travellers (particles or waves). The cylinders, shown in Fig. 7.1 in three different orientations, are essentially small gravitational wave antennas of the existing type. The flat plates between and around the cylinders are ionization-sensitive devices, i.e., sandwiches of scintillators and some high density material, conventionally used in the detection of charged particles or electromagnetic showers. All of the gaps between gravitational antenna-like moduli should in principle be equipped with these devices. The whole ensemble would be bathing in a cryostat.

The characteristic frequencies, ω_n , of the low order vibrational modes of a resonant detector (of typical size a few tens of centimeters) are in the tens of thousands of Hertz. The time resolution of such detectors cannot be smaller than the corresponding period $\Delta t_n \gtrsim 2\pi/\omega_n$. At these frequencies, the prevalent seismic motions are well below thermal noise - only an occasional microseism or

local disturbance can mock up an interesting signal. In gravitational wave antennas, these backgrounds are suppressed by good suspension and buffering, and they are put into anticoincidence with sensitive accelerometers. Our detector is in a sense a modular gravitational wave antenna. For a fixed total mass of resonator material, the gravitational signal to thermal noise is fixed. But for the detection of gravitational waves, there are two obvious advantages to the modular structure of Fig. 7.1. First, the amplitude ratios of a coincident signal in all the differently oriented cylinders may help to disentangle the direction of a possible gravitational wave. Second, it is unlikely for a travelling vibrational disturbance, or mechanical "crack" in the detector to excite all the independent modules simultaneously: for a sufficiently good time resolution, noise other than thermal is very efficiently suppressed.

Fig. 7.1: A possible geometry for a direction-sensitive gravitational wave antenna with the capability of detecting monopoles and newtorites.

A major problem of superconducting-loop monopole detectors concerns the screening against variations of the ambient terrestrial magnetic field. The larger the loop, the smaller is the change in ambient field necessary to induce a current corresponding to a quantum unit of flux, and thereby to create a spurious signal. In the detector proposed here, the corresponding "loop size" is the inherent size of the vortices ($10^{-6} - 10^{-4}$ cm), so only a very large change in the external field (e.g., $\Delta H \sim H_{c1}$ in Type II superconductors) could be a source of spurious fluxons. A change ΔH in the external field, on the other hand, would produce a pressure change $(\Delta H)^2/8\pi$ on the surface of a superconductor, see Eq. (4.9). This may excite the eigenmodes if $\Delta H(\vec{x}, t)$ has sufficiently large Fourier components corresponding to the mode's wave number and frequency. This high frequency background can be suppressed, if necessary, with use of standard radio frequency shielding techniques.

There is yet another source of spurious signals common to superconducting loops and superconducting acoustic detectors: random motion of trapped flux, that diminishes as the temperature approaches absolute zero. This background, as well as possible releases of mechanical stress, could in a modular detector be measured module by module, allowing one to predict the probability of a spurious signal along a line of detectors (a false monopole) or in all detectors in coincidence (a false gravitational wave).

One limitation that an acoustical detector such as that of Fig. 7.1 shares with an array of superconducting loops is the difficulty in measuring monopole velocities. The problem is probably not as serious as with an array of superconducting loops. The time resolution of a sonic antenna module is at best of the order of $\Delta t \sim \ell/c_s$, with ℓ the module's dimension in the direction of the monitored vibrational mode and c_s the appropriate sound speed. For two modules at distance L to react to the passage of a monopole of speed c at distinguishable times ($t' - t = L/\beta c$), it is necessary that $\beta < Lc_s/\ell c$. Even for a large choice of the ratio L/ℓ , say 30, this implies $\beta < 5 \cdot 10^{-4}$, for a typical sound velocity $c_s \sim 5$ km/sec. Even small improvements towards a better time resolution, such as the detection of higher vibrational modes, could be helpful.

A minimum ionizing track, such as a cosmic ray muon, should be visible in the counters that have been represented in Fig. 7.1 as square plates. The energy loss of such a particle in the sonic resonators is $dE/dx \sim 2.6(\rho/\rho_0)\text{MeV/cm}$, where ρ/ρ_0 is the density of the material relative to water. The energy loss Eq. (7.1) induces a thermo-acoustic sonic pulse that is typically smaller than that expected for a monopole, which loses energy to heating at a rate of the order of $\sim 500 (\text{MeV/cm}) (n_c/3)^{2/3} (\beta/10^{-3})$, see Eq. (3.5).

In an ideal situation the effective noise temperature of the sonic antenna would be so low, that even minimum ionizing tracks could be detected: this would allow a continuous calibration of the thermo-acoustic response of the detector. If the detector material is not superconducting, as in the case of a chromium antenna, this is essentially all the calibration one needs. We have illustrated in Fig. 6.1 the signal temperature of a minimum ionizing track in Cr ($\rho = 7.2\rho_0$); such signals should be visible in "tomorrow's" detectors.

In a detector of a few cubic meters a cosmic ray neutrino could, at most a few times per year, interact and produce a sufficiently energetic ionizing shower to mock up a monopole signal. It is highly unlikely for such an event to excite several aligned sonic modules, and not to register in the ionization detectors surrounding them. The same argument applies to natural radioactivity-induced signals.

To end this chapter with a flight of fancy, we analyze one of the conceivable unconventional signals that a detector such as that in Fig. 7.1 could be sensitive to. As progress is made in elementary particle physics, theorists feel compelled to introduce new particles. The unification of weak and electromagnetic interactions led to the prediction of charmed quarks and of intermediate vector bosons with a priori known masses. The same theory requires the existence of the top quark and of elementary scalars, yet to be found. Further unification with the strong interactions suggests the existence of monopoles and the decay of the proton, mediated by superheavy bosons. Similarly, new particles are likely to be required by a consistent future theory of quantum gravity, unified or not with the rest of the fundamental forces. Among the new particles, there may well be some that interact with ordinary matter only gravitationally^{*)}. We refer to these "particles" as *newtorites*, to encompass the possibility that they exist not only in "elementary" form but also in meteorite-like aggregates held together by their own self-interactions. Much of this gravitationally interacting matter could remain in our Universe, having "decoupled" from ordinary matter at some very early "Planck" time. Most of the gravitational mass of galaxies, clusters of galaxies, and the Universe as a whole is unaccounted for. It could all be in the form of newtorites: particles, dust, meteorites or other ghostly aggregates of this new type of matter. We could be continuously bombarded by newtorites as heavy as enormous conventional meteorites, without even noticing. A gravitational wave antenna, modular or not, may "notice".

^{*)} S.L. Glashow informs us that this possibility has been entertained by A.D. Linde, who, following national tradition, dubbed the new matter "Type II". We avoid this nomenclature that could be confusing in our context.

Newtonites if they exist, could be responsible for the invisible halo in our own galaxy, or be slowly moving in the universal reference system in which the 3°K black-body radiation is isotropic. In both cases they would be expected to cross terrestrial detectors at a speed $\beta = v/c \sim 10^{-3}$. We therefore feel justified in using Newtonian gravitational mechanics in the estimate of newtorite signals in a resonant detector. This estimate, as we shall see, is quite similar to that of the effect of a monopole, discussed in Chapter 5.

The general equation for undamped vibrations of a solid object subject to a gravitational force density $\vec{f}$ is analogous to Eqs. (5.3) and (5.4):

$$\ddot{\vec{u}} + \vec{\nabla} \cdot \vec{u} = \vec{f} / \rho \quad (7.1)$$

We expand the displacement $\vec{u}$ in normal modes as in Eq. (5.7), to obtain the eigenmode amplitude equation of motion Eq. (5.8), with a source

$$s_n(t) = \frac{1}{\rho V} \int_V \vec{u}_n \cdot \vec{f} d^3x \quad (7.2)$$

The solution to Eq. (5.8) takes again the form of Eq. (5.18), with a vanishing $s_n(\infty)$ (the passage of the newtorite leaves behind no static deformation of the target), and with A_n given by

$$A_n = \int_{-\infty}^{\infty} s_n(t) dt \quad (7.3)$$

This time the integral starts at minus infinity since the newtorite affects the body long before it hits it. The signal energy and temperature in each eigenmode are given by Eq. (5.19) as

$$E_n = \frac{1}{2} \rho V A_n^2 \quad (7.4)$$

$$T_n = E_n / k_B \quad (7.5)$$

Next, we must compute A_n with use of the explicit form of a newtonian force per unit volume $\vec{f}$. For simplicity we assume the newtorite's size to be much smaller than the detector's dimensions, in which case the gravitational pull is that of a point particle of mass m :

$$\vec{f} = - \frac{G m \rho}{r^3} \vec{r} \quad (7.6)$$

$$r^2 = (\vec{x}_T - \vec{x}_T^0)^2 + (x_L - \beta c t)^2 \quad (7.7)$$

Here $G = 6.67 \cdot 10^{-8} \text{ cm}^3 \text{g}^{-1} \text{sec}^{-2}$, is the gravitational constant, $\vec{x}_T - \vec{x}_T^0$ are the transverse co-ordinates of a volume element of the detector relative to an arbitrarily chosen fixed point along the monopole track, and $x_L - \beta c t$ is the corresponding longitudinal co-ordinate relative to the instantaneous position of the monopole (both a value of x_L and the $t = 0$ instant may be arbitrarily chosen). Substitute Eqs. (7.2) into Eq. (7.3) with the force given by Eqs. (7.6) and (7.7) and perform the time integral to obtain:

$$A_n = -2 \frac{G m}{V \beta c} \int_V \frac{\vec{u}_n \cdot (\vec{x}_T - \vec{x}_T^0)}{(\vec{x}_T - \vec{x}_T^0)^2} d^3 \vec{x} \quad (7.8)$$

The signal is a fairly complicated function of the newtorite's trajectory. To estimate it in detail we again refer to an asymmetric parallelepipedical target as that of Fig. 5.1a, and take the newtorite, for the sake of illustration, to hit the target at a large angle: $\theta \approx 90^\circ$ in Fig. 5.1a. We concentrate on the "longitudinal" vibrations along the x direction. With use of the explicit modes of Eq. (5.22) we obtain, after some algebra

$$A_n = + \frac{2 \sqrt{2} G m}{\beta c} \frac{\pi}{\ell_1} (I_1 \sin k_n \bar{x}_1 + I_2 \cosh k_n \bar{x}_1) \quad (7.9a)$$

$$I_1 = 1 - \frac{1}{\pi} [\text{si}(k_n(\ell_1 - \bar{x}_1)) + \text{si}(k_n \bar{x}_1)] \quad (7.9b)$$

$$I_2 = \frac{1}{\pi} [\text{ci}(k_n(\ell_1 - \bar{x}_1)) - \text{ci}(k_n \bar{x}_1)] \quad (7.9c)$$

where $\bar{x}_1$ is the position where the newtorite hits, and si and ci are sine and cosine integrals. Even in our simplified example the signal is a fairly complicated function of $\bar{x}_1$ and n . For $n = 1$ and central incidence $I_1 = 0.87$, $I_2 = 0$. The corresponding signal temperature is obtained by substitution of Eq. (7.9a) into Eqs. (7.4) and (7.5):

$$T_s (n=1, \bar{x}_i=0, \theta \approx 90^\circ) \approx 30 \frac{\rho G^2 m^2}{k_B \beta^2 c^2} \frac{l_2 l_3}{l_1} \quad (7.10)$$

Notice that the result is independent of the elastic parameters of the medium and of the sound speed within it. Also, the signal is of the same order of magnitude for all modes, apart from geometrical factors of order unity that we have not specified. For a body that is not as asymmetric as the parallelepiped in our example, it is clear from the foregoing derivation that the result should also be of the form of Eq. (7.10), apart from an overall factor of order unity. To give an example consider an antenna module for which the ratio of lengths in Eq. (7.10) is $l_2 l_3 / l_1 = 10$ cm. The numerical result is

$$T_s = 2.9 \times 10^{-5} \text{ K} \left[\frac{\beta}{10^{-3}} \right]^{-2} \left[\frac{m}{1 \text{ kg}} \right]^2 \quad (7.11)$$

The condition $T_s > T_{\text{eff}}$, with T_{eff} the effective temperature noises of today and tomorrow discussed in Chapter 6, yields for the minimum mass of a detectable newtorite:

$$m(T_{\text{OD}}) > 3.7 \text{ kg} \cdot [\beta / 10^{-3}] \quad (7.12a)$$

$$m(T_{\text{OM}}) > 0.1 \text{ kg} \cdot [\beta / 10^{-3}] \quad (7.12b)$$

Gravitation is a weak force; even a sophisticated detector like the one we are discussing could only detect rather hefty newtorites.

8. - CONCLUSIONS

The results of this paper are summarized in Fig. 6.1, to which we now add a few more comments.

The thermo- and magneto-acoustic process of energy transfer from a monopole to the vibrational eigenmodes of a resonant detector is particularly inefficient. In a 10cm-long voyage through a superconducting metal, at speed $\beta = v/c \sim 10^{-4}$, a monopole would lose about a milli-erg of its kinetic energy. Most of this lost energy is employed to establish a flux tube, and to heat the material. Only

some 10^{-18} to 10^{-17} ergs end up as coherent vibrations in each of the eigenmodes, a fifteen orders of magnitude inefficiency, or worse. This should immediately discourage the hypothetical monopole-listener, were it not because of the fabulous noise-reduction techniques that can be reached in "sonic" detectors similar to existing gravitational wave antennas; an achievement based on a combination of large resonating quality factors, low temperatures, and quiet electronics. At the temperatures that such detectors are to be operated in the near future, and with no externally applied magnetic field, most metals and alloys are superconducting. In superconductors, as we have seen, there is a magneto-acoustic monopole signal that survives to zero monopole velocity. The magneto-acoustic effect is bigger than the estimated thermo-acoustic effect for β smaller than a material dependent $\beta_0 \sim 10^{-4}$. Our estimate of the magneto-acoustic effect is ultimately based on the existence of the quantized, conserved magnetic flux of the monopole, and on known properties of flux tubes and of volume charges in the phase transitions of superconductors. (Direct experimental information about the flux tubes is only available for alloys and the few pure metals that are also Type II superconductors. The high Q materials that would presumably be used in actual practice are metal alloys.) Because of this connection with well established physics, we expect our estimates of the magneto-acoustic effect to be quite reliable. The same cannot be said of the thermo-acoustic effect, which we have conservatively tried to underestimate, and which could be in error by an order of magnitude. For most of the pure metal superconductors we have analyzed in detail, the thermal and magneto-acoustic effects cancel at a particular monopole velocity that happens to lie in the interesting range. Perhaps a purely thermo-responsive material, like chromium, is the most sensible choice for a monopole-listening device.

We have seen that a combination of a sonic monopole detector and a gravitational wave antenna suggests itself. Essentially we envisage making the gravitational wave antenna "modular". For fixed total target mass (and cryogenic equipment), the modularity of the antenna does not change the gravitational wave's signal-to-thermal-noise ratio; but it would help fight noise other than thermal, and make a single antenna modestly directional for single gravitational bursts.

The major considerations (other than price and technical difficulty) in the comparison of a monopole detector of the type we suggest, and other existing or planned detectors, depend on the expected monopole velocities and the conceivable surface areas. Conventional ionization detectors can be deployed in large areas but may let slow monopoles escape undetected. The secondary-ionization detectors based on the mechanism suggested by Drell et al.¹⁷⁾ have a good chance of decreasing the monopole-velocity detectability threshold to $\beta \sim 0(10^{-4})$, perhaps

covering all the range of conceivable velocities of cosmic monopoles. But for monopoles of the lowest speeds that can be entertained in certain theoretical scenarios, such as the heliocentric one¹²⁾, the secondary-ionization detectors would still be operating close to the threshold of detectability, β_{thr} . The precise value of β_{thr} , and the actual shape of the rapidly-varying sensitivity curve close to it, may be subject to a significant theoretical uncertainty. A negative result of a monopole search with a secondary-ionization detector would - in our opinion - not constitute an absolute proof of the absence, to a certain flux level, of slow relic monopoles. The sonic detector we have discussed, on the other hand, is "infallible", in the same sense as superconducting loop detectors: it can "hear" monopoles of any β , and it has the extra advantage of not requiring a sophisticated magnetic shielding. Though it is easy to defend the virtues of hypothetical detectors in their comparison with the limitations of existing ones, we believe that a sonic device could be made much larger than a superconducting loop detector, without running into insuperable technical problems. Finally, a sonic monopole detector can be defended on grounds of its versatility: it may simultaneously operate as a gravitational wave antenna, an observatory for newtonites, and a general collector of surprises from the cosmos.

NOTE ADDED

As this manuscript was being completed, we received a communication on the same subject by A.M. Allega and N. Cabibbo, who briefly estimate the sonic monopole signal in $\mathcal{A}\ell$ gravitational wave antennas. Their results agree with ours in order of magnitude.

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