Ref.TH.3736-CERN

VACUUM ALIGNMENT IN SUPERSYMMETRIC GAUGE THEORIES

Graham M. Shore ^{*)}

CERN -- Geneva

ABSTRACT

The vacuum alignment problem is analyzed in the context of supersymmetric gauge theories with dynamical symmetry breaking. Three cases are distinguished, depending on whether the vacuum expectation value of the weak gauge current superfield in a vacuum characterized by the orientation Ω is zero for all Ω , for some Ω , or for no Ω . In the first case, a non-renormalization theorem is proved to all orders in the weak coupling, showing that the usual criterion of minimizing the vacuum energy density is insufficient to determine the alignment, and possible resolutions of the problem are discussed. The second case is similar, except that the possible alignments are restricted to the range of Ω giving a vanishing VEV and the weak gauge group may then be broken non-minimally. In the third case, supersymmetry is itself broken at the tree level by the weak interactions. The supersymmetric generalization of the Schwinger mechanism for dynamical mass generation is described.

^{*)} Address from 5 October 1983 : Department of Physics, Imperial College, Prince Consort Road, London SW7 2BZ, United Kingdom.

1. - INTRODUCTION

Composite models of quarks and leptons¹⁾, and technicolour theories of electroweak symmetry breaking²⁾ exhibit the following generic structure. Consider a system of massless fermions, "metaquarks", interacting through a confining gauge interaction, "metacolour", with gauge group G_S . This system possesses a large, global, chiral symmetry G , which is spontaneously broken to a subgroup H at energies below the confinement scale of G_S . Although the subgroup H is dynamically determined by the metacolour dynamics, its orientation within G is not. Any subgroup $\Omega H \Omega^{-1}$, where $\Omega \in G$, is equivalent. Now suppose that a further subgroup G_W of G is gauged. The new, "weak" interaction may be treated perturbatively at the energy scale of interest. Determining the relative orientation of the subgroups H and G_W is now an important physical question. This is the "vacuum alignment" problem. The situation is illustrated in Fig. 1. The weak bosons corresponding to broken, gauged generators (region 1) acquire mass through the Schwinger mechanism, while the Goldstone bosons corresponding to spontaneously broken symmetries which are also broken explicitly by G_W (region 2) may acquire mass through weak interactions. The vacuum alignment problem is therefore crucial in determining both the pattern of G_W breaking, and the light particle spectrum of the model.

In this note, we consider these questions in the context of supersymmetric theories. We assume from the outset that strongly interacting, supersymmetric gauge theories exhibit confinement and chiral symmetry breaking, and that this is achieved without spontaneous breaking of supersymmetry. We may then consider a supersymmetric metacolour theory of interacting metaquarks and metasquarks along the lines described above, and determine the effects of a weak gauging of a subgroup of the global chiral symmetry.

It turns out that the physics is very different depending on whether the lowest component of the weak gauge current superfield C^A , defined below, has a vanishing VEV in the unperturbed metacolour theory for all vacuum orientations Ω , for some Ω , or for no Ω .

In the first case, we find that the vacuum degeneracy is not lifted by the weak interactions, i.e., the usual criterion of minimizing the vacuum energy $V(\Omega)$ is not sufficient to determine the relative orientation of H and G_W . This is established to all orders in the weak coupling g_W on the basis of a new non-renormalization theorem, which we prove using superfield techniques.

In the second case, we show that the possible alignments are restricted by vacuum energy considerations to those Ω for which the VEV of C^A vanishes. Within this class, the results described for the first case apply.

In the third case, supersymmetry is spontaneously broken at the tree level by the weak interactions, and the vacuum energy $V(\Omega)$ is therefore non-zero at $O(g_w^2)$. Here, the cancellation mechanism ensures that the only surviving $O(g_w^2)$ contribution to $V(\Omega)$ comes directly from a tree-level term in the perturbed Lagrangian.

We then discuss, again in the superfield formalism, the Schwinger mechanism for the dynamical generation of masses for the gauge bosons and fermions of the weak group G_w . We conclude with a discussion of how the vacuum alignment problem is to be resolved in supersymmetric theories with $\langle C^A \rangle = 0$, and the observation that in some cases, the vacuum alignment can imply a non-minimal breaking of the weak gauge group.

2. - PRELIMINARIES

The Lagrangian for the metacolour interaction may be written as follows:

$$\mathcal{L} = \text{Tr} \frac{1}{g_s^2} \int d\alpha d\theta F^\alpha F_\alpha + \text{h.c.} + \int d\alpha d\theta d\bar{\theta} \bar{Q} e^{\frac{1}{2}} Q \quad (1)$$

Here, F_α is the usual field strength for the metacolour gauge field, Φ^r . The matter fields, which lie in the fundamental representation of the metacolour group (generators τ^r), form N_f chiral multiplets,

$$(Q_\pm)_k = \left(s_\pm + \theta^\alpha q_{\pm\alpha} + \theta^2 F_\pm \right)_k \quad (2)$$

For convenience, we have introduced a compact notation where $Q = (Q_+, Q_-)$ is a $2N_f$ vector, and thus in Eq. (1), $\Phi = \Phi^r T^r$, where

$$T^r = \left(\begin{array}{c|c} \tau^r & 0 \\ \hline 0 & -\tau^r \end{array} \right)$$

Generators of the chiral group G are then $2N_f \times 2N_f$ matrices. The generators of the unbroken subgroup H are denoted by T^i , and the broken generators by X^a .

We first consider the symmetry currents of this theory. These are defined, in functional superfield formalism, as follows, considering the broken currents as an example. The action of the global symmetry operator Ω^a on the action Γ is

$$\Omega^a \Gamma \equiv \int dx \omega^a \Gamma = -i \int dx d\theta \int^a Q \frac{\delta \Gamma}{\delta Q} - i \int dx d\bar{\theta} \int^a \bar{Q} \frac{\delta \Gamma}{\delta \bar{Q}} \quad (3)$$

Then,

$$\omega^a \Gamma = -2i \partial^{\alpha\dot{\alpha}} J_{\alpha\dot{\alpha}}^a \quad (4)$$

defines the current $J_{\alpha\dot{\alpha}}^a$. Explicitly, we find

$$\omega^a \Gamma = i \int dx [D^2, \bar{D}^2] (\bar{Q} e^{\frac{1}{2}} X^a Q) \quad (5)$$

and so,

$$J_{\alpha\dot{\alpha}}^a = -4i [D_\alpha, \bar{D}_{\dot{\alpha}}] (\bar{Q} e^{\frac{1}{2}} X^a Q) \quad (6)$$

Up to a numerical factor, this is just the $\theta\bar{\theta}$ component of the superfield

$$C^a = \bar{Q} e^{\frac{1}{2}} X^a Q \quad (7)$$

This superfield plays a central role in our analysis. We call C^a the "current superfield" corresponding to the generator X^a . Similarly, for the unbroken generators, $C^i = \bar{Q} e^{\frac{1}{2}} T^i Q$.

Now we introduce the weak interactions. Let G^A be the linear combinations of generators of the chiral group G which are to be gauged. A kinetic term for the weak gauge superfield ϕ^A is included in the Lagrangian in the standard way. In addition, the interaction term in $\mathcal{L}$ becomes:

$$\mathcal{L}_{int} = \int dx d\theta d\bar{\theta} \bar{Q} e^{\frac{1}{2}} e g_w \phi Q \quad (8)$$

where $\phi = \phi^A G^A$. Expanding to $O(g_w^2)$, the g_w dependent terms in $\mathcal{L}_{int}$ are

$$\mathcal{L}_{int}(g_w) = \int dx d\theta d\bar{\theta} \left(g_w \phi^A C^A + \frac{1}{4} g_w^2 \phi^A \phi^B C^{\{A,B\}} + \dots \right) \quad (9)$$

where C^A is the weak current,

$$C^A = \bar{Q} e^{\frac{1}{2}} G^A Q \quad (10)$$

and

$$C^{\{A,B\}} = \bar{Q} e^{\frac{1}{2}} \{G^A, G^B\} Q \quad (11)$$

3. - VACUUM ALIGNMENT

The key to resolving the vacuum alignment question was originally found by Dashen³⁾, although not in the context of gauge theories. Weinberg⁴⁾ later gave a general treatment of the problem for gauge theories with dynamical symmetry breaking, and his analysis has since been simplified and extended to technicolour models by Peskin⁵⁾ and by Preskill⁶⁾. Here we follow closely the latter two papers.

Dashen's theorem³⁾ states that the physically preferred vacuum is that which minimizes the contribution of the weak perturbation to the vacuum energy. Explicitly, denote by $|0\rangle$ the H invariant vacuum, and by $|0, \Omega\rangle = U(\Omega)|0\rangle$ the vacuum invariant under $\Omega H \Omega^{-1}$, where $U(\Omega) = \exp i \rho^a(\Omega) X^a$. (Since $|0\rangle$ is H invariant, it is possible to restrict U to depend only on the broken generators X^a). The correct vacuum is found by minimizing the potential

$$V(\Omega) = \langle 0, \Omega | \mathcal{H}_w | 0, \Omega \rangle \quad (12)$$

$$= \langle 0 | U^{-1} \mathcal{H}_w U | 0 \rangle \quad (13)$$

where $\mathcal{H}_w$ is the perturbing Hamiltonian.

In the spirit of Eq. (13), it is technically convenient to regard the subgroup H as fixed, and consider varying the orientation $\Omega^{-1} G_w \Omega$ of the weak gauge group within G . The ultimate physics is clearly the same, and we shall adopt this point of view from now on.

In this approach, we must distinguish the cases where the lowest component of the broken current superfield C^a does or does not have a VEV in the unperturbed metacolour theory. Consider first the case $\langle 0|C^a|0\rangle = 0$. In the supersymmetric gauge theory considered here, $V(\Omega)$ is given to lowest order by the $O(g_w^2)$ vacuum energy diagrams shown in Fig. 2. The first diagram contributes

$$V_1(\Omega) = -\frac{1}{2} g_w^2 \int dx_1 d\theta_1 d\bar{\theta}_1 \int dx_2 d\theta_2 d\bar{\theta}_2 \langle 0|\tau \phi^A(1) \phi^B(2)|0\rangle \times \langle 0,\Omega|\tau C^A(1) C^B(2)|0,\Omega\rangle \quad (14)$$

$$= -\frac{1}{2} g_w^2 \int dx_1 d\theta_1 d\bar{\theta}_1 \int dx_2 d\theta_2 d\bar{\theta}_2 \Delta_{\phi\phi}^{AB}(1,2) \langle 0|\tau C_\Omega^A(1) C_\Omega^B(2)|0\rangle \quad (15)$$

where $C_\Omega^A = \bar{Q} e^\Phi U^{-1}(\Omega) G^A U(\Omega) Q$, and τ denotes the covariant time-ordered product. The second diagram gives

$$V_2(\Omega) = \frac{i}{4} g_w^2 \int dx_1 d\theta_1 d\bar{\theta}_1 \Delta_{\phi\phi}^{AB}(1,1) \langle 0|C_\Omega^{\{A,B\}}|0\rangle \quad (16)$$

However, this is trivially zero since the superfield propagator $\Delta_{\phi\phi}$ is zero when the end-points coincide.

Notice that in these expressions for $V(\Omega)$, valid to lowest order in perturbation theory in g_w , the symbol $|0\rangle$ denotes the unperturbed, strong interaction vacuum. We now show that under the assumption that $|0\rangle$ does not break supersymmetry spontaneously, i.e., $Q_\alpha|0\rangle = 0$, $V(\Omega)$ is identically zero for all vacuum orientations Ω , provided $\langle 0|C^a|0\rangle = 0$.

To see this, work in the superfield Feynman gauge, where the propagator is

$$\Delta_{\phi\phi}^{AB}(1,2) = \Delta(x_1, -x_2) \delta(\theta_1, -\theta_2) \delta(\bar{\theta}_1, -\bar{\theta}_2) \delta^{AB} \quad (17)$$

Then,

$$V(\Omega) = -\frac{1}{2} g_w^2 \int dx_1 dx_2 d\theta d\bar{\theta} \Delta(x_1, -x_2) \langle 0|\tau C_\Omega^A(x_1, \theta, \bar{\theta}) C_\Omega^A(x_2, \theta, \bar{\theta})|0\rangle \quad (18)$$

Now define the operator

$$U(\theta, \bar{\theta}) = e^{-i(\theta^\alpha Q_\alpha + \bar{\theta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}})} \quad (19)$$

Then

$$C_R^A(x, \theta, \bar{\theta}) = U^{-1}(\theta, \bar{\theta}) C_R^A(x, 0, 0) U(\theta, \bar{\theta}) \quad (20)$$

Acting on a supersymmetric vacuum, $U(\theta, \bar{\theta})|0\rangle = 0$, since $Q_\alpha|0\rangle = 0$. So we find

$$\begin{aligned} & \int d\theta d\bar{\theta} \langle 0 | \tau C_R^A(x_1, \theta, \bar{\theta}) C_R^A(x_2, \theta, \bar{\theta}) | 0 \rangle \\ &= \int d\theta d\bar{\theta} \langle 0 | \tau U^{-1} C_R^A(x_1, 0, 0) U U^{-1} C_R^A(x_2, 0, 0) U | 0 \rangle \\ &= \int d\theta d\bar{\theta} \langle 0 | \tau C_R^A(x_1, 0, 0) C_R^A(x_2, 0, 0) | 0 \rangle = 0 \end{aligned} \quad (21)$$

since there is an extra integration $\int d\theta d\bar{\theta}$ left over. This completes the proof that the vacuum energy remains zero to $O(g_w^2)$.

This is the first step of a new non-renormalization theorem, valid to all orders in the weak interaction, with the strong metacolour interaction being treated exactly. The theorem states that $V(\Omega) = 0$ for all Ω , to all orders in g_w , provided $\langle C^a \rangle = 0$.

The proof is by iteration. By assumption, the unperturbed vacuum $|0\rangle$ used above is supersymmetric. Since we have shown that the vacuum energy is then zero to $O(g_w^2)$, we conclude that supersymmetry is not spontaneously broken to $O(g_w^2)$. At the next order of perturbation theory, this means that we may still use a supersymmetric vacuum state to compute the $O(g_w^4)$ contributions to $V(\Omega)$. A typical $O(g_w^4)$ diagram is shown in Fig. 3a. It gives

$$\begin{aligned} V_1^{(4)}(\Omega) &\propto g_w^4 \int dx_1 \dots dx_4 \int d\theta_1 \dots d\theta_4 \int d\bar{\theta}_1 \dots d\bar{\theta}_4 \Delta_{qq}^{AB}(1,2) \Delta_{qq}^{CD}(3,4) \\ &\times \langle 0 | \tau C_R^A(1) C_R^B(2) C_R^C(3) C_R^D(4) | 0 \rangle \end{aligned} \quad (22)$$

$$= g_w^4 \int dx_1 \dots dx_4 \int d\theta, d\theta_3 \int d\bar{\theta}, d\bar{\theta}_3 \Delta(x_1 - x_2) \Delta(x_3 - x_4)$$

$$\times \langle 0 | \tau C_Q^A(x_1, \theta_1 - \theta_3, \bar{\theta}_1 - \bar{\theta}_3) C_Q^A(x_2, \theta_1 - \theta_3, \bar{\theta}_1 - \bar{\theta}_3) C_Q^C(x_3, 0, 0) C_Q^C(x_4, 0, 0) | 0 \rangle \quad (23)$$

after substituting for the propagators, and using the property $U(\theta_3, \bar{\theta}_3)|0\rangle = |0\rangle$. Again this is zero, since after a change of θ integration variables, there remains a spare integration over $\theta, \bar{\theta}$. Further diagrams, such as Figs. 3b-3d, may be inspected, with similar results. Those with a closed superfield propagator vanish immediately, since $\Delta_{\phi\phi}(1,1) = 0$.

The structure is clear. For a vacuum bubble diagram, there is always an extra integration $\int d\theta d\bar{\theta}$ left over after rearranging the integrand. Such diagrams always, therefore, give zero.

Having thus proved that, in the case where the VEV of C^a vanishes, the weak interactions do not lift the degeneracy of the vacua $\Omega H \Omega^{-1}$, the question immediately arises as to what then determines the vacuum alignment. This will be discussed shortly, after we have described the breaking of the weak group G_W and the mechanism for the dynamical generation of mass for the gauge field multiplet.

First, however, we wish to collect a formal expression for the vacuum energy $V(\Omega)$ in a special case of particular physical interest, namely when the chiral symmetry breaking is such that the coset space G/H is a symmetric space. As shown by Peskin⁵⁾, this case includes the three main patterns of symmetry breaking likely to be realized by a strongly interacting gauge theory. The symmetric space assumption is simply the statement that it is possible to define a parity operation such that the broken generators are odd under parity, while the unbroken generators are even. That is, there exists a parity operator P , which preserves the Lie algebra of G , such that

$$P T^i P^{-1} = T^i \quad P X^a P^{-1} = -X^a \quad (24)$$

Equivalently, the commutators of the X^a and T^i have the structure

$$[T, T] = i T \quad [X, T] = i X \quad [X, X] = i T \quad (25)$$

As Peskin demonstrated⁵⁾, under these conditions the dependence on the vacuum orientation Ω may be factorized out of the strong interaction matrix element in Eq. (15). Following his treatment, it is straightforward to show that to $O(g_w^2)$,

$$V(\Omega) = \text{Tr} (U^{-1} G^A U)_X (U^{-1} G^A U)_X \quad (26)$$

$$\times \frac{1}{2} g_w^2 \int dx_1 d\theta_1 d\bar{\theta}_1 \int dx_2 d\theta_2 d\bar{\theta}_2 \Delta_{\psi\psi}(1,2) \langle 0 | \tau (C^T(1) C^T(2) - C^X(1) C^X(2)) \sigma \rangle$$

+ terms independent of $U(\Omega)$

where G_X^A is the projection of G^A onto the broken generators X^a , i.e.,

$$G^A = G_X^A + G_T^A = \text{Tr} [G^A, X^a] X^a + \text{Tr} [G^A, T^i] T^i \quad (27)$$

and $C^T(C^X)$ denotes any single unbroken (broken) current.

In conventional theories, minimizing the vacuum energy is thus reduced to a purely group-theoretical problem, provided only that we can determine the sign of the strong interaction matrix element, which may in fact be shown to be positive. The new feature of supersymmetric theories is that, due to a cancellation between the contributions to the vacuum energy from the weak gauge bosons and their fermionic partners, the strong interaction matrix element vanishes, provided $\langle C^a \rangle = 0$.

So far, we have considered the case where the VEV of C^a vanishes in the unperturbed metacolour theory. The situation changes if $\langle 0 | C^a | 0 \rangle \neq 0$. The main difference here is that the D component of the weak gauge superfield has a VEV proportional to $g_w \langle C_\Omega^A \rangle$, which is equal to $\text{Tr} [U^{-1} G^A U, X^a] \langle C^a \rangle$, since the vacuum is H invariant. If this is zero for some Ω , then, as we shall see, those Ω are energetically preferred and the situation resembles that just described. On the other hand, if $\langle C_\Omega^A \rangle \neq 0$ for all Ω , then supersymmetry is spontaneously broken at the tree level by the weak interactions. This supersymmetry breaking is manifested by an uncanceled $O(g_w^2)$ tree-level contribution to the vacuum energy $V(\Omega)$.

This mechanism is most clearly exhibited in terms of components, but to keep contact with the rest of this paper, we shall remain in the superfield formalism, leaving the component field treatment to a forthcoming paper⁷⁾. In superfields, the essential point is that if the gauge superfield acquires a non-vanishing VEV $\langle \phi^A \rangle \neq 0$, then the conventional covariant gauge-fixing condition breaks down⁸⁾. The resolution is simple. First shift the field, defining $\hat{\phi}^A = \phi^A - \langle \phi^A \rangle$, then introduce the conventional gauge-fixing in terms of the fluctuation field $\hat{\phi}^A$ only. The result of the shift is to modify the interaction part of the Lagrangian as follows:

$$\begin{aligned} \mathcal{L}_{\text{int}}(g_w) = \int dx d\theta d\bar{\theta} \left\{ g_w \hat{\phi}^A (C^A - \langle C^A \rangle) \right. \\ \left. + g_w \langle \phi^A \rangle (C^A - \langle C^A \rangle) + \frac{1}{2} g_w \langle \phi^A \rangle \langle C^A \rangle + \dots \right\} \end{aligned} \quad (28)$$

Here we have displayed only those terms involving a single explicit factor of g_w . As already seen, the omitted terms will not affect the vacuum energy to $O(g_w^2)$.

The first term in Eq. (28) gives rise to a connected $O(g_w^2)$ vacuum energy diagram as before. By the same arguments, this diagram gives zero. In terms of components, the contributions of the connected diagrams of $O(g_w^2)$ involving the weak gauge bosons, fermions and auxiliary fields still cancel. Since $\langle \phi^A \rangle$ is of $O(g_w)$, the second term in $\mathcal{L}_{\text{int}}(g_w)$ is already of $O(g_w^2)$ and so makes no contribution to the vacuum energy at this order. It will, however, contribute at $O(g_w^4)$. Finally, the third term, which no longer involves the dynamical fields, gives a residual $O(g_w^2)$ tree-level contribution to the vacuum energy. Explicitly we find:

$$\begin{aligned} V(\Omega) = \frac{1}{8} g_w^2 \int dx \quad \text{Tr} [U^{-1} G^A U, X^a] \text{Tr} [U^{-1} G^A U, X^b] \langle C^A \rangle \langle C^B \rangle \\ + O(g_w^4) \end{aligned} \quad (29)$$

Clearly, $V(\Omega) \geq 0$. Thus, if it is possible to find an orientation Ω such that $\text{Tr}[U^{-1} G^A U, X^a] \langle C^a \rangle = 0$, then that Ω is energetically preferred. For such Ω , $\langle C_\Omega^A \rangle = 0$ and the non-renormalization theorem just described applies directly.

However, if $V(\Omega) > 0$ for all Ω , then supersymmetry is spontaneously broken at $O(g_w^2)$. The vacuum energy arises from the tree-level term only, and the alignment is determined in the usual way by finding the global minimum of $V(\Omega)$. It is important to emphasize, however, that the cancellation mechanism we have described for the current superfield correlation function is crucial here also, in order to establish that the tree-level term is the only $O(g_w^2)$ contribution to survive.

In this case, the situation becomes more complicated beyond $O(g_w^2)$. First, there are new diagrams to consider, arising from the second term of Eq. (28). But also, since supersymmetry is now broken at $O(g_w^2)$, the vacuum state which must be used to calculate the $O(g_w^4)$ corrections in perturbation theory is no longer supersymmetric, and so the argument for the vanishing of the higher order vacuum bubble diagrams fails.

4. - DYNAMICAL MASS GENERATION FOR THE G_W BOSONS AND FERMIONS

The basic physics involved in the dynamical generation of masses for the weak gauge bosons in a conventional model is the Schwinger mechanism. The Goldstone bosons corresponding to broken currents which are gauged in G_W give rise to poles at $k^2 = 0$ in the vacuum polarization tensor for the weak gauge fields. This shifts the pole in the propagator away from $k^2 = 0$, and the weak bosons thereby acquire a mass. The extra degree of freedom required for the massive vector is provided by the Goldstone boson which disappears from the physical spectrum.

We present here the supersymmetric generalization of the Schwinger mechanism. Again, the result will be derived in the superfield formalism, and in the case $\langle C_\Omega^A \rangle = 0$. The case $\langle C_\Omega^A \rangle \neq 0$ will be discussed in Ref. 7). Consider the perturbation expansion in g_W of the superfield propagator ϕ , as always treating the metacolour interaction exactly. This is represented diagrammatically in Fig. 4. The first of the $O(g_W^2)$ diagrams gives, using Eq. (17) for the Feynman gauge propagator,

$$\begin{aligned} \text{Diagram} &= -g_W^2 \frac{1}{k^4} \langle 0 | \tau C^A(k, \theta_1, \bar{\theta}_1) C^B(-k, \theta_2, \bar{\theta}_2) | 0 \rangle \\ &= - \frac{1}{k^4} \pi_1^{AB}(k, \theta_1, -\theta_2, \bar{\theta}_1, -\bar{\theta}_2) \end{aligned} \quad (30)$$

(Convention-dependent factors of 2 have been suppressed throughout this section). Here, π_1^{AB} is the contribution of the diagram to the superfield vacuum polarization tensor, which may be shown as before to depend only on the differences $\theta_1 - \theta_2$, $\bar{\theta}_1 - \bar{\theta}_2$. Recalling that $\delta(\theta_1 - \theta_2) = \frac{1}{4}(\theta_1 - \theta_2)^2$, we may expand in components

$$\pi_1^{AB} = g_W^2 \langle 0 | D^A(k) C^B(-k) | 0 \rangle \delta(\theta_1 - \theta_2) \delta(\bar{\theta}_1 - \bar{\theta}_2) + \text{lower components} \quad (32)$$

The lower components are, in general, gauge dependent. We use here the notations D and C for the highest ($\theta^2 \bar{\theta}^2$) and lowest ($\theta = \bar{\theta} = 0$) components respectively of the current superfield C . Summing the first series of diagrams in Fig. 4, we thus find

$$\begin{aligned} \Delta_1^{AB} &= -i \delta(\theta_1 - \theta_2) \delta(\bar{\theta}_1 - \bar{\theta}_2) \left(k^2 1 + i g_W^2 \langle 0 | D C | 0 \rangle \right)_{AB}^{-1} \\ &+ \text{lower cpts.} \end{aligned} \quad (33)$$

If $\langle 0 | D^A(k) C^B(-k) | 0 \rangle|_{k=0} \neq 0$, then the pole in Δ_1^{AB} is shifted from $k^2 = 0$. The contribution of these diagrams to the mass matrix is thus

$$(m_1^2)^{AB} = -i g_w^2 \langle 0 | D^A C^B | 0 \rangle|_{k=0} \quad (34)$$

$$= -i g_w^2 k^2 \langle 0 | C^A C^B | 0 \rangle|_{k=0} - i g_w^2 \langle 0 | (D^A - \partial^2 C^A) C^B | 0 \rangle|_{k=0} \quad (35)$$

Now it is not difficult to check that the term $(D^A - \partial^2 C^A)$ is the $\theta=\bar{\theta}=0$ component of the superfield $\frac{1}{4}\{D^2, \bar{D}^2\} (\bar{Q} e^{\Phi} G^A Q)$. Comparing with Eq. (5), we see that it may be rewritten in the form $\frac{1}{4}(-\int dx d\theta \delta^A Q \delta\Gamma/\delta Q + \int dx d\bar{\theta} \delta^A \bar{Q} \delta\Gamma/\delta \bar{Q})$, i.e., as a sum of "counting operators" - the subset of Class IIa composite operators (gauge invariant operators which vanish by the equations of motion) of the general form $(-i \Phi \delta\Gamma/\delta\Phi)$. Such operators have special simplifying properties^{7),9)} which in particular allow the two-point correlation function above to be reduced to a VEV. We find

$$\langle 0 | (D^A - \partial^2 C^A)(k) C^B(-k) | 0 \rangle = -\frac{i}{4} \langle 0 | C^{\{A,B\}} | 0 \rangle \quad (36)$$

where $C^{\{A,B\}} = \bar{Q} e^{\Phi} \{G^A, G^B\} Q$. This term is, however, exactly cancelled by the second series of diagrams, which give a contribution

$$(m_2^2)^{AB} \propto g_w^2 \langle 0 | C^{\{A,B\}} | 0 \rangle \quad (37)$$

to the mass matrix.

We are thus left with the result that the mass matrix for the weak gauge superfield is proportional to the pole at $k^2 = 0$ in the strong interaction correlation function $\langle 0 | C^A(k) C^B(-k) | 0 \rangle$, i.e.,

$$(m^2)^{AB} = -i g_w^2 k^2 \langle 0 | C^A C^B | 0 \rangle|_{k=0} \quad (38)$$

To simplify the discussion, we again assume from this point on that G/H is a symmetric space. The broken currents $J_{\alpha\dot{\alpha}}^a$ couple to the Goldstone bosons π^b as follows:

$$\langle 0 | J_{\alpha\dot{\alpha}}^a | \pi^b \rangle = i k_{\alpha\dot{\alpha}} F_{\pi} f^{ab} \quad (39)$$

Now in this supersymmetric theory, the π belongs to a chiral multiplet. It is the pseudoscalar component, and we call its scalar partner the σ . There is also a Majorana fermion ψ . The pole in the above matrix element arises from the σ intermediate state. Knowing the coupling of $J_{\alpha\dot{\alpha}}^a$ to $|\pi^b\rangle$, we can deduce the coupling of the lowest component C^a of the broken current superfield to $|\sigma^b\rangle$, using the (anti-) commutation relations for the supersymmetry generators with components of a chiral supermultiplet. We find:

$$\langle 0 | C^a | \sigma^b \rangle = -i F_{\pi} \delta^{ab} \quad (40)$$

Evaluating the pole at $k^2 = 0$ in $\langle 0 | C^A(k) C^B(-k) | 0 \rangle$ by saturating with the σ thus gives

$$\langle 0 | C^A C^B | 0 \rangle_{\sigma} = -i \frac{1}{k^2} F_{\pi}^2 \text{Tr}(G_x^A G_x^B) \quad (41)$$

We therefore conclude that the common mass matrix for the gauge bosons and fermions is

$$(m^2)^{AB} = g_W^2 F_{\pi}^2 \text{Tr}(G_x^A G_x^B) \quad (42)$$

or, in the vacuum parametrized by Ω ,

$$(m^2)^{AB} = g_W^2 F_{\pi}^2 \text{Tr}(U^{-1} G^A U)_x (U^{-1} G^B U)_x \quad (43)$$

In terms of the Wess-Zumino gauge component fields $(A_{\alpha\dot{\alpha}}, \lambda_{\alpha}, \bar{\lambda}_{\dot{\alpha}}, D)$ of the weak gauge superfield ϕ , this mass generation mechanism is understood as follows. The gauge fields $A_{\alpha\dot{\alpha}}$ corresponding to broken generators couple derivatively to the π , the λ_{α} couples to the ψ , while the D couples to the σ and becomes dynamical. The poles in the respective propagators are all shifted to the common mass (43). The Goldstone chiral multiplet disappears from the spectrum, providing the additional degrees of freedom required by the massive vector supermultiplet, which comprises a massive pseudovector, a massive fermion and a massive scalar.

5. - DISCUSSION

Consider first the case where $\langle C^a \rangle = 0$ in the unperturbed metacolour theory. Comparing Eqs. (43) and (26), we see that there exists a formal proportionality between the vacuum energy and the trace of the gauge field mass matrix, i.e.,

$$V(\Omega) \propto \text{Tr} (m^2)^{AB} \quad (44)$$

In a conventional theory, this makes sense. The preferred vacuum is that which minimizes the trace of the vector boson mass matrix. This is the precise statement underlying the intuitive picture that the vacuum aligns so that the weak group G_W is broken "as little as possible".

However, in the supersymmetric theory, the constant of proportionality is zero, to all orders in the weak interactions. This leaves us with several possibilities. If the vacuum alignment is to be determined by vacuum energy considerations, then clearly some supersymmetry breaking must occur. This could arise in several ways:

- (i) the model could be changed to incorporate soft, explicit supersymmetry breaking at the metacolour level. This would (at least partly) lift the vacuum degeneracy found here. It would presumably be straightforward to extend our analysis to consider this case;
- (ii) spontaneous supersymmetry breaking by the metacolour interactions may occur, i.e., the unperturbed vacuum $|0\rangle$ would not satisfy $Q|0\rangle = 0$;
- (iii) spontaneous supersymmetry breaking by weak instantons may be possible. This is not excluded by our non-renormalization theorem.

These possibilities for modifying the theory, or invoking hypothetical supersymmetry breaking mechanisms, do not, however, address the question of how the vacuum aligns in the case $V(\Omega) = 0$ for all Ω . It appears to us that in such a case where the candidate vacua are not distinguished by the criterion of vacuum energy, the dynamically preferred vacuum will be that which in some sense gives rise to the lightest spectrum of excitations. After all, if we imagine, for example, raising the temperature of the system from zero, presumably particle excitations occur when the threshold is passed for particle production in the most favourable vacuum. It is therefore quite likely that in this more sophisticated analysis of the dynamics, the criterion for vacuum alignment would remain as before, i.e., minimizing the trace of the gauge field mass matrix, and so again minimizing the extent of G_W breaking.

In the second case, where $\langle C^a \rangle \neq 0$ but $\langle C_\Omega^A \rangle = 0$ for some Ω , similar results apply. However, in this case, the allowed values of Ω are already restricted by vacuum energy considerations to satisfy $\text{Tr}[U^{-1}G^A U, X^a] \langle C^a \rangle = 0$. It follows that unless this range of $U(\Omega)$ includes that required to minimize the trace of the gauge field mass matrix, the chosen vacuum will not be such as

to minimize the breaking of G_W . We therefore conclude that in supersymmetric theories the weak gauge group may be broken more than expected from the "minimal breaking" intuition.

Finally, in the supersymmetry breaking case, where $\langle C^a \rangle \neq 0$ and $\langle C_\Omega^A \rangle = 0$ for all Ω , the vacuum energy is non-zero at $O(g_W^2)$, and the alignment is determined by the usual Dashen criterion. Again, non-minimal G_W breaking is likely.

The analysis shown here has been entirely in terms of the superfield formalism. While this is elegant and indispensable for proving general results, such as the non-renormalization theorem, its very compactness may sometimes tend to obscure the underlying physics. To some extent, this is the case here. In an extended version of this paper⁷⁾, currently in preparation, we shall include explicit component field derivations of our main results, and a thorough description of the supersymmetric generalization of the Schwinger mechanism, as well as discussing a number of subtle points which have necessarily been glossed over in this short summary.

ACKNOWLEDGEMENTS

I should like to thank Gabriele Veneziano for many useful discussions on this topic.

Added Note:

As this work was being completed, we received a paper by Mattis¹⁰⁾ discussing the patterns of symmetry breaking in gauged, supersymmetric, non-linear σ models. Such models are interesting as the low-energy effective Lagrangians for the type of theory considered here. The relationship between these complementary approaches is not completely straightforward, but a superficial comparison between our results and those of Ref. 10) seems to indicate a general agreement on the basic physics.

REFERENCES

- 1) For a review, see:
M.E. Peskin, Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn (1981), Ed. W. Pfeil;
For recent supersymmetric developments, see, e.g.:
R. Barbieri, A. Masiero and G. Veneziano, Phys. Lett. 128B (1983) 179;
W. Buchmüller, R.D. Peccei and T. Yanagida, Phys. Lett. 124B (1983) 67, and Munich preprints MPI-PAE/Pth 28/83 and 41/83 (1983).
- 2) For a review, see:
E. Farhi and R. Jackiw, eds., "Dynamical Gauge Symmetry Breaking", World Scientific Publishing Co., (1982);
M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. B189 (1981) 575;
S. Dimopoulos and S. Raby, Nucl. Phys. B192 (1981) 353.
- 3) R. Dashen, Phys. Rev. 183 (1969) 1245 and Phys. Rev. D3 (1971) 1879.
- 4) S. Weinberg, Phys. Rev. D13 (1976) 974 and Phys. Rev. D19 (1979) 1277.
- 5) M.E. Peskin, Nucl. Phys. B175 (1980) 197.
- 6) J. Preskill, Nucl. Phys. B177 (1981) 21.
- 7) G.M. Shore, in preparation.
- 8) S. Ferrara and O. Piguet, Nucl. Phys. B93 (1975) 261.
- 9) G.M. Shore, CERN preprint TH. 3593 (1983), to appear in Nucl. Phys. B.
- 10) M.P. Mattis, SLAC-PUB-3132 (1983).

FIGURE CAPTIONS

- Fig. 1 : Diagram showing the relation of the groups G , H and G_W discussed in the text. The arrows indicate the possible orientations $\Omega H \Omega^{-1}$ of the unbroken subgroup within G . It has been assumed for simplicity of presentation that the global symmetry group left invariant by the weak interaction is simply G_W itself.
- Fig. 2 : Vacuum energy diagrams of $O(g_w^2)$. The shaded blobs denote strong interaction matrix elements of the current superfield.
- Fig. 3 : Some vacuum energy diagrams of $O(g_w^4)$.
- Fig. 4 : Diagrammatic expansion for the dressed gauge superfield propagator $\tilde{\Delta}_{\phi\phi}^{AB}(1,2)$.

FIG. 1

(a)

(b)

FIG. 2

(a)

(b)

(c)

(d)

FIG. 3

$$\tilde{\Delta}_{\Phi\Phi}^{AB}(1,2) = \begin{array}{c} \text{A} \quad \text{B} \\ \text{~~~~~} \text{~~~~~} \\ 1 \quad \quad 2 \end{array} + \begin{array}{c} \text{~~~~~} \text{~~~~~} \end{array} + \begin{array}{c} \text{~~~~~} \text{~~~~~} \end{array} + \dots$$

FIG. 4