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INCLUSIVE DECAYS OF HEAVY FLAVOURS

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1. INTRODUCTION

The study of weak decays of heavy flavoured particles is important as a source of information on many interesting aspects of the $SU(3) \otimes SU(2) \otimes U(1)$ standard theory. Strong and electroweak effects all contribute to the determination of the decay properties. Much light can be shed on important new and old problems such as the understanding of parton dynamics, the riddle of non-leptonic weak decays of strange particles, and the determination of fundamental parameters in the theory such as the K-M mixing angles. The inclusive (semileptonic and total) decay rates are especially important in that they are the simplest and imply the least amount of model dependency. Many interesting problems are already met at this level, some of them not yet clarified, so that we shall concentrate on inclusive rates in this lecture.

2. PRELUDE: THE τ LEPTON

The decays of the τ lepton offer an example in a simpler context of a two-step strategy which will be developed in the following for heavy flavour decays.

Step one is the prediction of the leptonic rate (the analogue of the semileptonic rate for quark decay). Neglecting the final lepton masses this is simply obtained by rescaling the muon-decay rate:

$$\Gamma(\tau \rightarrow \nu_\tau \ell \nu_\ell) = \left(\frac{m_\tau}{m_\mu} \right)^5 \Gamma_\mu . \quad (1)$$

If one takes from experiment the value of the leptonic branching ratio¹:

$$B_\ell(\tau)_{\text{exp}} = (0.176 \pm 0.016) , \quad (2)$$

the prediction of the leptonic rate can be translated into a value for the τ lifetime:

$$\tau(\tau) = \tau(\mu) \left(\frac{m_\mu}{m_\tau} \right)^5 B_\ell(\tau) = (2.81 \pm 0.25) \times 10^{-13} \text{ s} \quad (3)$$

to be compared with the measured value (presented at the Brighton Conference)

$$\tau(\tau)_{\text{exp}} = (3.20 \pm 0.41 \pm 0.35) \times 10^{-13} \text{ s} \quad [\text{MARK II}] . \quad (4)$$

Step two is the theoretical evaluation of the leptonic branching ratio, or equivalently of the total rate, which is more difficult due to strong interactions and final quark mass effects. For massless final quarks and leptons one has:

$$B_\ell(\tau) = \left[2 + 3 \left(1 + \frac{\alpha_s(m_\tau^2)}{\pi} + \dots \right) \right]^{-1} \approx 0.19 . \quad (5)$$

The factor of 3 in the quark channel is from colour (τ lepton decays thus measure the number of colours), $(1 + \alpha_s/\pi + \dots)$ is the same Quantum Chromodynamics (QCD) correction that applies to $R_{e^+e^-}$ (both referring to the production of a quark pair from a current). Finally we observe that relatively large values for the u and d quark masses are disfavoured by the data, because they would lead to $B_\ell \gtrsim 0.2$. Light quarks are rather indicated by experiment which we interpret as due to the fact that in the final state mostly light mesons are found.

3. SEMILEPTONIC WIDTH OF A HEAVY QUARK

According to the strategy outlined above, one first tackles the problem of the semileptonic width of a heavy flavoured particle. When the momentum of the lepton pair is sufficiently large with respect to the hadron binding energy a parton description suggests that the rate for semileptonic decay is reduced to the corresponding quark decay: $Q \rightarrow q + \ell + \nu_\ell$. Deviations from the parton model associated with interference with spectators and "annihilation" are absent or suppressed in this mode, so that quark decay is appropriate

in most cases. (One possible exception is the $F^\pm$ charmed meson: see in the following.) We thus concentrate on the decay of a heavy free quark. The semileptonic width is given by

$$\Gamma_{s\ell}(Q) = \sum_q |U_{Qq}|^2 I\left(\frac{m_q}{M_Q}, \frac{m_\ell}{M_Q}, 0\right) \left(\frac{M_Q}{m_\mu}\right)^5 \Gamma_\mu \times \left\{ 1 - \frac{2}{3} \frac{\alpha_s}{\pi} \left(\pi^2 - \frac{25}{4} \right) f\left(\frac{m_q}{M_Q}\right) + \dots \right\},$$

where U_{Qq} is the $Q \rightarrow q$ entry of the quark mixing matrix², $I(x,y,z)$ is the three-body phase-space factor³, and the last bracket arises from the QCD leading correction for real and virtual gluon emission⁴. The function $f(x)$ decreases monotonically from $f(0) = 1$ to $f(1) = 0.41$. The QCD correction for $\Lambda = 250$ MeV is $\sim 25\%$ for $c \rightarrow s$ and $\sim 15\%$ for $b \rightarrow c$.

The main theoretical uncertainties of $\Gamma_{s\ell}(Q)$ arise from a) the value of M_Q [as $\Gamma_{s\ell} \sim M_Q^5$] and b) the incidence of bound-state effects. Information on both can be obtained from a study of the lepton spectrum through a bound-state model⁵. According to Fig. 1 for Q bound in a meson, $\Gamma_{s\ell}$ is given by a convolution of the decay width in flight of Q (taken as off shell, with virtual mass W and momentum $\vec{q}$) with the probability of finding Q with momentum q in the meson:

Fig. 1

$$\Gamma_{s\ell} = \int d^3q |\psi(q)|^2 \Gamma_{s\ell}^Q(\vec{q}, W). \quad (7)$$

Treating the spectator as an on-shell particle one immediately obtains

$$W^2 = M_B^2 + m_{sp}^2 - 2M_B \sqrt{\vec{q}^2 + m_{sp}^2}, \quad (8)$$

where M_B and m_{sp} are the meson and spectator masses. Similarly, for the lepton spectrum:

$$\frac{d\Gamma_{s\ell}}{dE} = \int d^3q |\psi(q)|^2 \frac{d\Gamma_{s\ell}^Q}{dE}(\vec{q}, W, E) . \quad (9)$$

The QCD correction of order α_s to $d\Gamma_{s\ell}^Q/dE$ is included, as computed by Cabibbo et al.⁶ and Corb⁷.

The main virtue of this bound-state model is that it provides us with a parametrization of the Fermi motion of the heavy quark inside the meson in a way that fully respects the kinematics. We take a Gaussian form for the wave function:

$$\psi(q) = (\sqrt{\pi} p_F)^{-3/2} \exp\left(\frac{-\vec{q}^2}{2p_F^2}\right) . \quad (10)$$

p_F fixes at the same time the shape of the spectrum near the end point and the effective value of the Q mass: $M_Q \simeq \langle W \rangle$.

3.1 Charm Decay

For charm⁸ the electron spectrum in D decay has been measured by the DELCO Collaboration at SLAC. The results⁵ on p_F can be read from Figs. 2 and 3. The spectator and strange quark masses were taken in the ranges $m_{sp} = (0-150)$ MeV, $m_s = (300-500)$ MeV, which were chosen considering that the hadronic part of the final state of D semileptonic decays is mainly made up of a kaon plus pions. The value of α_s (0.38) is only important for the total width and not for the spectrum shape. Small values of p_F are indicated by the spectrum, in the range

$$p_F \lesssim 300 \text{ MeV} , \quad (11)$$

with preference for $p_F \simeq (0-200)$ MeV. The information obtained from the spectrum can be used to evaluate $\Gamma_{s\ell}$ for D mesons. With $\theta_C = 0$, $m_{sp} = (100-150)$ MeV, $m_s = (300-400)$ MeV, $p_F = (0-300)$ MeV, $\alpha_s = 0.20-0.36$ (corresponding to $\Lambda = 100-400$ MeV), one obtains:

$$\Gamma_{s\ell}^D = \cos^2 \gamma (1.1-3.4) \times 10^{11} \text{ s}^{-1} . \quad (12)$$

We follow the Maiani definition of mixing angles⁹:

$$\begin{aligned} u &\rightarrow \cos \beta d_c + \sin \beta b \\ c &\rightarrow \cos \gamma e^{i\delta} s_c + \sin \gamma (-\sin \beta d_c + \cos \beta b) \\ t &\rightarrow -\sin \gamma e^{i\delta} s_c + \cos \gamma (-\sin \beta d_c + \cos \beta b) \end{aligned} \quad (13)$$

Fig. 2 Charged lepton spectrum in D decay for $M_D = 1.866$ GeV, $m_S = 0.3$ GeV, $m_{sp} = 0.15$ GeV; $\alpha_S = 0.38$, $p_{cm} = 0.26$ GeV and $p_F = 0$ (solid), $p_F = 0.15$ GeV (dotted), $p_F = 0.3$ GeV (dashed). The normalization is fixed by the number of events.

Fig. 3 Charged lepton spectrum in D decay for $M_D = 1.866$ GeV, $m_S = 0.5$ GeV, $m_{sp} = 0$, $\alpha_S = 0.38$, $p_{cm} = 0.26$ GeV and $p_F = 0$ (solid), $p_F = 0.15$ GeV (dotted), $p_F = 0.3$ GeV (dashed). The normalization is fixed by the number of events.

leaving aside, for the time being, the recent SLAC results on the b lifetime (to be discussed later on), the existing limit on $\cos^2 \gamma$ is derived by the CERN-Dortmund-Heidelberg-Saclay (CDHS)¹⁰ Collaboration from the production of dimuons in ν and $\bar{\nu}$ deep inelastic scattering and is given by $\cos^2 \gamma \gtrsim 0.6$.

If one knows the semileptonic branching ratio $B_{s\ell}$ of D mesons, then the estimate in Eq. (12) of the semileptonic width leads to a predicted range for the total lifetime. The average $B_{s\ell}$ of 56% D^0 and 44% D^+ is known with good accuracy from experiments at the ψ'' resonance¹:

$$\bar{B}_{s\ell}(D^+, D^0) = (8.2 \pm 1.2)\% . \quad (14)$$

As the dominant transition $c \rightarrow s$ is isoscalar, $\Gamma_{s\ell}$ must be the same for D^+ and D^0 when isospin violations are neglected and $\theta_C \approx 0$. Then from Eqs. (12) and (14) one obtains

$$\cos^2 \gamma \bar{\tau}(D) = (2-8.5) \times 10^{-13} \text{ s} \quad (p_F \lesssim 300 \text{ MeV}) . \quad (15)$$

The present world averages of D^+ , D^0 , and F^+ lifetimes are given by¹¹

$$\begin{aligned} \tau_{\text{exp}}(D^0) &= \left(4.4 \begin{smallmatrix} + 0.6 \\ - 0.5 \end{smallmatrix} \right) \times 10^{-13} \text{ s} \\ \tau_{\text{exp}}(D^+) &= \left(8.8 \begin{smallmatrix} + 1.8 \\ - 1.0 \end{smallmatrix} \right) \times 10^{-13} \text{ s} \\ \tau_{\text{exp}}(F^+) &= \left(2.1 \begin{smallmatrix} + 1.3 \\ - 0.6 \end{smallmatrix} \right) \times 10^{-13} \text{ s} , \end{aligned} \quad (16)$$

which leads to

$$\bar{\tau}_{\text{exp}}(D) = \left(6.6 \begin{smallmatrix} + 1.2 \\ - 0.8 \end{smallmatrix} \right) \times 10^{-13} \text{ s} . \quad (17)$$

Similarly for $B(D^+) \approx 15\%$ (a value favoured by theory as we shall see) one obtains

$$\cos^2 \gamma \tau(D^+) \approx (4-14) \times 10^{-13} \text{ s} .$$

We also mention that from $B_{s\ell}(\Lambda_C) \approx 5\%$, as known from experiment, if one takes, for purposes of illustration, $\Gamma_{s\ell}(\Lambda_C) \approx \Gamma_{s\ell}(D)$ (as would be the case from free heavy quark decay) then one obtains

$$\cos^2 \gamma \tau(\Lambda_C) \approx (1.5-4.5) \times 10^{-13} \text{ s} ,$$

which is to be compared with the measured value¹¹:

$$\tau_{\text{exp}}(\Lambda_c) \approx \left(2.2 \begin{smallmatrix} + 0.8 \\ - 0.5 \end{smallmatrix} \right) \times 10^{-13} \text{ s} . \quad (20)$$

In conclusion, the theoretical estimates of $\Gamma_{s\ell}$ for charm lead to satisfactory agreement with the measured lifetime values, once the values of $B_{s\ell}$ are taken from experiment.

3.2 Beauty Decay

The electron spectrum of beauty decay is particularly interesting in that it is sensitive to the ratio of the $(b \rightarrow c) \approx \sin \gamma \cdot \cos \beta \approx \sin \gamma$ and the $(b \rightarrow u) \sim \sin \beta$ transitions:

$$\frac{d\Gamma_{s\ell}^b}{dE} = \sin^2 \gamma \frac{d\Gamma^c}{dE} + \sin^2 \beta \frac{d\Gamma^u}{dE} . \quad (21)$$

The possibility of separating the two components depends on the theoretical control of the hard tail of the charged lepton spectrum. This is sensitive to QCD corrections and to bound-state effects. In particular, the case of a final massless quark is of special interest and is relevant for the $b \rightarrow u + e + \nu$ channel. The electron spectrum is given by

$$\frac{d\Gamma}{dx} = \frac{d\Gamma^0}{dx} \left[1 - \frac{2\alpha_s}{3\pi} G(x,0) \right] , \quad (22)$$

where $x = 2E/M_Q$ is the electron energy fraction in the rest frame of the heavy quark of mass M_Q , $d\Gamma^0/dx$ is the uncorrected spectrum, while $G(x,0)$ is a complicated function which describes the QCD correction^{6,7}. Both $d\Gamma^0/dx$ and $G(x,0)$ are different for the $c \rightarrow s$ and the $b \rightarrow u$ transitions. In fact, as a consequence of the $V - A$ structure of the current, the ν spectrum in the $c \rightarrow s$ transition is the same as the charged lepton spectrum in the $b \rightarrow u$ transition and conversely. In particular,

$$\frac{d\Gamma^0}{dx} = \begin{cases} \sim x^2 (1 - x) & (c \rightarrow s) \\ \sim x^2 (2 - x) & (b \rightarrow u) . \end{cases} \quad (23)$$

In both cases⁵ near $x \rightarrow 1$

$$G(x,0) \xrightarrow{x \rightarrow 1} \ln^2 (1 - x) . \quad (24)$$

While in the $c \rightarrow s$ case the singular behaviour of $G(x,0)$ at $x = 1$ is damped by the corresponding vanishing of $d\Gamma^0/dx$, it cannot be ignored in the $b \rightarrow u$ case, where the bare spectrum is non-vanishing

at $x = 1$. In the latter case, first-order perturbation theory becomes inadequate near $x \rightarrow 1$, at values of $x = x_c$ such that

$$\frac{2\alpha_s}{3\pi} \ln^2 (1 - x_c) \approx 1 \Rightarrow M_{\text{QCD}} = M_Q (1 - x_c)^{1/2} \approx 0.55 \text{ GeV} \quad (25)$$

(for $M_Q \approx 5 \text{ GeV}$). M_{QCD} is the minimum invariant mass of the recoiling hadronic system at which first-order perturbation theory can still be applied. The double logs are of infrared origin and can be resummed⁵ at all orders leading to a Sudakov exponential:

$$\frac{d\Gamma}{dx} = \frac{d\Gamma^0}{dx} \exp \left[-\frac{2\alpha_s}{3\pi} \ln^2 (1 - x) \right] \left[1 - \frac{2\alpha_s}{3\pi} \tilde{G}(x,0) \right], \quad (26)$$

where

$$\tilde{G}(x,0) = G(x,0) - \ln^2 (1 - x). \quad (27)$$

The resulting spectrum now vanishes for $x \rightarrow 1$ and one cannot distinguish a massless final quark from a quark with mass $m \sim M_{\text{QCD}}$. Since, however, the c quark mass is much larger than M_{QCD} , the possibility of distinguishing the $b \rightarrow c$ and the $b \rightarrow u$ transitions is not spoiled. In practice, these delicate effects are concealed by the Fermi motion smearing near the end point related to p_F .

Bound-state effects can be taken into account by the same model as already described⁵. The same values for p_F as found from the analysis of D decays can be tentatively assumed. This is justified in a crude non-relativistic picture of c and b flavoured mesons, where the wave function is determined by the reduced mass which is insensitive to the heavy quark mass provided it is sufficiently large.

The results obtained by the CUSB and CLEO Collaborations at Cornell can be summarized as follows. The $b \rightarrow c$ transition is found to be dominant with respect to the $b \rightarrow u$ transition. From the ratio of rates $\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c) \lesssim 0.05$, we obtain:

$$\frac{\sin^2 \beta}{\sin^2 \gamma} \lesssim 0.04. \quad (28)$$

The total lifetime of a b flavoured meson can be written down in terms of the corresponding $B_{s\ell}$ as:

$$\tau(b) = \frac{B_{s\ell}}{0.131} \frac{10^{-14} \text{ s}}{\sin^2 \gamma [Z_c + (\sin^2 \beta / \sin^2 \gamma) Z_u]}. \quad (29)$$

For $p_F \lesssim 300$ MeV and $\Lambda = 250$ MeV one finds

$$Z_c = 2.5-3.5, \quad Z_u = 5.6-7.2 . \quad (30)$$

From the experimental value of B_{sl} measured at the γ''' , which refers to a weighted average of 60% B^+ and 40% B^0 ,

$$B_{sl}^{\text{exp}} = 0.131 \pm 0.012 , \quad (31)$$

and the bound in Eq. (28), one obtains

$$\tau(B) = \frac{1}{\sin^2 \gamma} (2.4-4.4) \times 10^{-15} \text{ s} . \quad (32)$$

On the experimental side an upper limit found by the JADE Collaboration at DESY is known since last year:

$$\tau(B) < 1.4 \times 10^{-12} \text{ s} \quad [\text{JADE}] . \quad (33a)$$

Quite recently two measured values have been reported from SLAC (presented at the Brighton and Cornell Conferences):

$$\tau(B) = (1.8 \pm 0.6 \pm 0.4) \times 10^{-12} \text{ s} \quad [\text{MAC}] , \quad (33b)$$

which implies by Eq. (32):

$$0.03 \leq |\sin \gamma| \leq 0.07 ,$$

and

$$\tau(B) = \left(10.4 \begin{smallmatrix} + 4.9 \\ - 3.9 \end{smallmatrix} \right) \times 10^{-13} \text{ s} \quad [\text{MARK II}] , \quad (33c)$$

corresponding to

$$0.4 \leq |\sin \gamma| \leq 0.08 .$$

Thus, if these results are confirmed, one is led to conclude that $\sin \gamma \sim O(\theta_C^2)$!

4. INCLUSIVE NON-LEPTONIC RATES

To proceed further toward a computation of B_{sl} and of non-leptonic inclusive rates we shall divide the discussion into two conceptually different points: first the set-up of an effective Hamiltonian for non-leptonic weak amplitudes and then the attempts at evaluating its matrix elements. We shall see that the first part

can be carried through on rather solid grounds, while the second step is more tentative.

Let us consider a non-leptonic weak process induced by charged currents. In lowest order in the weak coupling the transition matrix element is given by the time-ordered product of the two weak charged currents folded with the W propagator:

$$H_{fi} \approx g_W^2 \int d^4x D_W(x^2, M_W^2) \langle F | T [J^\mu(x) J_\mu^\dagger(0)] | I \rangle, \quad (34)$$

where M_W , g_W , and D_W are the W boson mass, coupling, and propagator, respectively. For flavour-changing amplitudes the leading contributions in the limit $M_W \rightarrow \infty$ arise from the four-fermion operators of dimension six in the short-distance operator expansion for the T-product¹². In the particular example of charm-changing processes the relevant terms are of the form

$$H_{fi}^{\Delta c=1} = \frac{G_F}{\sqrt{2}} \left\{ c_+(t, \alpha_s) \langle F | O_+(0) | I \rangle + c_-(t, \alpha_s) \langle F | O_-(0) | I \rangle + \dots \right\}, \quad (35)$$

where $t = \ln M_W^2/\mu^2$ and μ is a reference mass scale, $\alpha_s = \alpha_s(\mu)$ being the renormalized QCD coupling at the scale μ , and

$$\begin{aligned} O_\pm &= \frac{1}{2} [(\bar{s}'c)_L (\bar{u}d')_L \pm (\bar{u}c)_L (\bar{s}'d')_L] \\ &= \frac{N \pm 1}{2N} (\bar{s}'c)_L (\bar{u}d')_L \pm \sum_A (\bar{s}'t^A c)_L (\bar{u}t^A d')_L. \end{aligned} \quad (36)$$

The shorthand notations for left-handed (right-handed) currents,

$$(\bar{q}_1 q_2)_{L,R} = \bar{q}_1 \gamma_\mu (1 \mp \gamma_5) q_2 \quad (37)$$

were used here; s' and d' are the Cabibbo-like quark mixtures coupled to the c and u quarks respectively; t^A are the SU(N) matrices in the quark (fundamental) representation, with the normalization $\text{Tr}(t^A t^B) = \frac{1}{2} \delta_{AB}$. The second equality in Eq. (36) is obtained through Fierz rearrangement of $(\bar{u}c)(\bar{s}'d')$, according to the identity

$$\begin{aligned}
(\bar{q}_1 q_2)_L (\bar{q}_3 q_4)_L &= \frac{1}{N} (\bar{q}_1 q_4)_L (\bar{q}_3 q_2)_L \\
&+ 2 \sum_A (\bar{q}_1 t^A q_4)_L (\bar{q}_3 t^A q_2)_L .
\end{aligned} \tag{38}$$

In Eq. (35) the dots stand for non-leading terms from operators of higher dimension and also from "penguin" operators¹³⁾ of dimension six, which are present in some channels for non-degenerate quark masses, and will be considered later on. In the free field limit $c_+ = c_- = 1$ and

$$H^{\Delta c=1} = \frac{G_F}{\sqrt{2}} (\bar{s}' c)_L (\bar{u} d')_L \quad (\text{free fields}) . \tag{39}$$

In the leading logarithmic approximation O_+ and O_- are multiplicatively renormalizable, because anomalous dimensions are determined by the massless theory, and $O_{\pm}$ have definite and different transformation properties under the $SU(N)$ flavour symmetry of the massless theory. c_+ and c_- deviate from the free field value by a known amount:

$$c_{\pm} = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(M_W^2)} \right)^{\gamma_{\pm}} \tag{40}$$

$$\begin{aligned}
\gamma_+ &= \frac{-9(N-1)}{N(11N-2f)} \Big|_{\substack{N=3 \\ f=4}} = -\frac{6}{25} \\
\gamma_- &= \frac{9(N+1)}{N(11N-2f)} \Big|_{\substack{N=3 \\ f=4}} = \frac{12}{25} ,
\end{aligned} \quad (\gamma_+^2 = 1/\gamma_-) \tag{41}$$

where N and f are the numbers of colours and of flavours, respectively. Recently a calculation of the effective Hamiltonian at the next to the leading log accuracy was completed¹⁴⁾ with the result:

$$c_{\pm} = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(M_W^2)} \right)^{\gamma_{\pm}} \left\{ 1 + \frac{\alpha_s(\mu^2) - \alpha_s(M_W^2)}{\pi} \rho_{\pm} + \dots \right\} , \tag{42}$$

where $\rho_{\pm}$ is independent of the renormalization scheme for the operators $O_{\pm}$, but only depends on the definition of α_s . Numerically, in the $\overline{MS}$ definition of α_s , for $N = 3$ and $f = 4, 6$, one obtains

$$\rho_{\pm} = \begin{cases} -0.469 \\ +1.36 \end{cases} \quad f = 4$$

$$\rho_{\pm} = \begin{cases} -0.574 \\ +1.65 \end{cases} \quad f = 6 .$$
(43)

It is important to observe that the next to the leading corrections are of "normal" size, i.e. $\rho_{\pm} \sim 0(1)$, and follow exactly the same pattern of enhancement and suppression of the leading term. Actually most of the correction could be reabsorbed in a redefinition of α_s (by scaling up $\Lambda_{\overline{MS}}$ by a factor of about 1.8). As a consequence, the above results considerably reinforce the leading log predictions and put the status of the effective Hamiltonian on a much more solid basis.

It is well known that for strange-particle decays the structure of the effective Hamiltonian works in favour of the $\Delta I = 1/2$ rule¹⁵. In fact, for strangeness-changing amplitudes O_- is pure $\Delta I = 1/2$ and c_- is enhanced by gluon effects, while O_+ also contains $\Delta I = 3/2$ and c_+ is suppressed. However, $\mu \sim m_s$, where m_s is the strange quark mass, is too low to rely on massless perturbation theory in the descent from M_W to m_s . Below the charm mass m_c various effects connected with the no longer negligible c mass appear, such as penguin diagrams and separate anomalous dimensions for operators in different $SU(3)_f$ representations^{13,16}. On the other hand, the presence of still heavier quarks, such as b and t , is not of great relevance and can be effectively taken into account by adjusting the number f of excited flavours between 4 and 6. Penguin diagrams shown in Fig. 4 are zero for equal quark masses because of the Glashow-Iliopoulos-Maiani (GIM) mechanism. Their coefficient is rather small: $(c_+ + c_-)(\alpha_s/12\pi) \ln(m_c^2/m_s^2)$; (the argument of the log is the heavy

Fig. 4

quark mass in the loop divided by the energy scale for the process. The value of the K_L - K_S mass difference shows that the presence of the t quark is unimportant.). But the claim, originally made in Ref. 13, is that the matrix element is particularly large, owing to a low-energy effect connected with the smallness of the current mass of u and d quarks. In conclusion, the $\Delta I = 1/2$ rule observed in K and hyperon decays corresponds to a factor $A_{1/2}/A_{3/2} \sim 20$ in amplitude. The QCD effect in the coefficients $c_{\pm}$ can account for the square root of this factor. The remaining factor is presumably due to low-energy effects in matrix elements, also including perhaps those of penguin diagrams. This view is to some extent supported by the fact that the $\Delta I = 1/2$ rule is less pronounced in Ω^- decays, where low-energy effects are presumably less important.

For c decays the penguin diagrams, in the limit $m_b \gg m_d \sim m_s \sim 0$, are given by

$$H = \frac{G_F}{\sqrt{2}} (c_+ + c_-) U_{cb} U_{ub} \frac{\alpha_s(m_c)}{12\pi} \ln \frac{m_b^2}{m_c^2} (\bar{c}t^A u)_L (\bar{q}t^A q)_{L+R} . \quad (44)$$

The existing bounds¹⁷ on mixing angles give $|U_{cb}U_{ub}| \lesssim 0.04$ (even if we disregard Eqs. (33) for the b lifetime). Thus, as the matrix elements are not expected to be large, penguin diagrams are certainly irrelevant for charm decays. At most they can contribute in Cabibbo suppressed channels, but their contribution is expected to be small even at that level. For b decays, in the limit $m_t \gg m_u \sim m_c \sim 0$, one similarly obtains

$$H = \frac{G_F}{\sqrt{2}} (c_+ + c_-) U_{tb} U_{ts} \frac{\alpha_s(m_b)}{12\pi} \ln \frac{m_t^2}{m_b^2} (\bar{b}t^A s)_L (\bar{q}t^A q)_{L+R} . \quad (45)$$

With little information on the mixing angles ($|U_{tb}U_{ts}| < 0.6$) one can only say that this contribution is presumably small, but could be not entirely negligible.

As a first conclusion, we have seen that the structure of the effective non-leptonic Hamiltonian is known in the form

$$H = \frac{G_F}{\sqrt{2}} [c_+ O_+ + c_- O_- + \text{penguins}] \quad (46)$$

and that penguin diagrams are negligible in the $|\Delta c| = 1$ sector where the Hamiltonian reduces to

$$H = \frac{G_F}{\sqrt{2}} \left[\frac{c_+ + c_-}{2} (\bar{c}s')_L (\bar{d}'u)_L + \frac{c_+ - c_-}{2} (\bar{c}u)_L (\bar{d}'s')_L \right]. \quad (47)$$

Rather confusing is the status of matrix elements, in particular the relative importance of the quark decay with spectator(s) and the annihilation mechanism which we are now going to discuss.

Initially it was assumed, according to parton ideas, that the c quark decay was the main mechanism, with the other constituents in the hadron acting as spectators.

This obviously leads to approximately the same B_{sl} or lifetimes for all charmed hadrons. Since the discovery of a longer lifetime for D^+ than for D^0 it was clear that quark decay cannot be the only effect. After the first data of the DELCO Collaboration followed by new evidence from MARK II, now there is also an increasing amount of data from emulsions and bubble chambers that led to the lifetimes quoted in Eqs. (16) and (20). In particular, the best value for the ratio of D^+ and D^0 lifetimes appears now to be:

$$\frac{\tau(D^+)}{\tau(D^0)} \approx 2.0 \pm 0.4. \quad (48)$$

This more recent value is much closer to 1 than previously reported and more understandable theoretically as we shall see.

Although quark decay cannot be, at least for charm, the only effect, however it certainly is there. It is thus important to summarize what would happen for the decay of a free heavy quark. We shall later discuss the possible role of the spectators.

In the leading log approximation the total non-leptonic width of a heavy quark Q arises from $Q \rightarrow q + q + \bar{q}$ and is given by¹⁸

$$\Gamma_{nL} = \frac{G_F^2 M_Q^5}{192\pi^3} \sum_{q, q_\alpha, \bar{q}_\beta} |U_{Qq}|^2 |U_{q_\alpha q_\beta}|^2 \times I \left(\frac{m_q}{M_Q}, \frac{m_{q_\alpha}}{M_Q}, \frac{m_{q_\beta}}{M_Q} \right) \frac{2c_+^2 + c_-^2}{3}, \quad (49)$$

where the last factor is larger than 1 as follows from Eqs. (40) and (41) for c_+ . It arises as follows. By writing H in Eq. (47) in terms of one single ordering of quark fields, obtained by Fierz rearrangement of the second term according to Eq. (38), one gets

$$\begin{aligned}
H \approx \frac{G_F}{\sqrt{2}} \left\{ \left(\frac{2}{3} c_+ + \frac{1}{3} c_- \right) (\bar{c}s')_L (\bar{d}'u)_L \right. \\
\left. + (c_+ - c_-) (\bar{c}t^A s')_L (\bar{d}'t^A u)_L \right\}.
\end{aligned} \tag{50}$$

The incoherent sum of the rates into colour singlet and colour octet $d'u$ final states leads to

$$\left(\frac{2}{3} c_+ + \frac{1}{3} c_- \right)^2 + \frac{2}{9} (c_+ - c_-)^2 = \frac{2c_+^2 + c_-^2}{2}, \tag{51}$$

where the factor of $2/9$ arises from the ratio of $\text{Tr } I \cdot \text{Tr } I = 9$ for the singlet-singlet contribution and $\sum_{AB} [\text{Tr}(t^A t^B)]^2 = 2$ for the octet-octet contribution. The overall normalization is chosen so as to reproduce the free field result for $c_+ = c_- = 1$.

Ignoring phase-space distortions and finite corrections of order α_s , this leads to:

$$B_{s\ell} \approx \frac{1}{2 + 2c_+^2 + c_-^2} \approx \begin{cases} 20 \text{ free fields} \\ (13 - 16)\% \text{ leading logs.} \end{cases} \tag{52}$$

Phase-space corrections are however of importance, because the three-body phase space is sensitive to masses. In particular, non-leptonic amplitudes could be somewhat suppressed compared to semi-leptonic modes, because of quark masses. It has been observed¹⁹ that by setting $m_s \approx 500$ MeV and $m_{u,d} \approx 300$ MeV the effect is quite large and could bring $B_{s\ell}(D^+)$ back into the 20% range. On the other hand, the effect is quite small for $m_s \approx 300$ MeV and $m_{u,d} \approx 100$ MeV, which looks more reasonable in view of the final state being made up of π and K , and because no analogue effect is seen in τ semileptonic decay. Thus I think that this effect is probably marginal.

Corrections of order α_s to the non-leptonic rate have also been computed recently¹⁴. In the $\overline{MS}$ prescription for α_s they amount to the replacement

$$\begin{aligned}
\frac{2c_+^2 + c_-^2}{3} \rightarrow \frac{2c_+^2 + c_-^2}{3} \left\{ 1 + \frac{2\alpha_s}{3\pi} \left(\frac{31}{4} - \pi^2 + \frac{19}{4} \frac{c_-^2 - c_+^2}{2c_+^2 + c_-^2} \right) \right. \\
\left. + 2 \frac{\alpha_s(m_c) - \alpha_s(M_W)}{\pi} \frac{2c_+^2 \rho_+ + c_-^2 \rho_-}{2c_+^2 + c_-^2} \right\},
\end{aligned} \tag{53}$$

where $\rho_{\pm}$ are defined in Eq. (42). Numerically for $\Lambda \simeq 0.25$ GeV the curly bracket is 1.21. The size of the correction is normal and its sign is once more in the same direction as that of the leading log effect. The prediction of a depletion of B_{sl} for quark decay below the free field value of 20% is thus confirmed by the corrections of order α_s , which lower the semi-leptonic width and increase the non-leptonic rate. We shall see that the annihilation mechanism cannot affect much the D^+ rate, and that spectator effects, which for D^+ could in principle be relevant, are found to be small. Thus the prediction of a small B_{sl} , obtained from the study of the decay properties of a heavy quark, is most probably relevant for D^+ decay.

Now naturally comes the question of possible spectator effects. An interesting point has been put forward²⁰. At the Cabibbo allowed level a c quark decays according to $c \rightarrow s\bar{d}$. The D^+ meson is unique in that the spectator $\bar{d}$ finds an identical antiquark in the final state of c decay. Note parenthetically that also for Λ_c there would be a u spectator identical with a u quark from c decay. Assuming for a moment that the D^+ wave function was a δ -function at the origin, then the two $\bar{d}$'s would be emitted in the same space-time point. The final state would be the same as produced from the vacuum by an interaction where $\bar{c}$ is replaced by $\bar{d}$: for example $(cs)_L (\bar{d}u)_L \rightarrow (\bar{d}s)_L (\bar{d}u)_L$, because the D^+ is colour singlet and spin singlet, which means that colour and helicity are just right for the replacement of an incoming c with an outgoing $\bar{d}$. Thus the analogue of O_- would vanish and the non-leptonic rate of D^+ would be reduced by a factor

$$\frac{4c_+^2}{2c_+^2 + c_-^2} \sim \frac{1}{2}.$$

Now, of course this is an upper bound because the wave function is not a δ at the origin and the overlap between spectator and active antiquark must be far smaller. This matter can be quantitatively studied²¹ by going back to the bound-state model developed in the analyses of semileptonic decays. We write the non-leptonic width of D^+ as a sum of a direct plus an interference term arising from the diagrams in Fig. 5 a and b, respectively:

Fig. 5

$$\Gamma = \Gamma_Q + \Gamma_{\text{int}} . \quad (54)$$

Omitting mixing angles for simplicity we have

$$\begin{aligned} \Gamma_Q = & (2c_+^2 + c_-^2) J_{2L} \int d^3q |\psi(q)|^2 \left(\frac{W}{E_W} \right) \frac{G_F^2 W^5}{192\pi^3} \\ & \times I \left(\frac{m_s}{W}, \frac{m_{sp}}{W}, \frac{m_{sp}}{W} \right) , \end{aligned} \quad (55)$$

where $(2c_+^2 + c_-^2)$ arises from QCD leading logs, J_{2L} is the two-loop QCD correction [Eq. (53)], W is defined in Eq. (8), W/E_W is the Lorentz factor for the decay in flight of a c quark of mass W , and I is the phase-space factor ($m_u \approx m_d \equiv m_{sp}$). Correspondingly:

$$\begin{aligned} \Gamma_{\text{int}} = & -(c_-^2 - 2c_+^2) \frac{G_F^2}{\pi} \int \frac{d^3q_1 d^3q_2}{(2\pi)^3} \psi(q_1) \psi^*(q_2) \\ & \times \left(\frac{W_1 W_2}{E_{W_1} E_{W_2}} \right)^{1/2} f(\vec{q}_1, \vec{q}_2) , \end{aligned} \quad (56)$$

where $f(\vec{q}_1, \vec{q}_2)$ is a kernel arising from matrix elements and phase space which is given in Ref. 21. The limits on p_F obtained from the lepton spectrum lead to rather strong upper bounds for the ratio $R = -\Gamma_{\text{int}}/\Gamma_Q$ which are shown in Fig. 6. One realizes that the interference mechanism, although working in the right direction, can only play a marginal role in explaining a difference in lifetime between D^+ and D^0 of a factor of 2-3 as observed. The theoretical expectation for $B_{s\ell}(D^+)$ is plotted in Fig. 7 as a function of p_F and α_s . The resulting prediction turns out to be

$$B_{s\ell}(D^+) = \left(15 \begin{smallmatrix} + \\ - \end{smallmatrix} 2 \right) \% . \quad (57)$$

We now consider the annihilation mechanism. First we recall that by this name we mean exchange of a W between constituents of a given hadron, in either the s - or t -channels. The name annihilation is, strictly speaking, only appropriate for an s -channel exchange.

The first point to recall is that annihilation without gluon emission cannot work, for two main reasons. The first is the helicity suppression factor. In the massless limit a pseudoscalar

Fig. 6 The ratio $R = -\Gamma_I/\Gamma_Q$ of the interference and the direct term in D^+ non-leptonic decay, as a function of p_F for different values of $\alpha_s(M_D)$ and of final quark masses. The upper curve in each shaded band is for $m_s = 300$ MeV, $m_{sp} = 150$ MeV, while the lower curve corresponds to $m_s = 400$ MeV, $m_{sp} = 100$ MeV.

Fig. 7 The semileptonic branching ratio B_{sl}^+ of D^+ as a function of p_F for different values of $\alpha_s(M_D)$ and of final quark masses. The upper curve in each shaded band is for $m_s = 300$ MeV, $m_{sp} = 150$ MeV, while the lower curve corresponds to $m_s = 400$ MeV, $m_{sp} = 100$ MeV.

meson cannot decay into a spin-one $q\bar{q}$ pair as produced by a $V - A$ interaction. Even for an s quark the suppression factor $m_s^2/m_c^2 \sim 1/10$ is too small. Also colour factors are completely unfavourable. The D^0 decay needs a $c\bar{u}$ pair in a colour singlet combination, which means an amplitude proportional to $X_- = \frac{2}{3}c_+ - \frac{1}{3}c_-$ [from Eqs. (38) and (47)], while F^+ and D^+ need $c\bar{s}$ or $c\bar{d}$ in a colour singlet state, with amplitude $X_+ = \frac{2}{3}c_+ + \frac{1}{3}c_-$. Since $X_+^2/X_-^2 \approx 20$ this would make the lifetime of F^+ much shorter than that of D^0 and destroy the Cabibbo suppression pattern for D^+ . Thus annihilation without gluon emission cannot work. However it exists and can provide a sizeable rate for $F^+ \rightarrow \tau^+ \nu_\tau$ and contribute an important fraction of the D^+ Cabibbo-suppressed modes.

With single gluon emission the helicity suppression factor is removed and the colour factors look all right. In fact, in the D^0 case the amplitude for a $c\bar{u}$ pair in a colour octet is proportional to $(c_+ + c_-)$, while in the D^+ and F^+ cases the amplitude corresponding to colour octet $c\bar{s}$ and $c\bar{d}$ is proportional to $(c_+ - c_-)$. Other signatures for the relevance of the annihilation diagrams are also observed. For example, final states with no $s\bar{s}$ quarks in F^+ decay were seen, as well as dominance of isospin $\frac{1}{2}$ amplitudes in D^0 decays and rather abundant Cabibbo-suppressed modes in D^+ decays⁸.

If on qualitative grounds the annihilation mechanism with gluon emission is a convincing candidate for explaining the shorter lifetime of D^0 , F^+ , and Λ_c , compared to D^+ , the quantitative estimate of the relative importance of annihilation versus quark decay for charm is a much more difficult problem²². A crude non-relativistic estimate leads¹² to:

$$\Gamma_{\text{ann}}^{D^0} \approx \cos^2 \gamma (c_+ + c_-)^2 \frac{G_F^2 M_D^2}{6\pi} |\psi^{(8)}(0)|^2 P^{(8)}, \quad (58)$$

where $P^{(8)}$ is the probability of finding a colour octet spin-one $c\bar{u}$ pair in the D^0 and $\psi^{(8)}$ is the corresponding wave function. A perturbative evaluation of $P^{(8)}$ leads to (see the first of Refs. 22)

$$P^{(8)} \approx \frac{\alpha_s}{18\pi} \left(\frac{M_P}{m_u} \right)^2 \sim 0.2 \quad \left(\begin{array}{l} \alpha_s \approx 0.3 \\ m_u \approx 300 \text{ MeV} \end{array} \right). \quad (59)$$

The dependence on m_u^{-2} is a warning that perturbation theory might be misleading and $P^{(8)}$ could perhaps be larger. What about $|\psi^{(8)}(0)|^2$?

If one assumes that $\psi_{J=1}^{(8)} \approx \psi_{J=0,1}^{(1)}$ then $\psi(0)$ can be related to f_D , the analogue of f_π for the D meson:

$$f_D = \frac{12}{M_D} |\psi(0)|^2 . \quad (60)$$

Theoretical estimates¹² of f_D from potential models²³, QCD sum rules²⁴, and bag models²⁵ all lead to values in the range $f_D = (150-300)$ MeV. From the hyperfine splitting of D^* and D one obtains¹²

$$\Delta M^2 = \frac{64\pi}{9} \alpha_s \frac{|\psi(0)|^2}{m} \Rightarrow f_D = 380 \text{ MeV} \quad \left(\begin{array}{l} \alpha_s = 0.3 \\ m = 300 \text{ MeV} \end{array} \right) . \quad (61)$$

In conclusion, from f_D estimates it is difficult to imagine a violation of the bound $\Gamma_{\text{ann}}/\Gamma_Q \lesssim 1-2$ for D^0 , so that one expects $\tau(D^+)/\tau(D^0) \lesssim 2-3$. The D^+ is almost unaffected, the c decay mechanism is dominant, and the prediction for $B_{sl}(D^+)$ in Eq. (57) is confirmed. For F^+ the decay $F^+ \rightarrow \tau^+ \nu_\tau$ is a good f_F -meter. For $f_F \sim 150$ MeV one expects $B(F \rightarrow \tau \nu) \approx 1\%$. In the non-leptonic channels annihilation with one-gluon exchange is proportional to $(c_- - c_+)^2 P^{(8)}$ for F^+ . If $P^{(8)}$ is large, then the probability of two-gluon emission could also be non-negligible. In this case the relevant factor is $3X_+^2 P^{(1)}$ (from a colour singlet, spin 1 $c\bar{s}$ pair), which is colour-wise more favourable. In this case the lifetime of F^+ could be shortened near or below the D^0 level, and there could be a further sizeable contribution from a final state made up of leptons plus gluons to the semileptonic width of F^+ beyond quark decay.

In Λ_c decays the cd pair can have spin one even with no gluon emission and the annihilation rate for Λ_c depends on the baryon wave function, which is unrelated to the meson one. The observed value of the Λ_c lifetime indicates a sizeable annihilation term.

For beauty decay, violations of the parton model are expected to go down compared to charm decay. The spectator effects are certainly completely negligible (below 1%) because more phase space leads to far less wave-function overlap. Mass effects are also negligible unless two or more τ and/or c are present in the final state at the same time. Finally, the annihilation mode is decreasing with some power of m_Q . In the non-relativistic approximation, if one neglects the variation of $P^{(8)}$ (which is bound by one), then the ratio $\Gamma_{\text{ann}}/\Gamma_Q$ decreases as $1/m_Q^3$. But this is very speculative. Neglecting the annihilation contribution one expects:

$$B_{sl}(B^0) \approx B_{sl}(B^-) \approx (13-15)\% \quad (\text{no annihilation})$$

while with annihilation the B^0 lifetime is shortened and

$$B_{sl}(B^-) \gtrsim B_{sl}(B^0) \quad (\text{annihilation}) .$$

5. CONCLUSION

The picture is getting clearer and a reasonable understanding of heavy flavour decays is slowly emerging. On the one hand the experimental information is getting into focus and the initial indications of a giant ratio of D^+ and D^0 lifetimes are now replaced by a more reasonable (for theory) value of two or so. On the other hand, this crucial factor is now attributed, by general consensus, to the contribution of the annihilation term, while all other proposed mechanisms can presumably be discarded. Although the annihilation term cannot be precisely predicted, it is convergently estimated to be for charm of the same order as the quark-decay contribution. Furthermore, since the D^+ rates are almost unaffected by the annihilation term they are calculable and allow a test of the QCD-improved parton model. In particular, the experimental verification of the prediction for the semileptonic branching ratio of D^+ , $B_{sl}(D^+) \approx 15\%$, would be a striking confirmation of the QCD enhancement factor for non-leptonic rates arising from short-distance effects. For this interpretation to be unambiguous the accompanying signatures on the lifetimes of all charmed particles and on the quantum numbers of the exclusive modes and their relative intensities should eventually be detected. It is also of the utmost importance to measure beauty decays and verify that in this case the parton approximation is working far better than in the case of charm decay. Finally, we observe that once heavy flavour decays can be considered as understood, which we hope now to be not so far in the future, then one can probably reconsider the unsolved aspects of strange-particle decays and take advantage of the valuable lessons which have been learnt in order to tackle this more difficult problem.

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