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A COMMON SCALE FOR THE INVISIBLE AXION,
LOCAL SUSY GUTs AND SAXINO DECAY

"Dedicated to the memory of Henry Primakoff"

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ABSTRACT

We identify the scale 10^{11-12} GeV as that of the invisible axion and the SUSY breaking scale of local SUSY GUTs. The saxino ($\equiv$ a SUSY partner of the axino; the other partner is the invisible axion) decay is also commented.

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Phenomenological and theoretical studies of grand unified theories (GUTs) ¹⁾ give a hint of a very interesting mass scale $10^{11}\sim 10^{12}$ GeV. The invisible axion ²⁾⁻⁴⁾, if it exists in nature, should not dissipate the stellar energy too fast ⁵⁾, which restricts the scale for the invisible axion v to being greater than 10^9 GeV. On the other hand, too large a v forbids fast enough dissipation of the energy stored in the classical axion field, which restricts ⁶⁾ v to being equal to or less than $10^{11}\sim 10^{12}$ GeV. The geometrical hierarchy models ⁷⁾ and supergravity grand unification models (SUGRA) ⁸⁾ also give the scale 10^{11} GeV as an indispensable one. Furthermore, it has been suggested ⁹⁾ that the invisible axion can account for the missing mass of galactic haloes if $v=10^{11}\sim 10^{12}$ GeV.

Theoretically, supersymmetric (SUSY) solutions to the gauge hierarchy problem in the context of elementary Higgs scalars give an upper bound of SUSY breaking scale of order $10^{11}\sim 10^{12}$ GeV ⁸⁾. For SUSY breaking at M_{SB} , the light fields feel through supergravity quadratic and cubic "soft terms" of the scalar potential which are usually written as ⁸⁾

$$\tilde{m}^2 \sum_a |y_a|^2 + \left[\{ (A-3) \tilde{m} W + \tilde{m} \sum_a y_a \frac{\partial W}{\partial y_a} \} + h.c. \right] \quad (1)$$

where W is a superpotential, y_a denotes a light scalar field, A is a numerical constant dependent upon specific models, and $\tilde{m}$ is roughly the gravitino mass. We can get an upper bound on SUSY breaking by considering the $SU(2)\times U(1)$ breaking in terms of two Higgs scalars since they cannot escape the fate of feeling the first term of Eq. (1) through the supergravity multiplet.

In this paper, we pursue an idea of unifying the Peccei-Quinn symmetry breaking scale ($\equiv$ the invisible axion scale) v and SUSY breaking scale M_{SB} at $10^{11}\sim 10^{12}$ GeV.

The invisible axion requires $SU(3)\times SU(2)\times U(1)$ singlet chiral superfield(s) X (and X'). We find that the minimal model which we will present in detail requires two left-handed chiral PQ singlets, X and X' , and two O'RaiFeartaigh (OR) singlets, Z and Z' . The vacuum expectation value (vev) of X and X' is characterized by v . Since $v \ll 10^{16}$ GeV (or 10^{17} GeV in the SUSY case) in the $SU(5)$ model, the electroweak singlets X and X' must not belong to 24_H (or 75_H) responsible for $SU(5)$ breaking at 10^{16} GeV. They can be $SU(5)$ singlets or another 24_H 's (or 75_H 's). Without specification, they can refer to any of these if proper group invariants are taken in the Lagrangian. In this paper, however, we assume that these are $SU(5)$ singlets.

We proceed to consider the superpotential of OR type hidden from low energy physics. The simple Polonyi form¹⁰⁾ $W_P = M_{SB}^2(Z+\beta)$ cannot relate the SUSY breaking scale M_{SB}^2 to v . The superpotential is taken as

$$W = (\lambda_1 X X' + m^2) Z + (\lambda_2 X X' + m'^2) Z' + \mu^3 + f Q \bar{Q}' X \quad (2)$$

where X and X' are PQ fields, Z and Z' are $SU(5)$ singlet OR fields, Q and $\bar{Q}'$ are left-handed chiral fields 5_L and $\bar{5}_L$, and μ^3 is introduced to make the cosmological constant zero. All chiral fields are left-handed. Note that we have not introduced the term $Q \bar{Q}' X'$. The mass parameters m and m' are of the same order and characterize the scales of the invisible axion and SUSY breaking.

Obviously, the superpotential W has two $U(1)$ symmetries whose phases are characterized by α and β ,

$$\begin{aligned} Z &\longrightarrow Z \\ Z' &\longrightarrow Z' \\ X &\longrightarrow e^{2i\alpha} X \\ X' &\longrightarrow e^{-2i\alpha} X' \\ Q &\longrightarrow e^{-i\alpha+i\beta} Q \\ \bar{Q}' &\longrightarrow e^{-i\alpha-i\beta} \bar{Q}' \end{aligned} \quad (3)$$

The $U(1)_\beta$ symmetry is reflecting the conservation of the Q fermion number, and the $U(1)_\alpha$ symmetry is identified as the desired Peccei-Quinn symmetry¹¹⁾. When X and/or X' develop vev's, the $U(1)_\alpha$ symmetry is broken and there will appear the invisible axion. The invisible axion is decoupled from the low energy sector at tree level, but it will couple at the two-loop level.

In SUGRA, the potential is

$$V = E \tilde{V} + \frac{1}{2} D_\alpha D^\alpha \quad (4)$$

$$E = \exp\left(\kappa^2 \sum_a |z_a|^2\right) \quad (5)$$

$$\tilde{V} = \sum_a \left| \frac{\partial W}{\partial Z_a} + \kappa^2 Z_a^* W \right|^2 - 3 \kappa^2 |W|^2 \quad (6)$$

where $\frac{1}{2}D_\alpha^2$ is the usual D term. In the limit $\kappa \rightarrow 0$, we obtain the global SUSY potential. In fact, we can use first the global SUSY potential for the determination of vev's and masses of scalars. Local SUSY corrects these values by tiny amounts. However, the corrections are the dominant ones for those parameters that are not determined by the global SUSY. We will use the full local SUSY potential for these parameters.

SUSY is broken spontaneously, because the following F terms cannot be simultaneously taken as zero,

$$\frac{\partial W}{\partial Z} = \lambda_1 X X' + m^2 \quad (7)$$

$$\frac{\partial W}{\partial Z'} = \lambda_2 X X' + m'^2 \quad (8)$$

$$\frac{\partial W}{\partial X} = X'(\lambda_1 Z + \lambda_2 Z') + f Q \bar{Q}' \quad (9)$$

$$\frac{\partial W}{\partial X'} = X(\lambda_1 Z + \lambda_2 Z') \quad (10)$$

$$\frac{\partial W}{\partial Q} = f X \bar{Q}' \quad (11)$$

$$\frac{\partial W}{\partial \bar{Q}'} = f X Q \quad (12)$$

where the fields represent scalar components of superfields. In general we require

$$\frac{\lambda_1}{\lambda_2} \neq \frac{m^2}{m'^2} \quad (13)$$

and both $\partial W/\partial Z$ and $\partial W/\partial Z'$ cannot be made zero. SUSY is broken.

Let us define

$$\tan \alpha = \lambda_2/\lambda_1, \quad \lambda^2 = \lambda_1^2 + \lambda_2^2 \quad (14)$$

$$\tan \beta = m'^2/m^2, \quad m_o^2 = m^2 + m'^2 \quad (15)$$

Then the minimum of the global SUSY potential takes the following vev's

$$\langle X X' \rangle = -\frac{m_o^2}{\lambda} \cos(\beta - \alpha) \quad (16)$$

$$\langle \lambda_1 Z + \lambda_2 Z' \rangle = 0 \quad (17)$$

$$\langle Q \rangle = \langle \bar{Q}' \rangle = 0 \quad (18)$$

From (7), (8), and (16), we obtain F terms of Z and Z',

$$Z_F = -m_o^2 \sin \alpha \sin(\beta - \alpha) \quad (19)$$

$$Z'_F = m_o^2 \cos \alpha \sin(\beta - \alpha) \quad (20)$$

which are non-zeros because of (13). These SUSY breaking scales are of order $m_o^2 \approx (10^{11} \sim 10^{12}) \text{ GeV}^2$. In SUGRA, the SUSY breaking scale is modified, which will be presented after evaluation of $\langle Z \rangle$ and $\langle Z' \rangle$. The fields feeling SUSY breaking directly are Z and Z'. It is important to note that vev's of Z and Z' are not determined by the scale m_o except the condition (17). To determine $\langle Z \rangle$ and $\langle Z' \rangle$, the full local SUSY potential is needed. The vev's $\langle Z \rangle$ and $\langle Z' \rangle$ are expected to be of order $\kappa^{-1} = (2.4 \times 10^{18} \text{ GeV})$. Also the relation (16) alone does not specify the magnitude of the individual vev of X and X'. If one vev is very large while the other is very small, we fail to have a correct scale for the invisible axion since the invisible axion scale is the larger one. We find, however, that they become the same when the local SUSY potential is taken into account,

$$x \equiv |X| = |X'| = \left| -(m_o^2/\lambda) \cos(\beta - \alpha) \right|^{1/2} \quad (21)$$

The reason is that the scalar fields X and X' have additional mass terms given in Eq. (1) [or Eq. (4)] due to the supergravity effect. At the vacuum given by (17), the second term of Eq. (1) is identically zero and the first term of Eq. (1) determines the individual vev as given by Eq. (21).

To determine vev's of Z and Z' , let us redefine the following vacuum values

$$\varepsilon \equiv \cos \alpha Z + \sin \alpha Z' \quad (22)$$

$$\mathfrak{z} \equiv \cos \beta Z + \sin \beta Z' \quad (23)$$

Certainly, ε and $\mathfrak{z}$ are not independent, but this definition is useful when $\varepsilon \ll \mathfrak{z}$. We have a condition for the vanishing cosmological constant

$$\tilde{V} = 0 \quad (24)$$

and the local minimum conditions

$$\frac{\partial \tilde{V}}{\partial \mathfrak{z}} = 0 \quad (25)$$

and

$$\frac{\partial^2 \tilde{V}}{\partial \mathfrak{z}^2} > 0 \quad (26)$$

where (25) is derived using (24), and (26) is derived using (24) and (25). These three relations become, respectively

$$\begin{aligned} \tilde{V} &= -\lambda^2 x^4 + m_o^4 + \frac{\kappa^4 \mathfrak{z}^2}{\sin^2(\beta - \alpha)} (m_o^2 \mathfrak{z} + \mu^3)^2 \\ &\quad + 2\kappa^4 x^2 (m_o^2 \mathfrak{z} + \mu^3)^2 - \kappa^2 (m_o^2 \mathfrak{z} + \mu^3) (m_o^2 \mathfrak{z} + 3\mu^3) \\ &= 0 \end{aligned} \quad (27)$$

$$\begin{aligned} \frac{\partial \tilde{V}}{\partial z} = & \frac{2\kappa^4 z}{\sin^2(\beta-\alpha)} (2m_o^4 z^2 + 3m_o^2 z \mu^3 + \mu^6) \\ & + 4\kappa^4 x^2 m_o^2 (m_o^2 z + \mu^3) \\ & - 2\kappa^2 m_o^2 (m_o^2 z + 2\mu^3) = 0 \end{aligned} \quad (28)$$

$$\begin{aligned} \frac{\partial^2 \tilde{V}}{\partial z^2} = & \frac{2\kappa^4}{\sin^2(\beta-\alpha)} (6m_o^4 z^2 + 6m_o^2 z \mu^3 + \mu^6) \\ & + 4\kappa^4 x^2 m_o^4 - 2\kappa^2 m_o^4 > 0 \end{aligned} \quad (29)$$

Let us note that $z = 0$ is a solution to Eq. (28). But we exclude this solution since it does not satisfy the local minimum condition (29). Because μ^3 is always accompanied by m_o^2 term, we rescale μ^3 by

$$\mu^3 = \xi m_o^2 z \quad (30)$$

where ξ is a dimensionless parameter. Also we neglect $\kappa^2 x^2$ term compared to other terms. By this approximation we have not exactly cancelled the cosmological constant. Its error is of the order $\kappa^2 x^2 m_o^4 \approx (10^{7.5} \text{ GeV})^4$ which can be excluded by a more accurate calculation. The relevant equations are then

$$\begin{aligned} \sin^2(\beta-\alpha) + \frac{\kappa^4 z^4}{\sin^2(\beta-\alpha)} (1+\xi)^2 \\ - \kappa^2 z^2 (1+\xi)(1+3\xi) = 0 \end{aligned} \quad (31)$$

$$\frac{\kappa^2 z^2}{\sin^2(\beta-\alpha)} (2+3\xi+\xi^2) - (1+2\xi) = 0 \quad (32)$$

$$\frac{\kappa^2 \mathcal{Z}^2}{\sin^2(\beta-\alpha)} (6+6\xi+\xi^2) - 1 > 0 \quad (33)$$

There exists only one solution to (31), (32) and (33),

$$\xi = \frac{1}{2} (-1 + \sqrt{3}) \quad (34)$$

$$\mathcal{Z} = \kappa^{-1} |\sin(\beta-\alpha)| \left(2\sqrt{3}/(3+2\sqrt{3}) \right)^{1/2} \quad (35)$$

Therefore, the vev's of Z and Z' are

$$Z = \frac{\sin \alpha}{\sin(\alpha-\beta)} \mathcal{Z}; \quad Z' = \frac{\cos \alpha}{\sin(\beta-\alpha)} \mathcal{Z} \quad (36)$$

which are of the order of the Planck mass κ^{-1} .

The SUSY breaking scale in SUGRA is given by

$$M_S^2 = \kappa^{-1} \sqrt{3} m_{3/2} = \sqrt{3} m_o^2 |\sin(\alpha-\beta)| \exp\left(\frac{1}{2+\sqrt{3}}\right) \quad (37)$$

Let us now proceed to discuss the mass spectrum of the model (2) and its phenomenological consequences. It has two PQ gauge singlet chiral fields, X and X' , two OR gauge singlet fields, Z and Z' , and two gauge non-singlet matter fields Q and $\bar{Q}'$. The Q and $\bar{Q}'$ acquire a Dirac mass $m_Q < 10^{11}$ GeV. The remaining PQ and OR fields contain four two-component fermions and eight real scalars.

In the leading order, the masses of the fermion partners are as follows

$$m_{\tilde{Z}_1 \tilde{X}_1} = \sqrt{2} \lambda x, \quad m_{\tilde{Z}_2} = 0, \quad m_{\tilde{X}_2} = 0 \quad (38)$$

where

$$\begin{aligned}
\tilde{Z}_1 &= \cos \alpha \tilde{Z} + \sin \alpha \tilde{Z}' \\
\tilde{Z}_2 &= -\sin \alpha \tilde{Z} + \cos \alpha \tilde{Z}' \\
\tilde{X}_1 &= \frac{1}{\sqrt{2}} (\tilde{X} + \tilde{X}') \\
\tilde{X}_2 &= \frac{1}{\sqrt{2}} (\tilde{X} - \tilde{X}')
\end{aligned} \tag{39}$$

The $\tilde{Z}_1$ and $\tilde{X}_1$ form a four-component Dirac fermion. The $\tilde{Z}_2$ field is the Goldstone fermion eaten by the gravitino through the super-Higgs mechanism. The two-component field $\tilde{X}_2$ is massless at the tree level and also does not acquire ¹²⁾ a soft mass of order $\frac{m_3}{2}$.

For the discussion of scalar masses, we redefine the PQ fields with a convenient phase convention as

$$\phi = \chi + \frac{1}{\sqrt{2}} (\rho_\phi + i a_\phi) ; \phi = X \text{ or } X' \tag{40}$$

Mass eigenstates are ρ_{X_1} , ρ_{X_2} , a_{X_1} , and a_{X_2} where

$$\begin{aligned}
X_1 &= \frac{1}{\sqrt{2}} (X + X') \\
X_2 &= \frac{1}{\sqrt{2}} (X - X')
\end{aligned} \tag{41}$$

Similarly, we define

$$\begin{aligned}
Z_1 &= \cos \alpha Z + \sin \alpha Z' \\
Z_2 &= -\sin \alpha Z + \cos \alpha Z'
\end{aligned} \tag{42}$$

To leading order, the masses are

$$\begin{aligned}
 m_{\rho_{X_1}}^2 &= m_{a_{X_1}}^2 = 2\lambda^2\chi^2, \\
 m_{\rho_{X_2}}^2 &= m_{3/2}^2, \quad m_{a_{X_2}}^2 = 0, \\
 m_{Z_1}^2 &= m_{Z_1^*}^2 = 2\lambda^2\chi^2 \\
 m_{Z_2}^2 &= m_{Z_2^*}^2 = m_{3/2}^2
 \end{aligned} \tag{43}$$

It is easy to check that $\text{Str}M^2$ is of order $\frac{m_{3/2}^2}{2}$.

The $a_{X_2} \equiv a$ field is the Goldstone boson of the PQ symmetry and becomes the invisible axion with tiny mass $\sim 10^{-5}$ eV. The Z_2, Z_2^* and ρ_{X_2} fields are massless at tree level, but acquire mass of order $\frac{m_{3/2}^2}{2}$ by the supergravity effect. The other scalars are heavy with masses of order 10^{11} GeV.

Among the heavy fields with masses of order 10^{11} GeV, the SU(5) non-singlets Q and $\bar{Q}'$ are presumably the lightest ones because of a small Yukawa coupling. In this case, there exists one stable lepton or slepton belonging to Q and $\bar{Q}'$. The existence of the stable particle corresponds to the conservation of the Q number. This stable particle might have annihilated with its antiparticle just below the cosmic temperature $T \lesssim m_Q$.

The light particles, $\tilde{Z}_2, \tilde{X}_2, Z_2, Z_2^*, \rho_{X_2}$, and a , need special attention for their cosmological roles.

The Goldstino field $\tilde{Z}_2$ and its scalar partner Z_2 are decoupled from other fields of the theory. Therefore, there does not exist a rationale that they were ever in equilibrium after the inflation. An inflationary universe can harmlessly accommodate these particles. This is the argument used in Ref. 13).

The cosmological and astrophysical implications of the invisible axion a have been extensively discussed in the literature (2)-6), 14). The axino, the fermion partner of the axion, does not acquire a soft mass term of order $\frac{m_{3/2}}{2}$ as mentioned before. However, it can acquire a mass at the three-loop order, in which case its mass can be made smaller than 100 eV. The axino does not cause a conflict between cosmology and particle physics.

The axino $\tilde{a} = \tilde{X}_2$ is a two-component Majorana fermion. The scalar partners of the axino are the axion a and the scalar field $s \equiv \rho_{X_2}$. We call this new particle s a saxino.

The main decay mode of the saxino is $s \rightarrow 2$ gluons (i.e., to pions) through the heavy quark loop with a partial lifetime $\tau_s(s \rightarrow 2g) = 64\pi^3 x^2 / \alpha_c^2 m_s^3$ which is $\sim 4 \times 10^{-4}$ sec for $\alpha_c = 0.13$, $x = 10^{11}$ GeV and $m_s = 100$ GeV. The cosmic temperature at this time scale is $\sim (90 N_{\text{rel}})^{1/4} / (\kappa t \pi)^{1/2} \sim 170$ MeV which is above the temperature of the nucleosynthesis. The decay $s \rightarrow 2$ photons does not have cosmological significance.

The saxino also decays to inert particles by the following effective interaction (see the Fig.)

$$\mathcal{L}_{\text{eff}} = \frac{1}{M} \tilde{a} \tilde{a} a s \quad (44)$$

where $M = 4m_Z / \lambda^2$ in our model. The partial lifetime is $3 \times 2^{12} \pi^3 M^2 / m_s^3$ which is 0.01 sec for $\lambda = 2$, $m_s = 100$ GeV and $x = 10^{11}$ GeV. (for a smaller λ , the partial lifetime is longer.) The B.R. is $\lambda^2 / (3 \times 2^{11} \alpha_c^2)$. The relativistic and inert axion and axino produced by (44) should not exceed the critical mass density, i.e.,

$$\text{B.R.}(s \rightarrow \tilde{a} \tilde{a} a) \cdot T_a \lesssim 10 T_\gamma \quad (45)$$

For $m_s = 100$ GeV and $T_\gamma = 0.17$ GeV, this condition gives $\lambda < 18 \alpha_c \approx 2.3$. This is the only condition we satisfy for the model to be consistent with cosmology.

We have obtained a common scale for the invisible axion and SUGRA, which is achieved by introducing two PQ singlets and two OR singlets. In this kind of model, the saxino decays relatively fast and its lifetime is of the order of 10^{-4} sec.

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FIGURE CAPTION

Saxino decays to an invisible axion and two axinos.