Ref.TH.3721-CERN

LOCAL HAMILTONIAN MONTE CARLO STUDY OF THE MASSIVE SCHWINGER MODEL,
THE DECOUPLING OF HEAVY FLAVOURS

J. Ranft ^{*)}

CERN - Geneva

A B S T R A C T

The massive Schwinger model with two flavours is studied using the local Hamiltonian lattice Monte Carlo method. Chiral symmetry breaking is studied using the fermion condensate as order parameter. For a small ratio of the two fermion masses, degeneracy of the two flavours is found. For a large ratio of the masses, the heavy flavour decouples and the light fermion behaves like in the one flavour Schwinger model.

^{*)} On leave from Sektion Physik, Karl-Marx-Universität, Leipzig, D.D.R.

The Schwinger model ¹⁾, that is, Quantum Electrodynamics of Dirac particles with masses μ_1 and charges g in $1 + 1$ dimensions, is a very interesting model for studying properties which are also properties of QCD in $3 + 1$ dimensions, like quark confinement and chiral symmetry breaking. The massive Schwinger model ²⁾ with one single flavour was studied on the lattice by Ranft and Schiller ³⁾ using the local Hamiltonian Monte Carlo method ⁴⁾. Here the same method is used for the massive Schwinger model on the lattice with two flavours. This model was also recently studied by Burkitt et al. ⁵⁾ using an Euclidean fermion Monte Carlo method. They remove the twofold remaining degeneracy of Susskind fermions in $d = 2$ Cartesian dimensions by the introduction of mass terms as recently proposed ⁶⁾. The successful working of such methods would open the way to a general use of the theoretically more attractive Susskind fermions ⁷⁾ in lattice calculations. Burkitt et al. ⁵⁾ find that both flavours are degenerate for a small ratio of the two fermion masses and that the heavy flavour decouples for a large ratio of the two masses.

In the present paper, the same problem will be studied using the local Hamiltonian method. In $d - 1 = 1$ spatial dimensions, Hamiltonian Susskind fermions are not degenerate and it is straightforward to give different masses to the two flavours. Therefore, the results of Burkitt et al. ⁵⁾ can be reproduced without using a method to lift the degeneracy. Further properties of the massive Schwinger model with two flavours, which will not be studied here, are

- i) the phase transition at a background field value $\alpha = 1/2$, which is expected ²⁾ to occur in the two flavour model in a similar way as was found in the one flavour lattice model by Ranft and Schiller ³⁾, and
- ii) the phase transition for vanishing masses of both fermions where, according to Coleman ²⁾, in contrast to the one flavour model, chiral symmetry is restored. This feature was demonstrated in the lattice calculations of Burkitt et al. ⁵⁾.

The lattice Hamiltonian of the massive Schwinger model with two flavours, using the same notations as in Ref. 3), has the form

$$H = \frac{1}{2} g^2 a \sum_n (L_n - \alpha)^2 + \frac{1}{2a} \sum_n (\chi_n^\dagger a_n^\dagger \chi_{n+1} + h.c.) + \frac{1}{2a} \sum_n (\phi_n^\dagger a_n^\dagger \phi_{n+1} + h.c.) + \mu_\chi \sum_n (-1)^n \chi_n^\dagger \chi_n + \mu_\phi \sum_n (-1)^n \phi_n^\dagger \phi_n \quad (1)$$

The Susskind fermions are represented by the fermion operators $\chi_n, \chi_n^\dagger$ (first flavour) and $\phi_n, \phi_n^\dagger$ (second flavour), satisfying

$$\begin{aligned} \{\chi_n^+, \chi_m\} &= \delta_{nm} \quad , \quad \{\chi_n, \chi_m\} = 0 \quad , \\ \{\phi_n^+, \phi_m\} &= \delta_{nm} \quad , \quad \{\phi_n, \phi_m\} = 0 \quad , \\ \{\chi_n^+, \phi_m\} &= 0 \quad , \quad \{\chi_n, \phi_m\} = 0 \quad . \end{aligned} \quad (2)$$

The gauge bosons are described in the occupation number representation by the operators L_n, a_n, a_n^+ , satisfying the commutation relations

$$[L_n, a_m^+] = \delta_{nm} a_m^+ \quad , \quad [L_n, a_m] = \delta_{nm} a_m \quad . \quad (3)$$

The state of the lattice at a given time t_i is given by the state vector

$$|t_i\rangle = \prod_n (\chi_n^+)^{l_n^i} (\phi_n^+)^{k_n^i} (a_n^+)^{m_n^i} |0\rangle \quad (4)$$

μ_χ and μ_ϕ are the masses of the two fermions in units of the spatial lattice spacing a . The index n runs over all spatial lattice sites $1 \leq n \leq N_x$. We will use the model here only for the vanishing background field $\alpha = 0$.

In the local Hamiltonian Monte Carlo method, the Hamiltonian is split into two parts H_1 and H_2 . H_1 contains all terms of (1) which refer to odd lattice sites n ; H_2 contains all terms of (1) with even n . Expectation values of operators O

$$\langle O \rangle = \text{Tr}(O e^{-\beta H}) / \text{Tr}(e^{-\beta H}) \quad , \quad (5)$$

where β is the inverse temperature, are calculated by inserting $2N_t$ complete sets of states at different times separated by $\delta t = \beta/N_t$.

$$\begin{aligned} \text{Tr}(e^{-\beta H}) &= \sum_{t_1, t_2, \dots, t_{2N_t}} \langle t_1 | e^{-\delta t H_2} | t_{2N_t} \rangle \langle t_{2N_t} | e^{-\delta t H_1} | t_{2N_t-1} \rangle \dots \\ &\quad \dots \langle t_3 | e^{-\delta t H_2} | t_2 \rangle \langle t_2 | e^{-\delta t H_1} | t_1 \rangle \end{aligned} \quad (6)$$

Up to terms of order $O(\delta t^2)$, the matrix elements in (6) factorize into elements referring only to the variables of two neighbouring lattice sites and the link in between. To this order, only the two site matrix elements given in the Table differ from zero. From these matrix elements, it is easy to work out all possible

transitions to be considered in the Monte Carlo calculation. We note that up to order $O(\delta t^2)$, no simultaneous transitions of both flavours in the same two-site block occur.

We perform our Monte Carlo calculation using a lattice of $N_x = 24$ spatial sites and $2N_t = 56$ time slices. The spatial lattice separation is $a = 1$ and the width of the time steps is $\delta t = 0.1$ or 0.2 . After a change of some lattice parameter like the fermion mass μ_i by a small increment, we heat the lattice again using typically 200 iterations. For each measurement, we average usually over 1000 to 1500 iterations of the full lattice. As initial lattice we choose a state where each odd site is occupied by fermions of both flavours and all even sites are unoccupied. At the beginning, all boson occupation numbers are equal to zero.

The fermion condensates of both flavours

$$\begin{aligned} \langle \bar{\psi} \psi \rangle_1 &= \langle \sum_n (-1)^n \chi_n^\dagger \chi_n \rangle, \\ \langle \bar{\psi} \psi \rangle_2 &= \langle \sum_n (-1)^n \phi_n^\dagger \phi_n \rangle \end{aligned} \quad (7)$$

are measured separately.

The results are presented in Figs. 1 to 3. In Fig. 1 we compare the fermion condensate in the model with two degenerate flavours $m = m_1 = m_2$ with the fermion condensate of the single flavour Schwinger model. In the degenerate two flavour model, we keep away from the massless case $m = 0$, where the chiral symmetry restoring phase transition should occur. It is clearly visible, that both models behave differently. In Fig. 2 we present the results for a fixed small mass of the second fermion $m_2 = 0.05$ as function of the mass m_1 of the first fermion. For masses differing by not more than a factor two, we find both fermions to be still degenerate, and both flavours are characterized by the same condensate as given in Fig. 1 for the model with $m_1 = m_2$. For growing mass of the first fermion $m_1 \geq 3m_2$ no degeneracy occurs. The condensate of the light fermion drops down to the value corresponding to the one flavour model with the mass $m = m_2$. The condensate of the heavy flavour rises. The same effect is also visible in Fig. 3 where we keep the mass of the second flavour large $m_2 = 1$ and plot both condensates as function of the mass m_1 of the first fermion. For $m_1 \leq 0.1 - 0.2$, the heavy fermion decouples and the light flavour has the same condensate as found in Fig. 1 for the massive one flavour Schwinger model.

Except for the sign of the fermion condensate, which should be negative using the lattice model as defined here and in Ref. 3), and which is also negative in the continuum model, our results agree qualitatively with the results of Burkitt et al.⁵⁾. Comparing our plots with the ones in Ref. 5), it should be noted that we do not subtract from our fermion condensate values the condensates obtained for free fermions. In the massive two flavour Schwinger model, the massive flavour decouples completely and the light flavour has the same properties as in the one flavour Schwinger model. Furthermore, for a small ratio of the two masses $m_1 \leq 2m_2$, both flavours are degenerate and behave like in the model with $m_1 = m_2$. From the agreement we might conclude that the method used⁵⁾ to lift the degeneracy of the Euclidean Susskind fermions by introducing mass terms⁶⁾ works satisfactorily.

ACKNOWLEDGEMENTS

The author acknowledges discussions with A.N. Burkitt giving details of his work and with A. Schiller together with whom the method used was applied to the one flavour model. It is a pleasure to thank the CERN Theory Division for its hospitality and support.

REFERENCES

- 1) J. Schwinger, Phys. Rev. 128 (1962) 2425;
J. Lowenstein and J. Swieca, Ann. Phys. (N.Y.) 68 (1971) 172.
- 2) S. Coleman, Ann. Phys. (N.Y.) 101 (1976) 339;
S. Coleman, R. Jackiw and L. Susskind, Ann. Phys. (N.Y.) 93 (1975) 267.
- 3) J. Ranft and A. Schiller, Phys. Lett. 122B (1983) 403;
A. Schiller and J. Ranft, Nucl. Phys. B225 [FS 9] (1983) 204.
- 4) J. Hirsch, D. Scalapino, R. Sugar and R. Blankenbecler, Phys. Rev. D26 (1982) 5033.
- 5) A.N. Burkitt, A. Kenway and R.D. Kenway, Edinburgh preprint (1983), submitted to the Brighton Europhysics Conference on High Energy Physics.
- 6) A.N. Burkitt, A. Kenway and R.D. Kenway, Phys. Lett. 128B (1983) 83;
P. Mitra and P. Weisz, Phys. Lett. 126B (1983) 355.
- 7) L. Susskind, Phys. Rev. D16 (1977) 3031;
P. Becher and H. Joos, Z. Phys. C15 (1982) 343.

FIGURE CAPTIONS

- Figure 1 : The fermion condensate $\langle \bar{\psi}\psi \rangle = \frac{1}{2}(\langle \bar{\psi}\psi \rangle_1 + \langle \bar{\psi}\psi \rangle_2)$ of the massive Schwinger model with two degenerate flavours $m_1 = m_2$ compared to the fermion condensate of the massive one flavour Schwinger model.
- Figure 2 : The fermion condensates $\langle \bar{\psi}\psi \rangle_1$ and $\langle \bar{\psi}\psi \rangle_2$ for a fixed small mass of the second fermion $m_2 = 0.05$ as function of the mass m_1 of the first fermion.
- Figure 3 : The fermion condensates $\langle \bar{\psi}\psi \rangle_1$ and $\langle \bar{\psi}\psi \rangle_2$ for a fixed large mass of the second fermion $m_2 = 1$ as function of the mass of the first fermion m_1 .

TABLE

Two site matrix elements of the massive Schwinger model with two flavours β and γ . The gauge boson occupation numbers at times t_i and t_{i+1} are m_i and m_{i+1} . In the graphs, occupied fermions β and γ are represented by solid lines and unoccupied ones by dotted lines. Within both parts of the table, each matrix element K of flavour β can be combined with any element L of flavour γ .

a)	K	flavour β	L	flavour γ	Matrix elements
	1		1		$\int_{m_i, m_{i+1}} \exp\left\{-\frac{1}{2}g^2 a \delta t (m_i - \alpha)^2\right\} \cdot U_\beta^K U_\gamma^L$
	2		2		$U_\beta^1 = \exp\left\{-(-1)^{\tilde{m}} \delta t \mu_\beta\right\}$
	3		3		$U_\beta^2 = 1$
	4		4		$U_\beta^3 = \exp\left\{-(-1)^{\tilde{m}} \delta t \mu_\beta\right\} \cosh\left(\frac{\delta t}{2a}\right)$
					$U_\beta^4 = \cosh\left(\frac{\delta t}{2a}\right)$
b)	K	flavour β	L	flavour γ	Matrix elements
	5		3		$\exp\left\{-\frac{\delta t}{4}g^2 a [(m_i - \alpha)^2 + (m_{i+1} - \alpha)^2]\right\} \cdot U_\beta^K U_\gamma^L$
	6		4		$U_\beta^5 = - \int_{m_i, m_{i+1}} \exp\left\{-\frac{\delta t}{2}(-1)^{\tilde{m}} \mu_\beta\right\} \cdot \sinh\left(\frac{\delta t}{2a}\right)$
					$U_\beta^6 = - \int_{m_i, m_{i+1}} \exp\left\{-\frac{\delta t}{2}(-1)^{\tilde{m}} \mu_\beta\right\} \cdot \sinh\left(\frac{\delta t}{2a}\right)$

Fig. 1

Fig. 2

Fig. 3