Ref.TH.3740-CERN

UNIFYING PHOTON AND PION ABSORPTION

W.M.Alberico *)

CERN - Geneva

M.Ericson

Université C.Bernard Lyon I, 69621 Villeurbanne, France

and

CERN - Geneva

and

A.Molinari

Istituto di Fisica Teorica, Università di Torino

and

Istituto Nazionale di Fisica Nucleare

Sezione di Torino, Italy

A B S T R A C T

We study the $2p-2h$ contribution to the deep inelastic inclusive electron scattering by including both the meson exchange currents and the nucleon - nucleon correlations. A satisfactory agreement with the available experimental data is achieved. In the same frame p -wave pion absorption at threshold is also examined; it is shown that a strong quenching is required to account for the π - nucleus absorptive potential deduced from pionic atoms data. Finally the conditions are explored, which allow to establish a link between magnetic photon and p -wave pion absorption.

*) On leave of absence from Istituto di Fisica Teorica, Università di Torino, Italy

A number of theoretical studies have been performed on the influence of mesons and Δ isobar in the spin - isospin nuclear response ¹⁾.

In previous works ²⁾⁻⁴⁾ we have considered the 1p-1h frame, stressing, in the energy region of the quasi-elastic peak, the importance of collective effects. The latter should entail, at intermediate momentum transfers, a marked contrast between the spin-transverse nuclear response function, quenched and hardened, and the spin-longitudinal one, enhanced and softened.

The transverse response, essentially due to the magnetic coupling of the photon, has been measured with deep inelastic, inclusive electron scattering ^{5),6)}. The data are compatible with the above predictions. Moreover in the region of deeper inelasticity they show an almost complete filling in of the dip between the quasi-elastic and Δ peaks. In this kinematical region the cross section arises mainly from two-particle two-hole (2p-2h) excitations.

This also holds for pion absorption at threshold, single nucleon emission being forbidden by energy-momentum conservation.

The aim of this letter is to establish a link ^{*)}, based on a microscopic approach, between magnetic photon absorption in the dip region and p-wave threshold pion absorption, which occurs at a similar energy.

Although in both processes the coupling is of spin-isospin nature (the isoscalar γ -absorption being negligible), the set up of such a connection is far from trivial since the photon couples transversally to the nucleonic spins ($\vec{\sigma} \times \hat{q}$), while the pion couples longitudinally ($\vec{\sigma} \cdot \hat{q}$).

*) The existence of a link between magnetic photon and p-wave π absorption has been envisaged by T.Ericson ⁷⁾.

Actually, precisely this difference lies at the origin of the pronounced contrast we predict ³⁾ between the transverse and longitudinal 1p-1h spin-isospin responses.

A similar contrast, which would hamper a simple connection between the two cross sections, is expected in the 2p-2h domain as well.

The way to overcome this difficulty has been shown, in a different context, by T.Ericson and Bernabeu ⁸⁾ with their assumption of axial locality. The idea is to split the system in small subunits (nucleons), so small that at this local level the nature of the coupling is no longer relevant, while differences persist at the level of the global system.

Let us elucidate this concept in the 1p-1h frame. In a Hartree-Fock approach, where only one nucleon is involved, the transverse and the longitudinal spin responses coincide. In an RPA approach, instead, the two responses display the above mentioned striking contrast due to their collective nature.

To be more precise in the spin-isospin longitudinal channel ($\sigma = \tau = 1$, $\sigma_z = 0$), the response of infinite nuclear matter reads

$$\mathcal{R}(q, \omega) = -\frac{4}{\pi \rho} \text{Im} \Pi(q, \omega) \quad (1)$$

where

$$\Pi^L(q, \omega) = \frac{\Pi^0(q, \omega)}{1 - 4 \frac{f_\pi^2}{\mu_\pi^2} \left(g' - \frac{q^2}{q^2 + \mu_\pi^2 - \omega^2/c^2} \right) \Pi^0(q, \omega)} \quad (2)$$

and ρ is the nuclear matter density.

In the spin transverse channel ($\sigma_z = \pm 1$), instead, neglecting the weak influence of the rho exchange, one has

$$\Pi^T(q, \omega) = \frac{\Pi^0(q, \omega)}{1 - 4 \frac{f_\pi^2}{\mu_\pi^2} g' \Pi^0(q, \omega)} \quad (3)$$

As a consequence the link we are searching for can only be envisaged when the latter currents do not play a major role. This depends upon the momentum q : for small q , as for instance in the real photon absorption, the graphs of Fig.2 are dominant (thus ruling out the connection with p-wave pion absorption), whereas the opposite occurs at larger momenta, as in deep inelastic electron scattering.

Let us therefore focus on the latter process. For the contributions to it arising from the two-body meson exchange currents (MEC), including the Δ one, we utilize the expression of Donnelly and Van Orden ⁹⁾, where only the pion is retained as the major carrier of the MEC.

Concerning the current associated with the nucleon-nucleon correlations, we have derived it by keeping only a spin-isospin force consistent with the one obtained by Dickhoff ¹⁰⁾ in a G-matrix calculation. This we achieve by considering pion exchange only, with a softened πNN vertex form factor (of monopole type with $\Lambda_\pi = 800$ MeV) and by incorporating, in the central term, the Landau-Migdal parameter g' .

The non-relativistic reduction of the Feynman diagrams (a - d) of Fig.1 then reads ¹¹⁾:

$$\begin{aligned} \vec{J}^{\text{corr}}(\vec{p}_1', \vec{p}_1, \vec{p}_2', \vec{p}_2) = & \frac{1}{V^2 k_C} \frac{f_\pi^2}{\mu_\pi^2} \left\{ \frac{1}{[2M\omega/\hbar - \vec{q} \cdot (\vec{p}_1' + \vec{p}_1 - \vec{k}_2)]} \right. \\ & \times [\chi_{s_1'}^\dagger i(\vec{\sigma} \times \vec{q})(\vec{k}_2 \cdot \vec{\sigma}) \chi_{s_1} \{ \mathcal{F}_E(\mu_s + 2g_2\mu_v) + g_2 \mathcal{F}_D(\mu_v + 2g_1\mu_s) \} + \\ & + (\vec{p}_1' + \vec{p}_1 - \vec{k}_2) \chi_{s_1'}^\dagger (\vec{k}_2 \cdot \vec{\sigma}) \chi_{s_1} \{ \mathcal{F}_E(1 + 2g_2) + g_2 \mathcal{F}_D(1 + 2g_1) \}] + \\ & + \frac{1}{[2M\omega/\hbar - \vec{q} \cdot (\vec{p}_1' + \vec{p}_1 + \vec{k}_2)]} [-i \chi_{s_1'}^\dagger (\vec{k}_2 \cdot \vec{\sigma})(\vec{\sigma} \times \vec{q}) \chi_{s_1} \{ \mathcal{F}_E(\mu_s - 2g_2\mu_v) + \end{aligned}$$

The two responses turn out to be quite dissimilar: nevertheless the transverse one is directly linked to a pionic quantity, namely the p-wave pion self energy or optical potential

$$2\hbar\omega \langle \vec{q} | U_{opt} | \vec{q} \rangle = -4 \frac{g_\pi^2}{\mu_\pi^2} q^2 \Pi^T(q, \omega) \quad (4)$$

which is manifestly of local character.

So we have illustrated how the principle of axial locality works in the problem of the 1p-1h nuclear response. Let us pursue the same scope in the 2p-2h frame, with the aim of connecting π and γ absorption.

Before doing this it should be observed that axial locality solves the problem stemming from the coupling, but not the one associated with kinematics. Indeed the two absorption processes take place, for a given energy (near $m_\pi c^2$), at different momenta: almost vanishing for threshold pions, sizeable for virtual photons (data are available for $q \cong 200 \div 400$ MeV/c).

The conjecture of a straightforward extrapolation in momentum between the two, which would allow an easy prediction of deep inelastic inclusive electron scattering starting from the p-wave pion optical potential, turns out to be inadequate.

The link, if it exists, is more subtle since the extrapolation, at a fixed energy, is not a smooth function of the momentum. This is the major difficulty we have to face.

To deal with it, let us start with a microscopic approach which accounts for all the Feynman diagrams entering into the γ or π absorption. They are displayed in Fig.1, where the graphs (a - d) correspond to the absorption by a pair of correlated nucleons and the graphs (e - h) to the absorption by the two-body current associated with the virtual Δ excitation.

For the photons one should add, of course, the contribution arising from the pion-in-flight and the contact currents, shown in Fig.2.

$$\begin{aligned}
 & + g_2 \mathcal{F}_D (\mu_V + 2g_1 \mu_s) \} - (\vec{p}'_1 + \vec{p}_1 + \vec{k}_2) \chi_{s'_1}^+ (\vec{k}_2 \cdot \vec{\sigma}) \chi_{s_1} \{ \mathcal{F}_E (1 - 2g_2) + \\
 & + g_2 \mathcal{F}_D (1 + 2g_1) \} \Big] \frac{1}{(k_2^2 + \mu_\pi^2)} \chi_{s'_2}^+ (\vec{k}_2 \cdot \vec{\sigma}) \chi_{s_2} + \\
 & + \{ 1 \rightleftharpoons 2 \} , \tag{5}
 \end{aligned}$$

where V is the volume enclosing the system, $\mu_s = \mu_p + \mu_n = 0.88$ and $\mu_V = \mu_p - \mu_n = 4.70$ the isoscalar and isovector magnetic moments and

$$\vec{k}_i = \vec{p}'_i - \vec{p}_i \quad (i = 1, 2) \tag{6}$$

$$\begin{aligned}
 \mathcal{F}_E &= (1 - \delta_{g_1 g_2}) (1 - \delta_{g'_1 g_1}) (1 - \delta_{g'_2 g_2}) \equiv \\
 &\equiv -(i/4g_1) \chi_{g'_1}^+ \chi_{g'_2}^+ (\vec{\tau}_1 \times \vec{\tau}_2)_3 \chi_{g_1} \chi_{g_2} \tag{7}
 \end{aligned}$$

$$\mathcal{F}_D = \delta_{g'_1 g_1} \delta_{g'_2 g_2} \tag{8}$$

the $g_i (= \pm \frac{1}{2})$ being the isospin variables of the nucleons.

The above current is usually ignored in the standard definition of the MEC, since it is naturally embodied in the impulse approximation when correlated nuclear wave functions are used.

In the present nuclear matter calculation, however, we utilize an uncorrelated plane wave basis and the diagrams of Fig.1 must be consistently included if all the terms at the level of one pion exchange are to be accounted for.

Moreover, since the pion propagators in (5) are taken in the static limit, the well-known cancellation¹²⁾ between the so-called recoil term and the renormalization of the nucleonic wave function does not occur.

Notice also that, at variance with the MEC, the correlation diagrams contribute, already in leading order of the non-relativistic expansion, to the charge longitudinal nuclear response. Therefore gauge invariance is not guaranteed for the electric component of (5).

The latter however is small in the kinematic domain we are concerned with, a necessary condition for establishing the link with pion absorption.

The 2p-2h transverse nuclear response in inelastic electron scattering can be calculated from the vacuum - (2p-2h) matrix elements of the two-body currents illustrated in Figs.1 and 2.

In Born approximation and neglecting Pauli exchange contributions, the standard procedure yields ⁹⁾ for the structure function

$$4\pi S_T(\vec{q}, \omega) = \frac{AV^5Mc^2}{2(2\pi)^9} \int d\vec{p} \theta(p-k_F) \int d\vec{p}' \theta(p'-k_F) \int d\vec{k} \theta(k_F-k) \cdot \\ \cdot \int d\vec{k}' \theta(k_F-k') \delta^{(3)}(\vec{q}+\vec{k}+\vec{k}'-\vec{p}-\vec{p}') \delta\left\{\hbar\omega - \frac{\hbar^2}{2M}(\vec{p}^2+\vec{p}'^2-\vec{k}^2-\vec{k}'^2)\right\} \cdot \\ \cdot \left(\delta_{ij} - \frac{q_i q_j}{q^2}\right) \sum_{\text{all } s} \sum_{\text{all } s'} J_i^\dagger(\vec{p}, \vec{k}; \vec{p}', \vec{k}') J_j(\vec{p}, \vec{k}; \vec{p}', \vec{k}'). \quad (9)$$

This formula can be interpreted in terms of the imaginary part of the bare 2p-2h polarization propagator. Here we account altogether for 121 perturbative contributions to it, some of which are displayed in Fig.3. The explicit expression of all the terms in eq.(9) can be found in ref.11.

In dealing with the nucleon-nucleon correlations we kept the intermediate nucleon states off the mass shell. This amounts to neglecting second order self-energy insertions on the nucleonic lines, which seems consistent, as we have already left out the Hartree-Fock effects in the 1p-1h nuclear response.

In Fig.4 the transverse nuclear response measured in deep inelastic electron scattering on ⁵⁶Fe ⁵⁾ is displayed for $q = 370$ MeV/c. The data are compared with the results of our calculation, which includes, beside the 2p-2h response of eq.(9), the 1p-1h RPA transverse response already considered in ref.3. ⁺)

⁺) The apparent cusp of the corresponding line is due to the sharp cutoff of the free Fermi gas momentum distribution. A more realistic one would result in a smoother global response.

The inclusion of 2p-2h excitations remarkably improves the agreement with the experiment, particularly in the energy domain of the dip. We notice that the correlations among nucleons yield a substantial contribution, but the one arising from the pion-in-flight and contact currents does not exceed 12%.

Turning now to pion absorption we evaluate the absorptive piece of the p-wave optical potential and compare it with the experimental value.

The simplest parametrization of this quantity reads

$$2\hbar\omega \text{Im} \langle \vec{q} | U_{\text{opt}} | \vec{q} \rangle = q^2 4\pi g^2 \text{Im} C_0 \quad (10)$$

where the value of $\text{Im} C_0$ extracted from the pionic atoms data is^{13),14)}

$$\text{Im} C_0 \approx (0.05 \div 0.06) \mu_\pi^{-6}. \quad (11)$$

However, to account for the dressing of the pion, quite often a different parametrization is proposed, namely

$$2\hbar\omega \langle \vec{q} | U_{\text{opt}} | \vec{q} \rangle = \frac{-q^2 \alpha(q, \omega)}{1 - g' \alpha(q, \omega)} \quad (12)$$

with

$$\alpha(q, \omega) = 4 \frac{f_\pi^2}{\mu_\pi^2} \Pi^0(q, \omega) - 4\pi g^2 \mathcal{E}_0 \quad (13)$$

where the complex quantity $\mathcal{E}_0$ accounts for the bare 2p-2h excitations.

The imaginary part of (12) is then

$$\begin{aligned} 2\hbar\omega \text{Im} \langle \vec{q} | U_{\text{opt}} | \vec{q} \rangle &= \\ &= \frac{q^2 4\pi g^2 \text{Im} \mathcal{E}_0}{\{1 - g' [4(f_\pi^2/\mu_\pi^2) \text{Re} \Pi^0 - 4\pi g^2 \text{Re} \mathcal{E}_0]\}^2 + \{g' 4\pi g^2 \text{Im} \mathcal{E}_0\}^2} \quad (14) \end{aligned}$$

which shows that the value of $\text{Im} \mathcal{E}_0$ crucially depends on both g' and $\text{Re} \mathcal{E}_0$. Indeed, at a given density, the real and imaginary parts of the optical potential do not uniquely determine these three parameters.

Tauscher has made an attempt ¹⁴⁾ to extract separately $\text{Re } \epsilon_0$, $\text{Im } \epsilon_0$ and g' from the whole set of the pionic atoms' data, by exploiting the different density dependence of the various terms in eq.(14). His analysis yields $g' = 0.4 \pm 0.17$, $\text{Re } \epsilon_0 = 0.2 \mu_\pi^{-6}$ and $\text{Im } \epsilon_0 = 0.11 \mu_\pi^{-6}$, the extreme value of g' (~ 0.6) being consistent with

$$\text{Im } \epsilon_0 = 0.18 \mu_\pi^{-6}, \quad \text{Re } \epsilon_0 \cong 0.4 \mu_\pi^{-6}. \quad (15)$$

These estimates for g' and $\text{Im } \epsilon_0$ are also in agreement with the fit of Stricker et al. ¹⁵⁾.

The large difference between (11) and (15) can be ascribed, as it clearly appears from expression (14), to processes of vertex renormalization for the pion absorbed by a pair of nucleons [Lorentz-Lorenz-Ericson-Ericson (LLEE) effect]. Some of the corresponding diagrams, irreducible with respect to pionic lines, are shown in Fig.5.

Our microscopic approach refers to the "bare" quantity $\text{Im } \epsilon_0$, which we have evaluated starting from the graphs of Fig.1, the photon lines being replaced by pionic ones ¹⁶⁾. In terms of the 2p-2h polarization propagator, $\Pi^{2p-2h}(q, \omega)$, the parameter ϵ_0 is given by

$$4\pi g^2 \epsilon_0 = -4 \frac{g_\pi^2}{\mu_\pi^2} \Pi^{2p-2h}(q=0, \hbar\omega = m_\pi c^2). \quad (16)$$

A direct computation yields

$$\text{Im } \epsilon_0 = 0.23 \mu_\pi^{-6} \quad (17)$$

which is reduced to

$$\text{Im } \epsilon_0 \approx 0.18 \mu_\pi^{-6} \quad (18)$$

when a binding energy of ~ 10 MeV for each nucleon absorbing the pion is accounted for.

These values are numerically compatible with the estimates of Tauscher ¹⁴⁾ and of Stricker et al. ¹⁵⁾. Moreover the comparison

of the above, bare, theoretical quantity with the dressed experimental value (11) confirms the existence of a strong LEE renormalization of the absorptive optical potential.

The importance of this result is further stressed by the recognition that the p-wave pion absorptive potential provides directly the high energy tail of the Gamow-Teller (GT) strength ¹⁷⁾. Therefore the quenching affecting this potential applies to the GT strength in the same energy region as well. It has been suggested ¹⁸⁾ that the same quenching mechanism occurs also at small frequencies.

Its origin, as it appears from formula (14), is associated with both the Δ -hole and the 2p-2h polarization propagators entering into the vertex renormalization graphs of Fig.5. To ascertain the relative importance, in different energy regimes, of these two components of the polarization propagator, the explicit evaluation of $\text{Re } \Pi^{2p-2h}$ (i.e. of $\text{Re } \mathcal{E}_0$) is needed.

This program has been partially carried out by Oset et al. ¹⁹⁾. These authors in fact have already pointed out the importance of the vertex renormalizations for the real part of the p-wave pion nucleus optical potential, although their calculation is restricted to the contributions to $\text{Re } \mathcal{E}_0$ arising from the Δ current only.

It is important to emphasize the role of the 2p-2h polarization propagator, the key quantity underlying the previous considerations. It is not only the convenient tool to account for the above mentioned vertex renormalization in the absorption of threshold pions, but also it allows to establish a simple connection between γ and π absorption.

Indeed the 2p-2h transverse structure function in inelastic electron scattering, eq.(9), can be recast in the form

$$S_T(q, \omega) = -\frac{A^2}{8\pi^2 q} \frac{\hbar^2 q^2}{M} \mu_V^2 G_M^2(q_\lambda^2) \text{Im } \Pi^{2p-2h}(q, \omega) \quad (19)$$

where G_M is the electromagnetic vertex form factor of the nucleon. Then the comparison between eqs.(16) and (19) permits to relate $\text{Im } \epsilon_0$ with the transverse nuclear structure function at an energy close to the pionic mass. The simplicity of this link stems from the near constancy in momentum, for a given energy, of $\text{Im } \Pi^{2p-2h}$, as we have numerically tested.

However this connection refers to bare processes and not to physical ones. Indeed the unrenormalized 2p-2h excitations seem to be adequate, as we have seen in Fig.4, for describing the inelastic electron scattering in the dip region, not so to deal with pion absorption. Thus the vertex renormalizations of Fig.5 are important for one process (π absorption) and not for the other (γ absorption). As a consequence they must display a rapid momentum dependence, being substantial at small q , but almost vanishing in the deep (e, e') domain.

This can be understood already in the 1p-1h frame: indeed $\text{Re } \Pi^N$ and $\text{Re } \Pi^\Delta$ almost cancel each other in the dip region, in contrast with the threshold pion domain (q small and $\hbar\omega \approx m_\pi c^2$) where $\text{Re } \Pi^N$ is vanishing, but $\text{Re } \Pi^\Delta$ is large and negative, thus yielding a substantial LLEE effect and illustrating the non-smooth momentum dependence of the vertex renormalization.

For the exact magnitude of the quenching effect, definitive conclusions can be drawn only after evaluating $\text{Re } \Pi^{2p-2h}$. Once this is done, the hope is that pion and virtual photon absorptions can be reconciled in a unique theoretical frame, namely a RPA with a kernel including both 1p-1h and 2p-2h excitations.

We thank Prof. T.E.O.Ericson and E.Oset for useful discussions; we also acknowledge L.Tauscher for communicating his results. A.M. wishes to thank the Theory Division of CERN for the kind hospitality.

References

- 1) E.Oset, H.Toki and W.Weise, Phys.Rep. 83 (1982)281 and references therein.
- 2) W.M.Alberico, M.Ericson and A.Molinari, Phys.Lett. 92B (1980) 153.
- 3) W.M.Alberico, M.Ericson and A.Molinari, Nucl.Phys. A379 (1982) 429.
- 4) W.M.Alberico, M.Ericson and A.Molinari, Nucl.Phys. A386 (1982) 412.
- 5) J.S.McCarthy, Nucl.Phys. A335 (1980) 27;
R.M.Altemus, thesis, Dept. of Physics, Univ. of Virginia (1980)
- 6) P.Barreau et al., Nucl.Phys. A402 (1983) 515.
- 7) T.E.O.Ericson, Proceed. of the Workshop "From Collective States to Quarks in Nuclei", ed. H.Arenhövel and A.M.Saruis, Bologna 1980, p.403
- 8) T.E.O.Ericson and J.Bernabeu, Phys.Lett. 70B (1977) 176.
- 9) J.W.Van Orden and T.W.Donnelly, Ann.Phys. 131 (1981) 451.
- 10) W.H.Dickhoff, Nucl.Phys. A399 (1983) 287.
- 11) W.M.Alberico, M.Ericson and A.Molinari, preprint CERN-TH 3635, to appear in Ann.Phys.
- 12) M.Chemtob and M.Rho, Nucl.Phys. A163 (1971) 1;
M.Gari and H.Hyuga, Z.Phys. A277 (1976) 291.
- 13) C.J.Batty et al., Nucl.Phys. A322 (1979) 445.
- 14) L.Tauscher, private communication.
- 15) K.Stricker, H.McManus and J.Carr, Phys.Rev. C19 (1979) 229.
- 16) K.Shimizu and A.Faessler, Nucl.Phys. A306 (1978) 311; Nucl. Phys. A333 (1980) 495.
- 17) M.Ericson, Phys.Lett. 120B (1983) 285.

- 18) B.Desplanques, 7^e Session d'Etudes Biennale de Physique Nucléaire
Aussois, 14-18 Mars 1983, Lycen 8302, S.6; Communication 7.5
at the Intern. Symposium, HESANS, Orsay 1983
- 19) E.Oset and W.Weise, Nucl.Phys. A319 (1979) 477; Nucl.Phys. A329
(1979) 365;
E.Oset, W.Weise and R.Brockmann, Phys.Lett. 82B (1979) 344.

Figure Captions

- Fig. 1 : The current associated with nucleon-nucleon correlations (diagrams a - d) and with the excitation of a Δ isobar (diagrams e - h).
- Fig. 2 : Diagrams representing the pion-in-flight and contact currents.
- Fig. 3 : Some examples of the diagrams contributing to the 2p-2h polarization propagator: in a) the photon photoproduces a pion or interacts with a pion in flight between two nucleons, in b) the photon annihilates a Δ already present in the ground state of nuclear matter or creates a Δ -h excitation out of the medium; graphs c) are associated with the absorption of a virtual photon by a correlated pair of nucleons while d) represent some interferences between the above processes.
- Fig. 4 : The separated transverse magnetic response in ^{56}Fe at $q = 370 \text{ MeV}/c$ as a function of $\hbar\omega$. The experimental points (bars) are taken from ref.5. The dot-dashed line is the 1p-1h RPA response; the dotted line is the global 2p-2h response, the dashed line its contribution from the MEC alone and the continuous line represents the total 1p-1h, 2p-2h transverse response. The Fermi wave number is $k_F = 1.20 \text{ fm}^{-1}$.
- Fig. 5 : Diagrams representing the vertex renormalization for the absorption of a pion in the nuclear medium.

FIG. 1

FIG. 2

Fig. 3

Fig. 4

Fig. 5