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YANG-MILLS THEORIES COUPLED TO $n = 2$
SUPERGRAVITY: HIGGS AND SUPER-HIGGS EFFECTS
IN ANTI DE SITTER SPACE

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A B S T R A C T

All stationary points of the scalar potential of pure Yang-Mills theories minimally coupled to $n = 2$ gauged supergravity are found. For arbitrary Yang-Mills groups G , the supersymmetry-preserving extremum does not break G . All other stationary points break both the Yang-Mills group and $n = 2$ supersymmetry in anti de Sitter space. For $G = SU(N)$, we explicitly construct locally stable solutions with maximal residual symmetry as well as solutions with unbroken $SU(3) \otimes SU(2) \otimes U(1)$.

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Spontaneously broken supersymmetric gauge field theories [1] have recently become of interest as possible candidates for unified theories of fundamental interactions. Particular attention has been devoted to the local version of supersymmetric models as candidates for unifying gravity to the other basic forces [2]. In fact, when supersymmetry is gauged, the Einstein action is unified with non-gravitational interactions, and massive gravitinos, through the super-Higgs effect [3], make possible a bridge between gravity and particle physics at the energy scale of electroweak interactions. Consistent schemes for unification of supergravity with electroweak and strong interactions have been proposed in the framework of $n = 1$ supergravity coupled to matter [4] with a massive gravitino of typical mass $m_{3/2} = O(M_W)$. These schemes [5] assume a supersymmetry-breaking scale M_S of order [6]

$$M_S = O(\sqrt{M_W M_P}) = O(10^{11} \text{ GeV})$$

intermediate between the weak interaction scale M_W and the Planck mass M_P .

On the other hand, Grand Unified Theories (GUTs) of electroweak and strong interactions introduce a new scale M_X , in general smaller than M_P , and larger symmetries may indeed be present at such high scales. In particular, if supergravity is believed to be the ultimate local field theory unifying the currently known fundamental interactions, it is tempting to believe that larger supersymmetries should be relevant at very high energies to make the ultraviolet behaviour of the quantum theory as mild as possible.

The ultimate candidate for a unified theory manifesting extended supersymmetry is believed to be $n = 8$ supergravity [7] or some version of Kaluza-Klein eleven-dimensional supergravity [8]. Since for the moment these theories seem far from correct in their description of the presently known low-energy physics, it is of interest to explore the consequence of the simplest extension of $n = 1$ supersymmetry to a higher symmetry, with the aim of drawing definite conclusions about the symmetry breaking, particle content, and mass spectrum. The simplest candidate for such a higher symmetry is at present $n = 2$ supersymmetry [9], which implies in its

local version the couplings of $n = 2$ matter and gauge multiplets to $n = 2$ supergravity, the latter describing the graviton, two Majorana gravitinos, and a massless vector field [10].

In the present work, we confine ourselves to considering the interaction of $n = 2$ gauge (vector) multiplets coupled to $n = 2$ supergravity in the absence of $n = 2$ matter hypermultiplets [11-12].

In particular, when $n = 2$ vector multiplets, gauging an arbitrary Yang-Mills group G , are coupled to the gauged version of $n = 2$ supergravity [13], spontaneous breaking of the gauge group G as well as of $n = 2$ supersymmetry can simultaneously occur, because of the scalar fields of the vector multiplets acquiring non-vanishing vacuum expectation values.

When matter is absent and when the Yang-Mills $n = 2$ Lagrangian is "minimally"*) coupled to $n = 2$ supergravity, it will be shown that general features emerge that will enable us to establish a general classification of all stationary points of the scalar potential [12] of the corresponding Lagrangian field theory. As a by-product it is possible to classify all gauge symmetry-breaking patterns purely induced by $n = 2$ supergravity couplings and thus obtain the quantum numbers of any given matter multiplet with respect to the unbroken gauge group.

Our basic results are the following: the general scalar potential of $n = 2$ Yang-Mills theories "minimally" coupled to $n = 2$ supergravity depends on two arbitrary parameters (beyond the Planck mass $M_P = \sqrt{8\pi/k}$), g and g' being respectively the Yang-Mills gauge coupling and the gauge coupling constant of the $SO(2)$ group of gauged $n = 2$ supergravity. We study the stationary points of the scalar potential as a function of g and g' .

If $g' = 0$ there are no extrema which break supersymmetry. We find a set of degenerate vacua which can induce the Brout-Englert-Higgs mechanism and break G in Minkowski space, as was the case of global supersymmetry.

If $g' \neq 0$, the situation is completely different. There is a unique stationary point (a local maximum) which breaks neither supersymmetry nor the gauge

*) We call "minimal" the coupling of $n = 2$ vector multiplets to $n = 2$ supergravity as defined in ref. 12.

group G . It corresponds to massless multiplets in anti de Sitter space. All other stationary points break both $n = 2$ supersymmetry and the gauge group G . They correspond to the combined Higgs and super-Higgs effects of $n = 2$ supergravity. We recall that since the gauge multiplets are not charged with respect to the $SO(2)$ gauge symmetry of $n = 2$ supergravity, this latter symmetry remains unbroken, and consequently both supersymmetries are simultaneously broken. In the absence of matter multiplets with non-vanishing vacuum expectation values, the cosmological constant is always negative; so the spontaneous breakdown takes place in anti de Sitter space [14].

For $G = SU(N)$, we give all gauge symmetry-breaking patterns. They are characterized by all sets of (non-negative) integers $m_1, m_2, \dots, m_N$ such that

$$\sum_{n=1}^N n m_n = N \quad (1)$$

The residual gauge symmetry which corresponds to such a set is

$$H = SU(m_1) \otimes SU(m_2) \otimes \dots \otimes SU(m_N) \otimes U(1)^{p-1}$$

where p is the number of non-zero m_n 's. The $n = 2$ supersymmetry is broken, except in the case $m_1 = N$, where $H = G$. The maximal unbroken symmetry is then $H_{\max} = SU(N-2) \otimes U(1)$.

To check the stability of these non-trivial stationary points, we computed the scalar masses for $H = H_{\max}$, from the part of the potential quadratic in the fields. We then used the criterion of stability in anti de Sitter space obtained by Breitenlohner and Freedman [15]. It turns out that these stationary points are stable if the ratio g'^2/g^2 is large enough.

For $N \geq 7$, the breaking pattern $SU(N) \rightarrow SU(3) \otimes SU(2) \otimes U(1)$ is possible^{*)}. Interestingly enough, such an unconventional embedding of the usual low-energy gauge group, obtained from the $n = 2$ supergravity potential, still allows us

^{*)} The $N = 10$ case is an exception as will later become clear from the study of the general breaking pattern of $SU(N)$. See Eq. (1).

[for instance in the case of SU(7) and SU(8)] to obtain spin $\frac{1}{2}$ states with the correct assignments of colour and electroweak quantum numbers, when matter hypermultiplets [9] are added. These fields, which could be identified with quarks and leptons, are unavoidably accompanied by mirror particles and by exotic states, making the fermions not completely realistic at the present stage.

We now proceed to derive and discuss our results.

The $n = 2$ vector multiplet consists of a vector field W_μ , two Majorana spinors $\Omega_\pm$, and a complex scalar field z , all transforming according to the adjoint representation of G [12]. The usual (global) D-term [1] reads

$$D = g[z, z^+] \quad (2)$$

(we adopt a matrix notation $z = a + ib$ with a and b Hermitian). The scalar potential reads [12]

$$V(z, z^*) = X_0^4 \text{Tr}(D^2) - 16 \frac{g'^2}{k^4} X_0^2 - 8 \frac{g'^2}{k^4} \quad (3)$$

where

$$X_0 = \frac{1}{\sqrt{1 - k^2 \text{Tr}(zz^*)}} \quad (4)$$

This non-polynomial function of z may be rewritten as a fourth order polynomial by defining*)

$$X = X_0 z \quad (5)$$

Then:

$$\begin{aligned} V(X, X^*) &= g^2 \text{Tr}([X, X^+]^2) \\ &\quad - 16 \frac{g'^2}{k^2} \text{Tr}(XX^*) - 24 \frac{g'^2}{k^4} \end{aligned} \quad (6)$$

*) In ref. 12 the Lagrangian was obtained in terms of the X variables. In the variable z the scalar fields have standard kinetic terms with a Kähler potential as given by Eq. (20) below.

The first term of this potential corresponds to the gauge potential of $n = 1$ supersymmetry; it is positive definite and vanishes in directions for which $[X, X^+] = 0$, i.e. D is zero [16]. Thus, in the absence of g' , supersymmetry remains unbroken since the vacuum expectation value (v.e.v.) of D is zero. The scale of gauge symmetry breaking is, however, arbitrary and the cosmological constant is zero.

When the g' coupling constant is introduced, $V(X, X^+)$ becomes unbounded from below. The v.e.v.'s are given by stationary points of the potential which satisfy^{*)}

$$0 = \frac{dV}{dX_i} = -4g^2 f_{ijk} f_{lmk} X_j^* X_l X_m^* - 16g'^2 X_i^* \quad (7)$$

These stationary points correspond to locally stable solutions in anti de Sitter space if the following stability condition is fulfilled [15]

$$\frac{m^2}{\langle V \rangle} < 3/4 \quad (8)$$

where m^2 is the most negative eigenvalue of the scalar mass matrix given by the quadratic part of the potential, and $\langle V \rangle$ is the value of the potential for this solution. Equation (7) leads immediately to

$$\text{Tr} (\langle A \rangle \langle B \rangle) = 0 \quad (9)$$

*) We use the following conventions: $X = A + iB$ (with A and B Hermitian) is developed on a set of Hermitian generators T_i , $X = (1/\sqrt{2}) X_i T_i^1$, normalized with $\text{Tr} (T_i T_j) = 2\delta_{ij}$ and $[T_j, T_k] = 2if_{jkl} T_l$. We then have

$$\begin{aligned} \text{Tr} (X X^+) &= X_i X_i^* = A_i A_i + B_i B_i \\ [X, X^+] &= i f_{jkl} X_j X_k^* T_l = -2i f_{jkl} A_j B_k T_l \end{aligned}$$

From now on, we set $k = 1$.

$$\text{Tr} (\langle A \rangle^2) = \text{Tr} (\langle B \rangle^2) \quad (10)$$

$$\begin{aligned} \langle X_0 \rangle^4 \text{Tr} (\langle D \rangle^2) &= 8g'^2 \text{Tr} (\langle X \rangle \langle X \rangle^\dagger) \\ &= 16g'^2 \text{Tr} (\langle A \rangle^2) \end{aligned} \quad (11)$$

$$\langle V \rangle = -24g'^2 \left(1 + \frac{2}{3} \text{Tr} (\langle A \rangle^2) \right) \quad (12)$$

for the v.e.v.'s. Two kinds of solutions can occur. First, all v.e.v.'s are zero. Supersymmetry and gauge symmetry remain unbroken and $\langle V \rangle = -24g'^2$. The quadratic term in the potential gives a universal squared "mass" $m_u^2 = -16g'^2$ to all scalar fields which verifies $m_u^2 = (2/3)\langle V \rangle$. This result is precisely what is expected from an actually massless particle in an anti de Sitter space. Moreover, unbroken supersymmetry and gauge symmetry indicate that all scalars have to remain massless. This stable solution is the only stationary point of V in the case where there is no D^2 term, either because g is zero, or if the gauge group is Abelian ($f_{ijk} = 0$).

If, however, some v.e.v.'s are non-zero, supersymmetry and gauge symmetry will be broken. From Eq. (7), it is apparent that non-zero v.e.v.'s of scalar fields will be of order g'/g . The supersymmetry-breaking scale $\sqrt{D}$ will then be of order $\sqrt{g'^2/g}$ and the cosmological constant is always negative.

The Goldstone fermions, which after the super-Higgs effect are eaten up by the two gravitinos, are given by

$$\Omega_{G\pm} = \text{Tr} (\langle D \rangle \Omega_{\pm})$$

($\Omega_{\pm}$ are the two multiplets of Majorana fermions belonging to the $N = 2$ Yang-Mills vector multiplet).

Equation (7) allows us to work out the general stationary points of $V(X, X^\dagger)$, for a given gauge group G . In the case $G = \text{SU}(N)$ we obtain the following

solution: $\langle X \rangle = \langle A \rangle + i\langle B \rangle$ can be written in block-diagonal form, with m_n blocks of dimension n , n being an arbitrary integer, and with the obvious constraint [Eq. (1)] $N = \sum_{n=1}^N n m_n$. Each possible v.e.v. is then characterized by the N integers

$$m_1, m_2, \dots, m_N$$

Each of these blocks of dimension n , in the basis where $\langle A \rangle$ is diagonal, reads

$$\langle A \rangle = \frac{g'}{g} \text{diag}(n-1, n-3, \dots, -(n-1)) \quad (13)$$

$$\langle B \rangle = \begin{pmatrix} 0 & B_1 & 0 & \dots & 0 & 0 \\ B_1^* & 0 & B_2 & \dots & 0 & 0 \\ 0 & B_2^* & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & B_{n-1} \\ 0 & 0 & 0 & \dots & B_{n-1}^* & 0 \end{pmatrix} \quad (14)$$

with

$$|B_j|^2 = \frac{g'^2}{g^2} j(n-j) \quad (15)$$

$\langle A \rangle$ and $\langle B \rangle$ are obviously traceless in each block and verify:

$$\text{Tr}(\langle A \rangle^2) = \text{Tr}(\langle B \rangle^2) = \frac{g'^2}{3g^2} (n+1)n(n-1) \quad (16)$$

The invariance of a given solution characterized by $m_1, m_2, \dots, m_N$ is

$$SU(m_1) \otimes SU(m_2) \otimes \dots \otimes SU(m_M) \otimes U(1)^{p-1}$$

where p is the number of non-zero m_n and $M = [N/2]$ is the largest value for which m_M can be larger than 1. Let us give two simple examples.

G = SU(2): There are two solutions:

$$\text{I: } m_1 = 2 \quad m_2 = 0 \quad : \quad \langle X \rangle = 0 \\ \text{SU(2) unbroken.}$$

$$\text{II: } m_1 = 0 \quad m_2 = 1 \quad : \\ \langle A \rangle = \frac{g'}{g} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \langle B \rangle = \frac{g'}{g} \begin{pmatrix} 0 & e^{i\alpha} \\ e^{-i\alpha} & 0 \end{pmatrix} \\ \text{SU(2) completely broken.}$$

G = SU(3): There are three solutions:

$$\text{I: } m_1 = 3 \quad m_2 = 0 \quad m_3 = 0 \quad : \quad \langle X \rangle = 0 \\ \text{SU(3) unbroken.}$$

$$\text{II: } m_1 = 1 \quad m_2 = 1 \quad m_3 = 0 \quad : \\ \langle X \rangle = \frac{g'}{g} \begin{pmatrix} 1 & ie^{i\alpha} & 0 \\ ie^{-i\alpha} & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{SU(3)} \longrightarrow \text{U(1)}$$

$$\text{III: } m_1 = 0 \quad m_2 = 0 \quad m_3 = 1 \quad : \quad \text{SU(3)} \\ \langle X \rangle = 2 \frac{g'}{g} \begin{pmatrix} 1 & ie^{i\alpha/\sqrt{2}} & 0 \\ ie^{-i\alpha/\sqrt{2}} & 0 & ie^{i\beta/\sqrt{2}} \\ 0 & ie^{-i\beta/\sqrt{2}} & -1 \end{pmatrix} \quad \begin{matrix} \text{completely} \\ \text{broken} \end{matrix}$$

Notice that the solution $0 = m_1 = m_2 = \dots = m_{N-1}, m_N = 1$ always breaks SU(N) completely. Supersymmetry breaking is thus compatible with a large variety of gauge symmetry-breaking patterns. However, the embedding of the unbroken $\text{SU}(m_1) \otimes \text{SU}(m_2) \otimes \dots \text{SU}(m_M)$ subgroup into SU(N) is restricted to be such that

$$\begin{aligned} \underline{N} &= (\underline{m_1}, \underline{1}, \dots, \underline{1}) \\ &\oplus 2(\underline{1}, \underline{m_2}, \underline{1}, \dots, \underline{1}) \\ &\oplus \dots \\ &\oplus M(\underline{1}, \underline{1}, \dots, \underline{1}, \underline{m_M}) \end{aligned} \tag{17}$$

It is tempting at this point to look for $SU(N)$ breaking into $G_0 = SU(3) \otimes SU(2) \otimes U(1)$. It is clear, however, that all embeddings of G_0 , as defined by Eqs. (13) and (14), will lead to exotic quantum numbers when matter hypermultiplets [9] are considered. These multiplets contain a doublet of Weyl fermions and complex scalars transforming under G in conjugate representations. Let us consider the simplest candidate model which breaks into G_0 , based on $SU(7)$, with $m_1 = 3$ and $m_2 = 2$. The unbroken $SU(3) \otimes SU(2) \otimes U(1)$ subgroup is defined through the embedding

$$\begin{aligned}
 \underline{7} &= (\underline{3}, \underline{1}, 2/3) \oplus 2(\underline{1}, \underline{2}, -1/2) \\
 \underline{21} &= 2(\underline{3}, \underline{2}, 1/6) \oplus 3(\underline{1}, \underline{1}, -1) \\
 &\quad \oplus (\underline{\bar{3}}, \underline{1}, 4/3) \oplus (\underline{1}, \underline{3}, -1) \\
 \underline{35} &= 3(\underline{3}, \underline{1}, -1/3) \\
 &\quad \oplus 2(\underline{\bar{3}}, \underline{2}, 5/6) \oplus (\underline{3}, \underline{3}, -1/3) \\
 &\quad \oplus 2(\underline{1}, \underline{2}, -3/2) \oplus (\underline{1}, \underline{1}, 2)
 \end{aligned} \tag{18}$$

The antisymmetric tensors of $SU(7)$ contain all the fields necessary to classify quarks and leptons (with mixed chirality) and additional $SU(2)$ and $U(1)$ exotic states. Since $\underline{7}$ contains only one $SU(3)$ -triplet, there will be no colour exotics. Moreover, the electric charge of all leptonic states [i.e. $SU(3)$ -singlets] is an integer, whilst coloured states have charges $\pm 4/3, \pm 1/3, \mp 2/3$. It is not a trivial result that symmetry breaking precisely selects a $U(1)$ group which is a candidate for a weak hypercharge. This fact is indeed not a peculiarity of $SU(7)$. The same occurs using $SU(8)$ with, however, a few differences. The embedding of $SU(3) \otimes SU(2) \otimes U(1)$ is the following:

$$\begin{aligned}
 \underline{8} &= 2(\underline{3}, \underline{1}, 1/6) \oplus (\underline{1}, \underline{2}, -1/2) \\
 \underline{28} &= 3(\underline{\bar{3}}, \underline{1}, 1/3) \oplus (\underline{1}, \underline{1}, -1) \\
 &\quad \oplus (\underline{6}, \underline{1}, 1/3) \oplus 2(\underline{3}, \underline{2}, -1/3)
 \end{aligned}$$

$$\begin{aligned} \underline{56} = & 3(\underline{\bar{3}}, \underline{2}, -1/6) \\ & \oplus 2(\underline{8}, \underline{1}, 1/2) \oplus 4(\underline{1}, \underline{1}, 1/2) \\ & \oplus 2(\underline{\bar{3}}, \underline{1}, -5/6) \oplus (\underline{6}, \underline{2}, -1/6) \end{aligned}$$

$$\begin{aligned} \underline{70} = & 3(\underline{3}, \underline{1}, 2/3) \oplus 3(\underline{\bar{3}}, \underline{1}, -2/3) \\ & \oplus (\underline{6}, \underline{1}, -2/3) \oplus (\underline{\bar{6}}, \underline{1}, 2/3) \\ & \oplus 4(\underline{1}, \underline{2}, 0) \oplus 2(\underline{8}, \underline{2}, 0) \end{aligned} \quad (19)$$

We obtain states with quantum numbers of quarks and leptons, and colour and electric charge exotics. Leptonic and coloured states can have electric charges $\pm \frac{1}{2}$, whilst there are some SU(3)-triplets with charges $\pm \frac{1}{6}$ and $\pm \frac{5}{6}$. The weak hypercharge obtained from SU(8)-breaking seems less attractive than in the SU(7) case. The extension of this analysis to larger SU(N) groups is straightforward.

This discussion of course does not give any indication of the possibility of separating left-handed and right-handed fermions inside an $n = 2$ matter multiplet, whose coupling to $n = 2$ supergravity can certainly modify the analysis of both the symmetry-breaking pattern and the cosmological constant problem.

We would now like to discuss the problem of the stability of the symmetry-breaking stationary points of the potential. To check the stability we need the "mass" spectrum^{*)} of the scalar fields (or at least the most negative squared mass) obtained from the quadratic terms in the Taylor expansion of $V(z, z^*)$, and from the structure of the scalar kinetic terms which read [12]:

$$\mathcal{L}_{KIN} = - (M_{KIN})^j_i (\partial_\mu z^i) (\partial^\mu z_j^*) \quad (20)$$

*) Recall that in anti de Sitter space, these "masses" do not correspond to the physical masses of states.

with

$$(M_{KIN})_i^j = \langle 2 \frac{\partial}{\partial z^i} \frac{\partial}{\partial z_j^*} \ln X_0 \rangle \quad (21)$$

Equivalently,

$$(M_{KIN})_i^j = \langle X_0 \rangle^2 \left(\delta_i^j + \langle X \rangle^j \langle X \rangle_i^* \right) \quad (22)$$

Thus, apart from an over-all $\langle X_0 \rangle^2$ factor, the kinetic terms for directions with non-zero v.e.v.'s are not canonically normalized and the mass matrix has to be correspondingly rescaled.

However, the mass spectrum shows several universal features which do not depend on the choice of the gauge group G . Scalar fields which correspond to generators T^i of the little group of $\langle z \rangle$, $[T^i, \langle z \rangle] = 0$, do not receive any mass contribution from the D^2 part of the potential. They only have the "universal squared mass" $-16g'^2$ and always fulfill the stability requirement since

$$\frac{m^2}{\langle V \rangle} = \frac{2}{3(1 + \frac{2}{3} \text{Tr}(\langle A \rangle^2))} < \frac{2}{3} \quad (23)$$

Notice that the non-zero v.e.v.'s shift the ratio from $2/3$, so that the scalars are no longer massless states as the corresponding gauge bosons are. Other general results can be obtained. The "Higgs particle", i.e. the scalar state collinear to the v.e.v. $\langle z \rangle = \langle a \rangle + i\langle b \rangle$ has a squared mass $+32g'^2\langle X_0 \rangle^2$, whilst the orthogonal state spanned by $\langle b \rangle - i\langle a \rangle$ [see Eqs. (9) and (10)] is as usual a Goldstone state. The states collinear to $\langle b \rangle + i\langle a \rangle$ and $\langle a \rangle - i\langle b \rangle$ have squared "masses" $-32g'^2$, which satisfies the stability criterion if

$$\text{Tr}(\langle A \rangle^2) > 7/6 \quad (24)$$

This inequality leads to a lower bound of order 1 on the ratio g'^2/g^2 since

$$\text{Tr}(\langle A \rangle^2) = O(1) \cdot g'^2/g^2$$

These last general results solve completely the $G = \text{SU}(2)$ [or $\text{SO}(3)$] case for which Eq. (7) requires

$$\langle X \rangle_{\text{SU}(2)} = \frac{g'}{g} \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} \quad (25)$$

i.e. $\text{Tr} (\langle A \rangle^2) = \text{Tr} (\langle B \rangle^2) = 2g'^2/g^2$. This v.e.v. breaks $\text{SU}(2)$ completely; one thus obtains three Goldstone states, a Higgs particle with positive squared mass $32g'^2 (1 + 4g'^2/g^2)$, and two scalar states with squared mass $-32g'^2$. Stability is ensured as long as

$$g'^2/g^2 > 7/12 \quad (26)$$

The largest unbroken gauge invariance with broken supersymmetry in the case $G = \text{SU}(N)$ is $\text{SU}(N-2) \otimes \text{U}(1)$. It is obtained by breaking the $\text{SU}(2)$ factor in the maximal subgroup $\text{SU}(N-2) \otimes \text{SU}(2) \otimes \text{U}(1)$ of $\text{SU}(N)$, by using the $\langle X \rangle_{\text{SU}(2)}$ solution [Eq. (25)]. The scalar mass spectrum is given in Table 1 [states are classified according to $\text{SU}(N-2) \otimes \text{SU}(2)$]. Since the most negative squared mass is the same as in the $G = \text{SU}(2)$ case, stability will be ensured when the condition (26) holds.

We would like to make some concluding remarks. In the present paper we have found all stationary points and symmetry-breaking patterns of $n = 2$ $\text{SU}(N)$ -Yang-Mills theory minimally coupled to $n = 2$ supergravity. An interesting fact is that we found solutions with an unbroken $\text{SU}(3) \otimes \text{SU}(2) \otimes \text{U}(1)$, with matter multiplets having the right assignments of colour and electroweak quantum numbers to be identified with quarks and leptons. This is not a trivial results because of the unconventional embedding of $\text{SU}(3) \otimes \text{SU}(2) \otimes \text{U}(1)$ in $\text{SU}(N)$ obtained from the $n = 2$ supergravity potential. Obviously, at the present stage, the spectrum does not look realistic because of the presence of mirror and exotic states. Furthermore, the possibility of having naturally massless fermions in such theories remains to be discussed.

An obvious generalization would be to extend this classification to all groups other than $\text{SU}(N)$ and to groups under which the hypermultiplets transform in irreducible representations [12].

We have shown that all solutions have a negative cosmological constant. The only way to make the cosmological constant possibly vanishing is to add hypermultiplets whose scalars take a non-vanishing vacuum expectation value and (or) to depart from the minimal coupling of supergravity to the Yang-Mills action. In this case the breaking pattern of the gauge group could be rather different from the one discussed in the present article. Also hypermultiplets having non-vanishing v.e.v.'s would be needed in order to break the $SO(2)$ gauge symmetry of $n = 2$ supergravity. This would be a necessary condition to get different "masses" for the two gravitinos of $n = 2$ supergravity.

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Table 1

SU(N-2) $\otimes$ SU(2) states	Squared masses
$(\underline{1}, \underline{3})$	3 Goldstone states 3 states with squared masses $+32g'^2 (1 + 4g'^2/g^2)$ $-32g'^2$ $-32g'^2$
$(\underline{\text{Adjoint}}, \underline{1})$ $\oplus (\underline{1}, \underline{1})$	$2(N-2)^2$ real states with squared mass $-16g'^2$
$(\underline{N-2}, \underline{2})$ $\oplus (\underline{\overline{N-2}}, \underline{2})$	$4(N-2)$ Goldstone states $4(N-2)$ states with squared masses $-24g'^2$