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THE DEUTERON PROPERTIES

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A B S T R A C T

The low energy deuteron parameters are discussed in view of their physical information and their constraints on the description of the deuteron in terms of more exotic subsystems. Important recent progress has recently been achieved on the understanding of the root mean square radius, the quadrupole moment, the D/S parameter and the asymptotic S state amplitude. There is a strong model-independent interlinkage of various quantities, which holds independent of the detailed model. Strong evidence is found for quantitative dominance of pionic interactions, particularly for the D/S ratio and quadrupole moment.

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Our understanding of NN interactions and their electromagnetic effects have traditionally been very heavily influenced by a handful of basic quantities in the deuteron. These are: the binding energy ϵ , the magnetic moment μ_d , the quadrupole moment Q , the "deuteron radius" r_d , the asymptotic value of the S wave function A_s and the asymptotic D/S ratio η .

Since similar quantities exist also in other nuclear systems, it is worthwhile to enquire why the present ones have become such extraordinary touchstones in nuclear physics. There are three basic reasons for this:

- first, they are all known to an experimental precision which is very high;
- second, most of them are strongly interlinked between themselves; this imposes strong constraints in any description;
- third, because of the highly well-defined conditions, it is routine to compare the predictions of these quantities in any serious model and to compare them to the very precise experimental values.

The last condition may sound trivial, but it is not. The fact that theoreticians return to a description of the same quantities in depth means that a number of smaller corrections are well understood. In this manner, the significance of the high experimental precision becomes fully relevant. These quantities rapidly separate theoretical chaff from wheat, and holds new approaches up to a rapid examination of their practical worth. This is of particular importance at the present moment, when the nature of NN interactions is being re-examined in the light of QCD. The numerical experimental values of the various quantities are given in Table 1 [excellent previous versions are found in Refs. 1 and 2] and are commented on below. While the main present developments concern Q , r_d , A_s and η , I will also discuss, for completeness, ϵ and μ_d .

Before this, however, it is useful to examine briefly the deuteron in the potential approach which, of course, has its limits. The usual deuteron equations for the coupled S and D states are

$$\begin{aligned} u'' &= [\alpha^2 + U_{00}]u + U_{02}w \\ w'' &= [\alpha^2 + 6/r^2 + U_{22}]w + U_{20}u, \end{aligned} \quad (1)$$

where the $U_{ij} = MV_{ij}$ are the potentials. The asymptotic wave functions u and w are normalized so that

$$\begin{aligned} u &\rightarrow A_S e^{-\alpha r} \quad \text{as } r \rightarrow \infty \\ w &\rightarrow \eta A_S e^{-\alpha r} \end{aligned} \quad (2)$$

A crucial property of the potentials is the notoriously weak central interaction U_{00} . In fact, if one would substitute the attractive central OPEP interaction in the S state and ignore the D wave coupling, the πNN coupling constant must be increased by a factor of three to produce binding, quite apart from the fact that short-range interaction is repulsive. In the D state U_{22} is repulsive and does not produce binding. The deuteron binding is thus a crucial property of the S-D channel coupling. It is mainly produced by the OPEP tensor potential as was observed long ago by Glendenning and Kramer³. The asymptotic S and D wave normalizations are the deuteron "coupling constants". They are directly and externally measurable in various ways as a matter of principle. The S wave normalization governs in particular the radius r_d , while the quadrupole moment is proportional to A_S^2 and also, very roughly, to η .

The S wave amplitude is related to the deuteron effective range $\rho(-\epsilon, -\epsilon)$ by the relation

$$A_S^2 \approx A_S^2(1+\eta^2) \equiv \frac{2\alpha}{[1-\alpha\rho(-\epsilon, -\epsilon)]} \quad (3)$$

This quantity has proved difficult to determine directly with sufficient accuracy. On the other hand, the closely related value $\rho(-\epsilon, 0)$, the mixed effective range is directly measurable and is defined by

$$\rho(-\epsilon, 0) \equiv \frac{2}{\alpha} [1 - (\alpha a_t)^{-1}] \quad (4)$$

The triplet scattering length a_t is in practice fixed by the incoherent cross-section $\sigma_0 = \pi(3a_t^2 + a_S^2)$. The experimental values by Houk⁴ and Dilg⁵,

2043.6(2.2) fm² and 2049.1(1.4) fm² are inconsistent within error bars. Since there is no obvious reason to discard any one of them, we use them both to obtain

$$\begin{aligned} \rho_{\text{Houk}}(-\epsilon, 0) &= 1.750(5) \\ \rho_{\text{Dilg}}(-\epsilon, 0) &= 1.760(5) \\ \hline \rho_{\text{av}}(-\epsilon, 0) &= 1.755(4) \text{ (2ppt)} \end{aligned} \quad (5)$$

Models for the deuteron incorporating OPEP characteristically give the effective range $\rho \approx 1.76$ fm to within some per cent, but the result has some model-dependence, and is far less accurate than experiments. The *difference* $\delta\rho = \rho(-\epsilon, -\epsilon) - \rho(-\epsilon, 0)$ is, however, nearly zero, and a magnitude smaller than might have been dimensionally expected. This is a natural consequence of OPEP as pointed out by Noyes⁶. Klarsfeld et al.⁷ in a recent manuscript, have found that this difference correlates very accurately with the exact value for the triplet scattering length a_t in numerous models (see Fig. 1).

FIGURE 1

The deuteron effective range $\rho(-\epsilon, -\epsilon)$ and the mixed effective $\rho(-\epsilon, 0)$ versus a_t^{-1} in various models according to Klarsfeld et al.⁷. Note that the *difference* $\delta\rho = [\rho(-\epsilon, -\epsilon) - \rho(-\epsilon, 0)]$ is very well determined by a_t^{-1} and that this difference is very model-independent in this variable.

Let us now return to experimental deuteron parameters and their interpretation in more detail.

*Binding energy*⁸

$$\epsilon = 2.224\,579(9) \text{ MeV (4 ppm)} \quad (6)$$

This quantity is known to a precision far outside any theoretical possibility, and it is in this sense irrelevant. It is, however, extremely important that theoretical deuteron descriptions have this quantity correct, even if this means some minor phenomenological adjustments at short range. The reason is that the deuteron scale parameter is the inverse wave number $R \equiv \alpha^{-1}$.

$$\alpha^{-1} \equiv R = 4.31896(2) \text{ fm (5 ppm)} \quad (7)$$

using relativistic kinematics. The non-relativistic value often used

$$\alpha_{NR}^{-1} \equiv R_{NR} = 4.31768(2) \text{ fm} \quad (8)$$

differs by 0.3 parts per thousand. This difference is nearly, but not quite irrelevant, since it does contribute about $2 \times 10^{-3} \text{ fm}$ to the experimentally determined mixed effective range parameter $\rho(-\epsilon, 0)$ known to about $\pm 5 \times 10^{-3} \text{ fm}$.

*Magnetic moment*⁹

$$\mu_d = 0.857\,406(1) \text{ n.m. (1 ppm)} \quad (9)$$

Once more this quantity is known to precision far beyond theoretical power. The most remarkable feature of the magnetic moment is that it is given to high accuracy as the sum of the neutron and proton values

$$\mu_n + \mu_p = 0.879\,696 \text{ n.m.} \quad (10)$$

The discrepancy is only

$$\Delta\mu_d = -0.02229 \text{ n.m.} \quad (11)$$

i.e., -2.6%. This, however, does not give full justice to the comparison, since there is a very substantial cancellation between μ_n and μ_p , which eliminates the isovector nuclear magnetic moment. A better measure of the scale would seem to be

$$\Delta\mu_d / (|\mu_n| + |\mu_p|) = -0.5\% \quad (12)$$

This result is most remarkable, since it means that whatever further descriptions are chosen, the nucleonic results must be preserved to within a fraction of a per cent.

Let us briefly examine various sources of corrections to the nucleonic sum. These fall into three categories.

A) Corrections assuming point nucleons. Since the magnetic moment occurs from currents, it is clear that contributions from orbital motion of the nucleons are important in principle. The fact that the deuteron is part of the time in a D state will generate a well-defined correction in any potential description, and this is the basis for the classical determination of the D state probability of the deuteron. However, even for pointlike nucleons, there are several additional, well-defined, corrections generated by minimal coupling from the LS and LL potentials. The physics of these corrections has been very well discussed by Scheerbaum¹⁰. It is in practice of very short range and therefore quite model-dependent with contributions of a few per cent.

B) Isobar (Δ, N^*) components enter the deuteron wave function. This depletes the nucleon content and the nucleon contributions to μ_d . At the same time it adds the isobar contributions proportional to their probability and their magnetic moments. Both of these quantities are imperfectly known.

C) At short distances, the deuteron may be in a six-quark or more complex state. Like for the isobar case, the contribution will depend both on the probability and the magnetic moment of this state.

In view of the fact that the magnetic moment is only one single number, there are clearly too many possibilities for a reliable test of any of these contributions. In fact, it would not even be trivial to test the point nucleon assumption well. As a consequence, I cannot but agree with the statement of Thomas² in the Vancouver Conference: "An apparently innocuous, well-understood property has turned into a nightmare".

In spite of the many possibilities, the magnetic moment is still a major restriction. It is far from trivial to accommodate more than about 2% at most of any exotic admixture into the deuteron wave function without actively provoking a discrepancy in the magnetic moment (a fortuitous agreement could, of course, occur, but should then be explained).

Let us now consider the remaining quantities for which substantial experimental and theoretical progress has taken place recently.

The deuteron radius r_d

This quantity is determined from highly accurate values for the q^2 term in the deuteron form factor as measured in the ratio of eD/ep scattering in the low q^2 region. The deuteron charge radius $\langle r_c^2 \rangle^{\frac{1}{2}}$ is in a sense the primary

number, but this is corrected for neutron and the proton form factors (or charge radii) so as to have an "intrinsic" deuteron radius. This defines the deuteron radius as

$$r_d^2 = \langle r_c^2 \rangle - \langle r_p^2 \rangle - \frac{3}{4} \frac{1}{M_p^2} + 6(dG_{en}/dq^2)_{q^2=0} \quad (13)$$

Both the proton and neutron form factors are known to a sufficient accuracy so that no new uncertainties of significance is introduced by this procedure. There are two determinations of r_d , inconsistent within the quoted accuracy^{*}). The determination of Bérard et al.¹² gives $r_d = 1.9650(45)$ fm, while Simon et al.¹¹ find $1.9576(68)$ fm. There is no obvious preference of one with respect to the other, so that the average value is $r_d = 1.9627(38)$ fm. This number includes meson-exchange contributions, etc., since no attempt has been made to include any many-body corrections.

One may well ask whether it is appropriate to introduce a definition of r_d , which implicitly assumes that the deuteron consists of nucleons. The nucleon size correction is about 7.5%, while the experimental numbers are quoted to a precision of 2 to 3 parts per thousand. While the mathematical definition of r_d causes no problem, it might well be questioned whether it has much sense to correct for nucleon size in the deuteron to a precision of a few per cent in the correction term. The answer to this is yes: the spatial regions which are affected by extended nucleon corrections extend to 4 to 8 fm which is very large. To a precision of a few per cent and even beyond in the correction, it is appropriate simply to subtract off the nucleon size. I have, however, not found a discussion about this point in the literature.

The deuteron radius is very closely related to the asymptotic S wave amplitude A_s in a nearly model-independent way, at least on the level of present accuracy.

The radius is related to the wave function by

$$r_d^2 = \frac{1}{4} \int_0^\infty r^2 (u^2 + w^2) dr + \text{MEC} + \text{ISOBAR} + \text{QUARKS, etc.} \quad (14)$$

The overwhelming bulk of the contribution comes from the asymptotic S state $u \rightarrow v = A_s e^{-\alpha r}$ which gives

$$r_c^{\text{as}} = \frac{1}{4} \alpha^{-\frac{3}{2}} A_s \approx 1.98 \text{ fm} \quad (15)$$

^{*}The definition of r_d by Simon et al.¹¹ differs from that of Bérard et al.¹² in that the Darwin-Foldy term $3/4 M_p^{-2}$ has not been eliminated. In the numbers quoted below, the modern value for the neutron form factor has been used, introducing a small correction to Ref. 12.

i.e., the experimental value to about 1%. The following corrections must be considered: a) the D state contribution of $+0.023$ fm, i.e., $+1.2\%$; this term is model-independent to present accuracy; b) the shape correction to $u(r)$ inside the asymptotic region ($-2 \pm 0.2\%$); this correction is not well investigated, but seems to be quite stable in different approaches (see below for a more detailed discussion); it is the main source of uncertainties; c) meson exchange corrections are of the order of $+0.002$ to 0.004 fm¹³, i.e. $+0.1$ to $+0.2\%$, and are not yet of importance in present discussions; d) isobar and quark effects contribute $<0.1\%$, the reason being that there are of short range and contribute only indirectly by a change in A_s ^{14,15}; this, however, is automatically included by using the *experimental* value for A_s .

In order to visualize the dependence of r_d with A_s in various well-known deuteron models or models for NN interactions, some of these results are displayed in Fig. 2 (unfortunately, r_d is unavailable in several models). The linear relation implied by the asymptotic wave function is clearly seen to be a

FIGURE 2

The variation of r_d with A_s for a number of current NN models. Note the very exact linear correlation independent of potential details. I am indebted to D.W.L. Sprung for several of the numerical values and to R. Machleidt for the information on the Bonn potential (HM). The same relation has been independently observed by Sprung, Klarsfeld and Martorell (private communication)¹⁶.

general feature nearly independent of other dynamical features. In order to see the shape correction more clearly, I show in Fig. 3 the percentage shape correction to r_d/A_s versus A_s . Since the correction is small, and not much precision is needed, a good guide to the functional dependence is given even by very crude S wave functions. From Hulthén wave functions, the correction is expected to be given by $-\frac{5}{9}(\alpha\rho)^3$ to leading order in terms of the effective range. Indeed, the various points scatter closely around this line. In view of the precision, it is quite possible that the scatter reflects numerical inaccuracy rather than model dependence.

From these results, it is possible to obtain the first direct determination of the asymptotic S wave amplitude using Fig. 2, as indicated in Table II. This result, taken together with the result of Eq. (5) establishes *experimentally*

$$\delta\rho = -0.018(13) \text{ fm} \quad (16)$$

According to Fig. 1, the expected value of $\delta\rho$ of $+0.012$ fm has opposite sign and is nearly model-independent. The discrepancy of $0.030(13)$ fm is slightly more than two standard deviations, so there is no major internal inconsistency in the data. In spite of this discrepancy, the present analysis establishes for the first time that direct experimental data give $\delta\rho \approx 0$.

FIGURE 3
The percentage size correction of r_d/A_s versus A_s for the same models. Note the small model-dependence. The solid line is the size correction using Hulthén wave functions.

as is expected from OPEP. The "natural" magnitude of $|\delta\rho|/\rho$ is about 5% while OPEP models in general give $|\delta\rho|/\rho \leq 0.5\%$. An elimination of the experimental inconsistencies would clearly be of considerable interest as a more accurate test.

The D/S ratio η

This quantity has been notoriously difficult to determine experimentally. In the past few years, three different methods of high precision have become available, so that this quantity now is known to a few per cent. The methods are all based on the orientation of the deuteron using tensor polarization, which singles out the D state. All but one of the experiments are first generation ones, so that it is not obvious that all precautions have been taken for high precision results both in the experiments and in the analysis. This was the topic of the deuteron workshop for one and a half days just before this Conference. The present results are given in Table III

Two words of caution must be given. First, the accuracy of the analytical extrapolation is not well established at the present high level of precision and may be a source of systematic error. This problem does not affect subCoulomb stripping, which therefore may be more useful for future improvements. Second, the accuracy of the average value is dominated by one single experiment¹⁸, and may give a too optimistic view of the actual precision once systematics are included.

Quadrupole moment Q

The standard literature value $Q = 0.2860(15) \text{ fm}^2$ (5 ppt) is due to Reid and Vaidya²¹. This is based on a theoretical analysis using variational techniques of highly accurate molecular beam experiments. A new value $Q = 0.28590(30) \text{ fm}^2$ (1 ppt) has just become available²⁰. This is based on an improved analysis by Bishop and Cheung²², as well as the realization that the variational techniques are exceptionally appropriate for the present case. The experimental value has now reached a precision which is presently outside theoretical reach.

With this survey of the experimental quantities, let us now turn to their significance.

As a first example, consider the question of the deuteron wave function as related to the experimental constraints. In a contribution to the Conference, Sprung et al.²³ explicitly construct the S and D wave functions using the following assumptions and procedures:

- take Q and r_d as constraints using inequalities (the MEC corrected value for Q , and the corrected r_d from Ref. 11 was used);
- assume η known;
- assume the long-range part of interaction known, whether OPEP or an improved OPEP based on theory;

- integrate u and w inwards from infinity consistent with the constraints to a value $r = R$;
- vary η and insist on the validity of the inequalities which follow from the constraints.

Their conclusion is that η is constrained to a small band of possible values. They also conclude that $u(r)$ is very well determined outside 1 fm, while $w(r)$ is rather ill-determined inside about 1.6 fm. As an example for the Turreil-Reuben-Sprung potential $\eta_{\text{TRS}}(R=1.6 \text{ fm}) = 2.67(5) \times 10^{-2}$. In Fig. 4, their results are shown using OPEP with the Paris potential for comparison. The interest of this method is that it does not enquire into the short-range behaviour of the interaction. It is clear that such an approach yields a value for η in very good agreement with experiments, which implicitly implies that Q and r_d fix η to a considerable degree. This approach permits one to visualize the model-dependence of these quantities far better than in the past. Unfortunately, the method used loses its power in the region inside about 1.5 fm, and cannot be systematically extended to shorter distances; still, it is most instructive outside this range.

FIGURE 4

The S and D wave functions as constructed by Sprung et al.²³ using OPEP in the external region. The Paris wave functions are given for convenient comparison.

A second example is a recent explicit construction of η and Q by Rosa-Clot and myself²⁰. Our purpose has been to obtain a very quantitative test of the accuracy of pionic effects in nuclear physics. For this we have developed a method that controllably eliminates contributions inside about 1 fm for these quantities.

The idea is most simply seen as follows, using a potential framework. The equation for the D wave

$$w^u = (\alpha^2 + 6/r^2 + U_{22})w + U_{20}u \quad (17)$$

has strong repulsion both because of the centrifugal barrier as well as due to the very important OPEP contribution to U_{22} . In this repulsive potential, the D wave is generated by the source function $U_{20}u$ from the S state via the tensor potential. An exact formal solution for η can now be expressed using the exact regular solution $\mathcal{J}_2(r)$ for the homogeneous D state equation.

$$\eta \equiv \int_0^\infty \eta(r) dr = M\sqrt{8} \int_0^\infty r \mathcal{J}_2(r) V_T(r) \bar{u}(r) dr \quad (18)$$

Here $\bar{u} \rightarrow e^{-\alpha r}$ as $r \rightarrow \infty$. The idea is now that one does not need to know much about the S wave, and that $\mathcal{J}_2(r)$ goes very fast to zero near the origin because of the strong repulsion.

In order to see the power of this formulation, Fig. 5 gives $\eta(r)$ versus r for iterated pure OPEP and the Paris potential using the same $\bar{u}$. The two agree to 4%, which is very remarkable as a starting approximation. Even the OPEP Born approximation is good to 20%. It is important that the region inside 1 fm is unimportant and controllable. The basic problem is not to improve the accuracy, but how not to spoil it.

FIGURE 5

The integrand for the D/S ratio with iterated OPEP and with the Paris potential according to Eq. (18). The same $\bar{u}$ is used. Note that about one third of the integral originates between 1 and 2 fm and very little inside 1 fm. The OPEP value is an excellent first approximation (from Ericson and Rosa-Clot²⁰).

The dependence of the exact S wave function is very weak indeed (see Fig. 6). Even a purely asymptotic wave gives poor but not unreasonable results, while realistic $\tilde{u}(r)$ introduce errors of less than 1% with the uncertainty coming in the peak region of the integrand. It is very important to note that u and w are not required to normalize to unity, so that isobar-quark admixtures are easily accommodated with no change.

The corrections and uncertainties are the following.

A) The 2π potential (-4.3%)

This contribution is theoretically well founded up to the exchange of ≈ 7 pion masses. The contributions are generated by a πN scattering (" Δ "), $\pi\pi$ phase shifts (of which the well-determined P wave exchange ($I=1, J=1$) is very dominant) and the crossed and uncrossed 2π exchange between nucleons. These contributions are all accurately fixed apart from an over-all (improvable) $\pm 15\%$ normalization uncertainty. The uncertainty in η from this source is about $\pm 0.6\%$. The theoretical contributions agree well with the results of the Paris potential even after the latter has had phenomenological adjustments inside 0.8 fm.

B) The πN coupling constants

To a precision of a few parts in a thousand the effective coupling constant is the weighted mean or charged and neutral pions

$$f^2 = \frac{2f_c^2 + f_0^2}{3} \quad \mu = \frac{2\mu_c + \mu_0}{3} \quad (19)$$

FIGURE 6

Dependence of the D/S integrand (18) on the S wave function. Even extremely poor wave functions like $\exp(-ar)$ give a fair result and normal wave functions introduce nearly negligible model dependence (from Ericson and Rosa-Clot²⁰).

Until very recently the experimental neutral coupling indicated a pseudovector coupling with $f_C^2 = f_0^2 = 0.0790^{24}$. A recent new value of the π^0 coupling constant from pp forward dispersion relations by Kroll²⁵ gives

$$f_{\text{exp}}^2 = 0.776(9) \quad (20)$$

This is -2% lower than previously and is consistent with the pseudoscalar coupling. With this new value, a major source of worry about consistency has been removed.

C) The πNN form factor

This quantity is the largest single uncertainty in the present analysis. The form factor can be parametrized for small t as

$$K(t) = \frac{\Lambda^2 - \mu^2}{\Lambda^2 + t} \quad (21)$$

corresponding to an equivalent uniform radius

$$R = \sqrt{t_0}/\Lambda \quad (22)$$

This parametrization is valid whatever the origin of the form factor, which could be due to extended quark bags. In a pion exchange language, the form factor requires at least a 3π exchange.

The effect on the integrand for η is seen in Fig. 7 for various Λ and R . In view of the close agreement of η with theory, it is very awkward to accept a phenomenological $\Lambda \lesssim 1000$ MeV, i.e., more than a -3% reduction. This indicates $R \lesssim 0.65$ fm.

Other information on the form factors is hard to obtain, but NN scattering data indicate large Λ ²⁶, consistent with the present observation. Dispersion relations for Λ indicate it to be ≥ 1000 -1400 MeV/c²⁷. On the other hand, quark bag models favour $\Lambda \lesssim 700$ MeV/c²⁸, indicating a conflict.

A similar analysis can be made of the quadrupole moment with $Q = Q_1 + Q_2$

$$\begin{aligned} Q_1 &= 1/\sqrt{50} \int_0^\infty r^2 u w dr \\ Q_2 &= -1/20 \int_0^\infty r^2 w^2 dr \end{aligned} \quad (23)$$

The D state contribution is -6% of the total and is easy to evaluate to the required accuracy.

With the previous Green function technique, one can write Q_1 identically as

FIGURE 7

Dependence of the D/S integrand on the π NN form factor. The integrand decreases by 3.1%, 8.5% and 19% for the three cases, which correspond to $R = 0.63$ fm, 0.84 fm and 1.26 fm. Note how the contributions are modified up to rather large r as soon as $\Lambda \lesssim 1000$ MeV (from Ericson and Rosa-Clot²⁰).

$$Q_1 \equiv A_S^2 \times (\text{constant}) \times \int_0^\infty F(r) \eta(r) dr \quad (24)$$

The integrand consists of a weight function $F(r)$, which is nearly model-independent apart from the very short-range region, and the previous integrand $\eta(r)$ (see Fig. 8). As a consequence, one expects Q and η to contain very similar

FIGURE 8

The quadrupole weight function $F(r)$ and the D/S integrand. The folded value is nearly model-independent (from Ericson and Rosa-Clot²⁰).

information, apart from the fact that the quadrupole moment emphasizes the intermediate range somewhat more. This is well borne out displaying Q/A_S^2 versus η in units which removes the over-all proportionality of the integrands to the πNN coupling constant. Figure 9 shows that indeed standard major models give a very exact linear relation between these quantities. In fact, the curve is useful, for an early version of the Paris model had a point far off the curve to the right: on investigation, we were informed by the Paris group that this was due to numerical inaccuracies, and the correct point is indeed on the universal line. Quark models with small Λ are also on the line, but they appear out of scale to the left. Using the same f^2 and A_S^2 any other variation of Q and η will obey the approximate relationship

$$\frac{\Delta Q}{Q} \approx 1.7 \frac{\delta \eta}{\eta} \quad (25)$$

The experimental point in the Figure is obtained using the experimental value $A_S 0.805^{29}$ and with $f^2 = 0.078$. The quadrupole moment is then corrected for MEC contributions according to Kohno¹³, giving the points indicated in the Figure. There is the possibility of a disagreement with any potential model, but this conclusion is based on only two standard deviations in η , which, in addition, may have an underestimated error. It is clearly important to increase the precision in η by a factor 2 to 3. In the meantime, accepting the validity of the

FIGURE 9

Variation of Q/A_S^2 versus η in various standard approaches. Both Q and η have been corrected for the trivial proportionality to the πNN coupling constant in Born approximation. Note the universal linear correlation to high precision independent of detailed assumptions (from Ericson and Rosa-Clot²⁰).

universal line, the quadrupole moment determines η more accurately. This leads to a value for $\eta \approx 2.65 \times 10^{-2}$ using $A_S = 0.880$. This is consistent with the previously quoted value of Klarsfeld et al.

Finally, how is this related to the problem of bag sizes? Apart from the Stony Brook little bag, there is disagreement with data using quark bag form factors straightforwardly. This indicates that some modification in the thinking about two bags in interaction may be needed. A possible way out has been indicated by Guichon and Miller²⁹. They observe that a bag is not an inert system. It is therefore logical to antisymmetrize the constituents of two interacting bags. In this way, they find that the corresponding Fock term substantially compensates the form factor effect. Using the condition of continuity in the probability current, they find that there is no problem even with the bag radii beyond 1 fm. In this case, the "form factor" becomes only an "effective form factor". This idea is an interesting one and is well worth to pursue further.

The conclusion of the present discussion is that we can now theoretically relate Q and η on the level of 1% and that the experimental precision in η must be improved to test this relation. As a by-product, the MEC contributions will be model-independently tested. Even as of now there can be no doubt that η and Q provide the best quantitative evidence for pions in nuclear physics, and this includes contributions both from rescattering and from the non-trivial region between 1 to 2 fm.

TABLE I
Deuteron properties*

$\epsilon = 2.24579(9)$ MeV	(4 ppm)
$\alpha^{-1} = R = 4.31896(2)$ fm	(5 ppm)
$\mu_d = 0.857406(1)$ n.m.	(1 ppm)
$r_d = 1.963(4)$ fm	(2 ppt)
$\rho_{av}(-\epsilon, -\epsilon) = 1.737(12)$ fm	(7 ppt) ⁺
$A_s(\text{average}) = 0.8802(20)$	(2 ppt)
$\eta = 0.0271(4)$	(13 ppt)
$Q_d = 0.28590(30)$	(1 ppt)
*Comments are given in the text.	
⁺ Note inconsistency with deduced value 1.767(4) from $\rho(-\epsilon, 0)$.	

TABLE II

$A_s^{\text{exp}}(\text{Bérard}) = 0.8813(24)$	$\rho_B(-\epsilon, -\epsilon) = 1.744(14)$ fm
$A_s^{\text{exp}}(\text{Simon}) = 0.8772(36)$	$\rho_S(-\epsilon, -\epsilon) = 1.720(21)$ fm
$A_s^{\text{exp}}(\text{average}) = 0.8802(20)$ (2 ppt)	$\rho_{av}(-\epsilon, -\epsilon) = 1.737(12)$ fm (7 ppt)
*Note that A_s is about three times more accurate than $\rho(-\epsilon, -\epsilon)$ which is of considerable practical use!	

TABLE III
Determinations of η by different methods

Method	$\eta \times 10^2$	Analysis
elastic ($\vec{D}, p$) ¹⁷	2.63(9)	Analytical extrapolation
$^2\text{H}(\vec{D}, p) \ ^3\text{H}$ ¹⁸	2.72(4)	"
SubCoulomb Stripping ¹⁹	2.71(8)	
Average ²⁰	2.71(4)	13 ppt

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