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HIGH ENERGY NUCLEUS-NUCLEUS COLLISIONS IN THE DUAL PARTON MODEL

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ABSTRACT

Average multiplicities, rapidity densities and dispersion in high-energy nucleus-nucleus collisions are computed in the framework of the dual parton model. Particular attention is devoted to the case of central collision, especially in connection with the possible production of a quark-gluon plasma. Our results are compared with recent cosmic ray data. A comparison of the predictions of different models is also given.

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1.- INTRODUCTION

The study of central collisions of heavy ions has become a very popular subject in the last few years, especially in connection with the possible observation of the transient production of a quark-gluon plasma⁽¹⁻⁴⁾. One of the main questions in this domain is to evaluate the energy density deposited in some phase space regions as a result of a central (head-on) nucleus-nucleus collision⁽⁵⁾.

In this paper we answer this question in the framework of the multi-chain dual parton model⁽⁶⁾ (DPM). This model has been able to describe with considerable success multiparticle production in hadron-hadron interactions⁽⁷⁾ (rise⁽⁸⁾ and shape⁽⁸⁾⁽⁹⁾⁽¹⁰⁾ of the plateau^{*}, multiplicity distributions⁽⁸⁾, inclusive cross sections⁽⁹⁾⁽¹¹⁾ and correlations⁽¹²⁻¹⁴⁾ including the detailed structure of the long-range forward-backward rapidity correlation⁽¹³⁾). The model can be extended in a straightforward way to hadron-nucleus⁽¹⁵⁻¹⁶⁾ and nucleus-nucleus⁽¹⁷⁾ interactions at high energies. The α - α data at $\sqrt{s} = 31$ GeV per nucleon have been described with great success⁽¹⁷⁾.

Here, we present calculations of average multiplicities, dispersion and rapidity densities in nucleus-nucleus collisions. Particular attention is devoted to the case of central (head-on) collisions, where we emphasize the dependence of the results on the "degree of centrality" of the collision. The calculation of the multiparticle observables requires the knowledge of several nucleon average values (number of wounded nucleons in the

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* The rise of the plateau is due to two effects. First there will be an increasing overlap between the two main chains; second at low energies the quark-antiquark sea chains are very narrow and of a negligible height due to phase space limitation. At ultra high energies, however, they will develop a full-size plateau. Up to ISR energies, the first effect is more important, and at higher energies the second effect begins to be more important.

beam and target, number of inelastic collisions, etc.). These are computed using an extension of the results of ref. 18.

The paper is organized as follows. A brief description of the model and its main formulae are given in section 2. In section 3 we present the numerical results and in section 4 the comparison with experimental data⁽¹⁹⁻²⁷⁾. A comparison of the predictions of different models^{(28-32)(33,34)(35,36)(37-39)} is given in section 5 and section 6 contains our concluding remarks. In the Appendix we outline the calculation of the nuclear averages appearing in our formulae.

2.- NUCLEUS-NUCLEUS COLLISIONS IN THE DUAL PARTON MODEL

In the dual parton model particle production in a nucleon-nucleon collision takes place in the form of chains of hadrons stretched between coloured fragments of the colliding hadrons with compatible quantum numbers. The nucleon splits into a valence quark and diquark and two chains of hadrons are formed $q_1^V - (qq)_2$ and $(qq)_1 - q_2^V$. This corresponds to a single inelastic collision. The rapidity densities $N_{q_1^V - (qq)_2(y)}$ and $N_{(qq)_1 - q_2^V(y)}$ of the chains are obtained by a convolution of momentum distribution functions and fragmentation functions of the hadron constituents and involve no free parameters. The (joint) momentum distribution functions are obtained from very general properties of the model (dominance of Regge singularities). The fragmentation functions, on the contrary, are an external input. Following previous work we use standard quark and diquark fragmentation functions⁽¹³⁾ obtained from e^+e^- and lp deep inelastic data since they lead to a good description of the nucleon-nucleon data. Alternatively, these fragmentation functions could be regarded as arbitrary functions to be determined from a fit to the nucleon-nucleon data. Obviously the same functions have then to be used in hadron-nucleus and nucleus-nucleus scattering.

The generalization to nucleus-nucleus collisions was done in reference 17 by neglecting the difference between valence and sea quarks. This

is quite harmless for α - α collisions where the generalization was applied but it could be too rough in the case of a collision involving heavy nuclei. Thus we propose here a somewhat improved version of the model.

Let us consider the particular situation in which $n_A(n_B)$ nucleons of nucleus A(B) are wounded (i.e. interact inelastically), with a total number of n inelastic collisions taking place. Let $\sigma_{n_A, n_B, n}$ be the corresponding cross section. Since each inelastic collision produces two chains of hadrons, $2n$ chains will be produced. The valence fragments available are: $n_A(n_B)$ valence quarks and diquarks in nucleus A(B). In the case $n_A < n_B$ the only chains one can form involving these valence constituents are n_A chains of type $(qq)_A - q_B^V$, n_A chains of type $q_A^V - (qq)_B$, $n_B - n_A$ chains of type $\bar{q}_A^S - q_B^V$ and $n_B - n_A$ chains of type $q_A^S - (qq)_B$, i.e. the valence constituents are contained in $2n_B$ chains. Here q^S and $\bar{q}^S$ are sea quarks and antiquarks. Since this takes care of all valence constituents and since the total number of chains is $2n$ (two chains for each inelastic nucleon-nucleon collision), the remaining $2n - 2n_B$ chains have to be of the type $\bar{q}_A^S - q_B^S$ (or $q_A^S - \bar{q}_B^S$). The rapidity density in a nucleus A-nucleus B collision is therefore given by

$$N^{A-B}(y) = \frac{1}{\sigma_{AB}} \sum_{n_A, n_B, n} \sigma_{n_A, n_B, n} \theta(n_B - n_A) \left[n_A (N^{(qq)_A - q_B^V}(y) + N^{q_A^V - (qq)_B}(y)) + (n_B - n_A) (N^{\bar{q}_A^S - q_B^V}(y) + N^{q_A^S - (qq)_B}(y)) + (n - n_B) (N^{\bar{q}_A^S - q_B^S}(y) + N^{q_A^S - q_B^S}(y)) \right] + \text{symmetric } A \leftrightarrow B \quad (1)$$

For $A \ll B$ a good approximation to eq. (1) is obviously to take

$$N^{AB}(y) = \langle n_A \rangle (N^{(qq)_A - q_B^V}(y) + N^{q_A^V - (qq)_B}(y)) + (\langle n_B \rangle - \langle n_A \rangle) (N^{\bar{q}_A^S - q_B^V}(y) + N^{q_A^S - (qq)_B}(y)) + (\langle n \rangle - \langle n_B \rangle) (N^{\bar{q}_A^S - q_B^S}(y) + N^{q_A^S - \bar{q}_B^S}(y)) \quad (2)$$

In fact this formula gives results remarkably close to those obtained from the exact expression (1) for $A \leq B$.

In the same way we can compute the double inclusive cross section. For simplicity we use eq. (2). Squaring this equation and assuming that particles produced in two different chains are uncorrelated, we obtain in a straightforward way

$$\begin{aligned}
 \langle N_{AB}^2 \rangle = & \langle n_A \rangle (2D_{qq-q_v}^2 - D_{qq-q_s}^2 - D_{\bar{q}_s-q_v}^2) \\
 & + \langle n_B \rangle (D_{qq-q_s}^2 + D_{\bar{q}_s-q_v}^2 - 2D_{\bar{q}_s-q_s}^2) \\
 & + 2\langle n \rangle D_{\bar{q}_s-q_s}^2 + 4\langle n^2 \rangle (N_{\bar{q}_s-q_s})^2 \\
 & + \langle n_B^2 \rangle (N_{qq-q_s} + N_{\bar{q}_s-q_v} - 2N_{\bar{q}_s-q_s})^2 \\
 & + \langle n_A^2 \rangle (2N_{qq-q_v} - N_{qq-q_s} - N_{\bar{q}_s-q_v})^2 \\
 & + 4\langle n n_B \rangle N_{\bar{q}_s-q_s} (N_{qq-q_s} + N_{\bar{q}_s-q_v} - 2N_{\bar{q}_s-q_s}) \\
 & + 4\langle n n_A \rangle N_{\bar{q}_s-q_s} (2N_{qq-q_v} - N_{qq-q_s} - N_{\bar{q}_s-q_v}) \\
 & + 2\langle n_A n_B \rangle (2N_{qq-q_v} - N_{\bar{q}_s-q_v} - N_{qq-q_s}) (N_{\bar{q}_s-q_v} + N_{qq-q_s} - 2N_{\bar{q}_s-q_s})
 \end{aligned} \tag{3}$$

where $D_{\text{chain}} = \sqrt{\langle N^2 \rangle_{\text{chain}} - \langle N \rangle_{\text{chain}}^2}$ are the dispersions of the individual chains.

Let us consider next the case of a central nucleus A-nucleus B collision. In the literature, there are many definitions of a central collision. Usually it refers to a collision at zero impact parameter. However, this definition is not suitable to compare with experimental data. We adopt here an experimental definition: central collision is the collision in which there is no fragment of the projectile inside a small veto angle ⁽⁴⁰⁾. This definition corresponds to the cross section of having A wounded nucleons in the projectile and all of them giving rise to a baryon in the final state which is slow enough not to be seen in the veto angle. Accor-

ding to this definition we have to replace in equations (1) to (3), n_A by A (all nucleons in the projectile are wounded) and $\sigma_{n_A, n_B, n}, \sigma_{AB}, \langle n_B \rangle, \langle n \rangle, \langle nn_B \rangle$ and $\langle n_B^2 \rangle$ by $\sigma_{A, n_B, n}^c, \sigma_{AB}^c, \langle n_B \rangle^c, \langle nn_B \rangle^c$ and $\langle n_B^2 \rangle^c$ respectively. All these quantities are computed using an extension of the methods developed in reference 18. The detailed expressions are given in the Appendix (this definition agrees with the name of "central" collision, in fact the shape of the central cross section is centered around $b=0^{(41)}$).

3.- NUMERICAL CALCULATIONS AND RESULTS

To obtain the average multiplicities, rapidity densities and dispersions we use eqs. (1) to (3). The calculation of the rapidity densities of the individual chains is done as in reference 13. Following this reference we also assume a Poisson distribution for the particles produced in the individual chains, i.e. $D_{\text{chain}}^2 = N_{\text{chain}}$. Other choices for D_{chain}^2 (for instance using e^+e^- data) lead to similar results⁽¹³⁾. The nuclear quantities in eqs. (1)-(3) are computed from the formulae in the Appendix. For the profile functions in these formulae we use a Gaussian form for helium,

$$T(b) = \frac{1}{\pi R^2} \exp(-b^2/R^2) , \quad R^2 = \frac{2}{3} \langle r^2 \rangle \quad (4)$$

where $\langle r^2 \rangle$ is the r.m.s. radius of the nucleus, and a standard Saxon-Wood function for the other nuclei considered ($A \geq 12$)

$$T(b) = \int \rho(b, z) dz , \quad \rho(x) = \frac{3}{4\pi R^3} \left[1 + \frac{a^2 x^2}{R^2} \right]^{-1} \left\{ 1 + \exp[(|x| - R)/a] \right\}^{-1} \quad (5)$$

with $R = 1.14 A^{1/3}$ and $a = .545$ fm.

For the nucleon-nucleon non diffractive cross section we take $\sigma_{NN} = 28$ mb in the energy range $\sqrt{s} = 31$ to 62 GeV. The results for the nuclear averages in eq. (2), both for central collisions and for all events, in the above energy range, are given in table 1.

In the numerical calculations we use the approximate eq. (2) but we have made sure that the results using the exact equation (1) are very similar. For instance, for $A \leq 64$ and $B \leq 64$ the results obtained in the two cases differ from each other by about 1%.

In figure 1 we present the height of the rapidity distributions at $y=0$ (normalized to nucleon-nucleon) for different beam and target nuclei at $\sqrt{s} = 31$ GeV. Figure 2 shows the multiplicity of negative particles for different nuclei at $\sqrt{s} = 31$ GeV. In figure 3 we plot the dispersion versus multiplicity for different beam and target nuclei at two different energies. We have drawn a straight line to guide the eye. Most of the points lie on this straight line, only the three last points are below this line. The slope of the straight line is almost 1, much larger than the usual Wroblewski slope for hadron-hadron interactions^{*}.

Let us turn next to central nucleus-nucleus collisions. In order to obtain the corresponding observables we need the nucleon-nucleon cross section σ_s for producing no fast baryon, i.e., no baryon in a small veto angle. We take $\sigma_s = 25.5$ mb. This value is rather arbitrary and corresponds approximately to a veto angle of 5° . To make sure of the correctness of this value we have used it to compute the cross section for central, semi-central and peripheral collisions (according to the definitions of reference 35) and compared the results with the experimental data⁽⁴⁰⁾ at 4.5 GeV/c. A very good agreement was obtained⁽¹⁸⁾⁽⁴¹⁾. We are aware that our definition of a central collision depends on the value of the veto angle. The higher the veto angle, the smaller σ_s and thus σ_{AB}^C . On the contrary, the multiplicity and height of the plateau will be larger. Experimentally, to obtain as many particles as possible (in order to produce the quark-gluon

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^{*} Notice that the experimental values of the D/N ratio for the reactions (d, He, C)C and (d,He)Ta at 4.2 GeV/c per nucleon are similar to the one for nucleon-nucleon. However C-Ta interactions at the same energy already indicate a deviation towards a larger slope⁽⁴²⁾⁽⁴³⁾.

plasma) it would be convenient to increase the veto angle. However the number of events will then be very small*. In fact even with a value $\sigma_s = 25.5$ mb we obtain for Ag-Ag, Pb-Pb and U-U central cross sections of less than 1 mb.

Our main results on central collisions are given in figures 4 to 6 and in tables 1 and 2. In figure 4 we show the multiplicity of produced negative particles for collisions between different nuclei at $\sqrt{s} = 31$ GeV and $\sqrt{s} = 62$ GeV. Notice that in a U-U collision at $\sqrt{s} = 62$ GeV 2288 negative particles are produced, i.e. about 7000 particles. This number will be increased due to the intranuclear cascade in the fragmentations regions which we have neglected. This number moreover increases very fast with the energy. (For U-U collisions the rise is 37 per cent from $\sqrt{s} = 31$ GeV to $\sqrt{s} = 62$ GeV).

In figure 5 we present the rapidity densities of negative particles at $y = 0$ for different nuclei. The straight line drawn at $\frac{dN_{AB}}{dy} = 100$ represents the limit above which the energy density is believed⁽⁴⁶⁾ to be sufficient to produce the phase transition to a quark-gluon plasma. In fact, assuming that the nuclei are compressed to a longitudinal thickness of about 1 fm and taking 0.5 GeV as the energy deposited per secondary, one obtains

$$E = \frac{.5 \text{ GeV } \frac{dN_{AB}}{dy} \big|_{y=0}}{1 \text{ fm } \pi R_B^2} \quad (6)$$

For $B = 64$ one obtains an energy deposition $E = 3 \text{ GeV/fm}^3$, clearly above the required⁽⁴⁶⁾ energy density of 1.5 GeV/fm^3 . Therefore, even admitting reasonable changes in the parameters, we can be sure that the central collisions which are above the straight line are going to produce sufficient energy deposition. We see in figure 5 that even collisions of

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* These correlations between "centrality" of cross sections and multiplicities are analogous to the situation encountered in hadron-nucleus collisions^(44,45).

not very heavy elements are above the straight line. For SPS energies, and oxygen beam we obtain an energy deposition slightly smaller than 1.5 GeV/fm^3 . However, as we said above, we can do that the collision becomes more central by increasing the veto angle giving rise to a energy deposition larger than 1.5 GeV/fm^3 . In this case the cross section of these events is very small. A detailed study of this point together with the situation at PS energies is under preparation.

In figure 6 we show the obtained dispersions of negative particles versus the negative multiplicities for different nuclei at two different energies. We have plotted also the straight line of figure 3 (broken straight line) found for all events, and the curve $D = \sqrt{N_-}$ corresponding to a Poisson distribution. From the figures 6 and 3 we see that the dispersions for central collisions are much smaller than the ones for the totality of events. For B fixed we observe a typical pattern which is pointed out in fig. 6 by drawing two curves (broken lines) for U and Pb at $\sqrt{s} = 62 \text{ GeV}$. We observe that for a fixed target the largest dispersion is not always the one obtained with the heaviest beam. For A fixed, the different central collisions tend to be grouped in straight lines (full straight lines of fig. 6) this behaviour can be easily understood in our model. In fact, at extremely high energies, the rapidity densities of all the chains are equal and the dispersion for central collisions can be written as

$$\frac{D}{N} = \frac{1}{\sqrt{2\langle n \rangle^c}} \frac{D_{q-\bar{q}}}{N_{q-\bar{q}}} \quad (7)$$

where $\langle n \rangle^c$ is the total number of inelastic interactions in a central collision. Since $\langle n \rangle^c$ depends strongly on A and only weakly on B (see table 1), the different points at fixed A will not be far from a straight line with a slope proportional to $1/\sqrt{\langle n \rangle^c}$.

4.- COMPARISON WITH COSMIC-RAY DATA

The multichain parton model describes rightly the α - α rapidity density and multiplicity distribution at $\sqrt{s} = 31$ GeV^(17,30,47). The comparison with cosmic ray information is summarized in table 2. This comparison must be taken with caution. The experimental figures are not quite consistent with each other. For example, the collision A = 11, B = 108 at .3 TeV/N produces more shower particles than the same collisions at 1.5 TeV/N and 1.7 GeV/N; the number of particles produced in collisions between A = 27 and B = 108 and A = 28 B = 108 differ from each other by a factor larger than 2. In general our values are not far from the experimental ones and follow their general trend, except for discrepancies in the last two lines of table 2. Let us comment on that. First of all, the experimental results on cosmic ray correspond to a limited statistics and, in many cases, to a single event. Thus, they may not represent the average situation. Moreover, as we said before, the concept of central collision is not an absolute one and there are collisions more central than others. For instance, we can increase the value of our veto angle and obtain a more central collision. In fact if we take $\sigma_s^{NN} = 20$ mb, $\sigma_F^{NN} = 15$ mb instead of $\sigma_s^{NN} = 25.5$ mb $\sigma_F^{NN} = 2.5$ mb (which would correspond to a larger veto angle), we obtain the following figures for Si-Ag collisions at 3.6 GeV/N: $\langle n_B \rangle^C = 74.2$, $\langle n \rangle^C = 123$, $2 \times N_{AB}^{(-)} = 789$, $\frac{dN^{(-)}}{dy} \Big|_{y=0} = 76$. These figures are close to the experimental value. Furthermore if one takes into account the incertitude in the energy of the collision (3.4 GeV/N to 4.7 GeV/N) which produces an incertitude in the number of shower particles of around 200, the discrepancy between theory and experiment goes away. The same considerations can be applied to the last line of Table 2. From these considerations, it is obvious that in comparing models with experimental data, it would be necessary at least to compare not only multiplicities, height of rapidity densities, etc., but also central cross sections. This would eliminate the freedom in changing the degree of centrality of the collisions*.

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* These considerations have also been applied to particle-nucleus collisions giving rise to useful results⁽⁴⁴⁾.

To complete the comparison with cosmic ray data, we present in figs. 7 and 8 the shape of the rapidity distributions. In fig. 7 we compare our results with B-Ag collisions (7a) and with Ca-Pb collisions (7b). In fig. 8 we present the comparison with the new published events of the JACEE collaboration⁽⁴⁸⁾. In fig. 8a we have plotted our results for Si-Ag, forcing the collision to be more central, choosing $\sigma_s^{NN} = 20$ mb and $\sigma_F^{NN} = 15$ mb. We observe a good agreement with the experimental data except in the observed rapidity fluctuations. We plot also our result (broken line) for $\sigma_s^{NN} = 25.5$ mb $\sigma_F^{NN} = 2.5$ mb. In fig. 8b we present our results for Ca-C central collisions with the two sets of input used before.

5.- COMPARISON WITH OTHER MODELS

a) Brodsky, Gunion and Khun model⁽³²⁾⁽⁴⁹⁾. In this model a gluon (color octet) is exchanged between a quark of an incident nucleon and a quark of a target nucleon. This gluon emits other gluons which transform into quark-antiquark pairs and give rise to new interactions with quarks of the target. The predictions for particle-nucleus collisions are very similar to the ones of the multichain dual parton model (DPM). For nucleus-nucleus collision the multiplicity is given by the formula

$$N_{AB} = N_{NN} \left[\langle n_A \rangle \left(1 - \frac{1}{\bar{v}_B + 1} \right) + \langle n_B \rangle \left(1 - \frac{1}{\bar{v}_A + 1} \right) \right] \quad (8)$$

$$\text{where } \bar{v}_B = B \frac{\sigma_{NN}}{\sigma_{NA}} \quad \text{and} \quad \bar{v}_A = A \frac{\sigma_{NN}}{\sigma_{NB}}.$$

Formula (8) can be written as

$$N_{AB} = N_{NN} \langle n \rangle \left(\frac{\langle n_A \rangle}{\langle n \rangle + \langle n_A \rangle} + \frac{\langle n_B \rangle}{\langle n \rangle + \langle n_B \rangle} \right) \quad (9)$$

In the case of central collisions one must change : $N_{NN} \rightarrow N_{NN}^C$, $\langle n \rangle \rightarrow \langle n \rangle^C$, $\langle n_A \rangle \rightarrow A$, $\langle n_B \rangle \rightarrow \langle n_B \rangle^C$.

In fig. 2 we have compared the total negative multiplicities ob-

tained from eq. (7) with our results. The Brodsky model predicts large multiplicity although for collisions of light nuclei. These differences are very hard to distinguish experimentally.

In fig. 4 we compare the central multiplicities. We see that the multiplicities are larger than ours, with the exception of Pb-Pb, Pb-U and U-U collisions at the highest energy, $\sqrt{s} = 62$ GeV. This is due to the fact that the density of particles produced in a $\bar{q}_s - q_s$ chain increases with energy and the number of these chains increases with the total number of inelastic collisions.

b) Additive quark model (AQM). This model^{(30) (33-34)} is one of the most successful ones in the description of particle-nucleus interactions, where many of its predictions are very similar to the predictions of the DPM, making it very difficult to disentangle both models with hadron-nucleus^(30, 50) experimental data.

In fig. 1 we show the rapidity densities obtained in AQM⁽²⁸⁾ together with our results. For collisions of heavy nuclei the AQM density is much lower than the one in DPM. For U-U collision the difference is as large as 50 per cent.

c) Collective tube models (CTM). This kind of models have very serious problems in reproducing the hadron nucleus data⁽⁵¹⁾. Its predictions⁽²⁹⁾ for central nucleus-nucleus collisions are shown in table 2. The obtained values are slightly larger than the ones in the DPM for very light nuclei and become smaller for heavier nuclei and for increasing energy.

d) Wounded nucleon model. In this model^{(4) (35) (36)} the rapidity density for AB collisions is given by

$$\frac{dN_{AB}}{dy} = \frac{dN_{NN}}{dy} (\langle n_A \rangle + (\langle n_B \rangle - \langle n_A \rangle) \beta(y)) \quad (10)$$

where $\beta(y)$ is a function extracted from particle-nucleus interactions. The formula (2) of the DPM can be written as

$$\begin{aligned} \frac{dN_{AB}}{dy} = \frac{dN_{NN}}{dy} \left[\langle n_A \rangle + (\langle n_B \rangle - \langle n_A \rangle) \frac{N^{\bar{q}_A - q_B^V}(y) + N^{q_A^S - \bar{q}_B}(y)}{N^{q_A - \bar{q}_B^V}(y) + N^{q_A^V - \bar{q}_B}(y)} \right] \\ + (\langle n \rangle - \langle n_B \rangle) (N^{\bar{q}_A^S - q_B^S}(y) + N^{q_A^S - \bar{q}_B^S}(y)) \end{aligned} \quad (11)$$

The difference between (10) and (11) resides in the additional term appearing in (11). The difference $\langle n \rangle - \langle n_B \rangle$ is only significant for $A \geq 64$ and $B \geq 108$ (see table 1) and the quark-antiquark sea densities are quite negligible outside the central rapidity region, and even there they are practically negligible up to energies of around $\sqrt{s} = 40$ GeV. Therefore only for collisions between heavy nuclei at very high energies (of the order of 2 TeV), the difference between (10) and (11) can be significant near $y = 0$. For central collisions the difference $\langle n \rangle^C - \langle n_B \rangle^C$ is larger and therefore this difference will be more important.

e) Multichain model (MCM). This model⁽³¹⁾⁽³⁷⁻³⁹⁾ is based⁽³⁷⁾ in the multiple scattering model of Capella and Krzywicki⁽⁵²⁾ and is therefore very similar to the multichain dual parton model (DPM). The differences come mainly from the different energy partition among the various inelastic collisions. The treatment of nucleus-nucleus collisions requires in this model very drastic approximations⁽³¹⁾. The computation of the observables in central collisions is done at fixed impact parameter b and afterwards an integration over low values of b is performed⁽³¹⁾. Therefore the definition of central collision is not the one used in our work.

The results of the model are given in table 2, the obtained multiplicities are larger than in the DPM. This fact arises mainly from the large value of $\langle n \rangle^C$. (See table 3, observe that the values of $\langle n_B \rangle^C$ are similar in both cases).

In figures 7a and 7b we show the rapidity densities obtained in reference 38 (broken line), compared with our values and with the experimental data. Notice that the rapidity densities in the MCM are symmetric with respect to $y = 0$ while the experimental data are not. From the definition of central collision used in our work there is no reason for symmetry and in fact our results are not symmetric.

Another large difference appears in the dispersions. According to reference 31 for Al-Ag or Si-Ag at .3 TeV the value of D obtained in the MCM is around 100. In the DPM this value is much smaller, around 20. Again, the difference must come mainly from the different definition of central collision used in the two models, although it may also be due to the drastic approximations in the MCM referred to above.

6.- CONCLUSIONS

We have developed the multichain dual parton model description of nucleus-nucleus collisions, with particular attention to the case of a central nucleus-nucleus collision. We emphasize the sensitivity of the results on the definition of what is a central collision. In this connection, it would be convenient that model predictions were given not only for the multiparticle observables but also for the cross sections. This would eliminate the ambiguity on the "degree of centrality" of the collision. Also, experimental data on these central cross section (for instance at 10 GeV/N) would be clearly welcome.

We have compared our predictions with the ones of different models. Some comments are in order: a) We have shown that in the multichain dual parton model even for collisions of not very heavy nuclei ($A = 64$, $B = 108$) at not extremely high energies (.5 TeV/N) the energy density is far above the one believed to be necessary in order to obtain the transient production of the quark-gluon plasma. The same statement is true in most models with minor modifications (see however reference 53). Therefore the

quark-gluon plasma is quite accessible as far as the energy deposition per unit volume in the central rapidity region is concerned . b) To test some models with particle-nucleus experimental data is very hard. Models with very different physical bases give rather similar predictions. However these models give sizeably different predictions for collisions between heavy nuclei. For instance, the AQM and the DPM both have been successful in the description of particle-nucleus experimental data, it being impossible up to now to discriminate between them due to their similar predictions. In nucleus-nucleus collisions the difference between the predictions of the two models becomes larger. Experimental data on the rapidity densities of collisions of even not very heavy nuclei at large number of N_g (grey tracks) could eliminate one of the models, even if the data are not at very high energies, say PS energies. c) MCM, DPM and Brodsky models are the ones which predict larger multiplicities and larger values of the central rapidity density and therefore seem to be favoured by the cosmic ray data. d) The large rapidity fluctuation shown by the data of fig. 8 have no easy explanation in any of the mentioned models. According to these models, these fluctuations will disappear when a sufficiently large number of events is considered. (Notice that individual events in $\bar{p}p$ collisions at $\sqrt{s}=540$ GeV show even more dramatic rapidity fluctuations⁽⁵⁴⁾). e) As far as the dependence of the average transverse momentum on the multiplicity shown by the $\bar{p}p$ collider data⁽⁵⁵⁾ has been explained in the DPM⁽⁵⁶⁾, we expect that the model gives predictions for the case of nucleus-nucleus collisions. Geometrical arguments give rise to a correlation between the ratio $\langle N \rangle / D$ and $d\langle P_T \rangle / dn$ ⁽⁵⁷⁾. For total nucleus-nucleus collisions the dependence of $\langle P_T \rangle$ on n is weaker than for $\bar{p}p$ collisions, however for central nucleus-nucleus collisions this dependence is stronger⁽⁵⁸⁾.

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APPENDIX

Using the methods developed in reference 18 and working in the optical approximation, one obtains

$$\sigma_{AB}^{n_A, n_B}(b) = \binom{A}{n_A} \binom{B}{n_B} (1 - \gamma_{AB}(b))^{B-n_B} \sum_{k=0}^{n_A} (-1)^k \binom{n_A}{k} [T_{W_A-k}(b)]^{n_B} \quad (A-1)$$

where

$$\gamma_{AB}(b) = 1 - (1 - \sigma T_{AB}(b))^A \quad (A-2)$$

$$T_L(b) = (1 - \sigma T_{AB}(b))^{A-L} - (1 - \sigma T_{AB}(b))^A \quad (A-3)$$

$$T_{AB}(b) = \int d^2s T_A(s) T_B(b-s) \quad (A-4)$$

T_A and T_B are the profile functions of nuclei A and B respectively and σ is the nucleon-nucleon inelastic non-diffractive cross section.

Equation (A-1) can be rewritten in the form

$$\sigma_{AB}^{n_A, n_B}(b) = \binom{A}{n_A} \binom{B}{n_B} \sum_{i=0}^{n_B} \sum_{j=0}^{n_A} \binom{n_B}{i} \binom{n_A}{j} (-1)^{n_A+n_B-i-j} (1 - \sigma T_{AB}(b))^{AB-i-j} \quad (A-5)$$

which is symmetric in A and B. From (A-5) it is easy to obtain the expression of $\sigma_{AB}^{n_A, n_B, n}$ which is given by

$$\sigma_{AB}^{n_A, n_B, n} = \binom{A}{n_A} \binom{B}{n_B} (\sigma T_{AB}(b))^n (1 - \sigma T_{AB}(b))^{AB-n} \sum_{i=1}^{n_B} \sum_{j=1}^{n_A} \binom{n_B}{i} \binom{n_A}{j} \binom{i+j}{n} (-1)^{n_A+n_B-i-j} \quad (A-6)$$

From (A-6) we can compute the different average values obtaining

$$\langle n \rangle = \frac{AB\sigma}{\sigma_{AB}} \quad \langle n_A \rangle = \frac{A\sigma_{NB}}{\sigma_{AB}} \quad \langle n_B \rangle = \frac{B\sigma_{NA}}{\sigma_{AB}} \quad (A-7)$$

$$\langle n n_A \rangle = \frac{AB\sigma}{\sigma_{AB}} \int d^2b T_{AB}(b) \left[(A-1) \sigma_{NB}(b) + 1 \right] \quad (A-8)$$

$$\langle n n_B \rangle = \frac{AB\sigma}{\sigma_{AB}} \int d^2b T_{AB}(b) \left[(B-1) \sigma_{NA}(b) + 1 \right] \quad (A-9)$$

$$\langle n_A^2 \rangle = \frac{A \int d^2b \sigma_{NB}(b) \left[(A-1) \sigma_{NB}(b) + 1 \right]}{\sigma_{AB}} \quad (A-10)$$

$$\langle n_B^2 \rangle = \frac{B \int d^2b \sigma_{NA}(b) \left[(B-1) \sigma_{NA}(b) + 1 \right]}{\sigma_{AB}} \quad (A-11)$$

$$\langle n^2 \rangle = \frac{AB\sigma}{\sigma_{AB}} \int d^2b T_{AB}(b) \left[(AB-1) \sigma_{AB}(b) + 1 \right] \quad (A-12)$$

For central collisions, following reference 18, in the optical approximation, the central A-B cross section with n_B wounded nucleons of B is given by the formula

$$\sigma_{AB}^{c, n_B} = \sum_{s=0}^A (-1)^s \binom{A}{s} \Gamma_{A, n_B}^s \quad (A-13)$$

where

$$\Gamma_{n_A, n_B}^s = \binom{B}{n_B} [T_{n_A}^s(b)]^{n_B} [1 - \sigma T_{AB}(b)]^{A(B-n_B)} \quad (A-14)$$

and

$$T_{n_A}^s(b) = [1 - \sigma T_{AB}(b)]^{A-n_A} [1 - \sigma_s T_{AB}(b)]^s - (1 - \sigma T_{AB}(b))^A \quad (A-15)$$

From eq. (A.13) we get

$$\langle n_B \rangle^c \equiv \sum n_B \sigma_{AB}^{c, n_B} =$$

$$B \int d^2b \left\{ \left[1 - (1 - \sigma_S T_{AB}(b))^B \right]^A - (1 - \sigma T_{AB}(b))^A \left[1 - (1 - \sigma_S T_{AB}(b))^{B-1} \right]^A \right\} / \sigma_{AB}^C \quad (A-16)$$

In (A-15) σ_S denotes the nucleon-nucleon cross section for processes without fast baryon, i.e., no baryon in the small veto angle which defines the central collision. We define σ_F by the expression $\sigma_F \equiv \sigma - \sigma_S$.

Summing over n_B in (A-13) we obtain the central AB cross section which is given by

$$\sigma_{AB}^C = \int d^2b \left[1 - (1 - \sigma_S T_{AB}(b))^B \right]^A \quad (A-17)$$

After some algebra eq. (A-13) can be expressed in terms of the numbers S and F of NN-collisions without (S) and with (F) baryon in the small veto angle. The corresponding formula is

$$\begin{aligned} \sigma_{AB}^{C, n_B} &= \binom{B}{n_B} \sum_{\substack{s=0 \\ F=0}}^{A n_B} (\sigma_S T_{AB}(b))^S (\sigma_F T_{AB}(b))^F (1 - \sigma T_{AB}(b))^{AB-S-F} \\ &= \sum_{k=0}^A (-1)^{A-k} \binom{A}{k} \sum_{i=0}^{n_B} \binom{n_B}{i} (-1)^{n_B-i} \binom{k}{S} \binom{A-k}{F} \equiv \sum_{\substack{s=0 \\ F=0}}^{A n_B} \sigma_{AB}^{C, n_B, S, F} \end{aligned} \quad (A-18)$$

From $\sigma_{AB}^{C, n_B, S, F}$ we can compute the different average values. We obtain

$$\langle S \rangle = \frac{AB \sigma_S}{\sigma_{AB}^C} \int d^2b T_{AB}(b) \left[1 - (1 - \sigma_S T_{AB}(b))^B \right]^{A-1} \quad (A-19)$$

$$\langle F \rangle = \frac{AB \sigma_F}{\sigma_{AB}^C} \int d^2b T_{AB}(b) \left[1 - (1 - \sigma_S T_{AB}(b))^B \right]^{A-1} (1 - (1 - \sigma_S T_{AB}(b))^{B-1}) \quad (A-20)$$

$$\langle n_f \rangle = \langle S \rangle + \langle F \rangle \quad (A-21)$$

$$\begin{aligned} \langle n_B^2 \rangle^c &= \frac{1}{\sigma_{AB}^c} \int d^2b \left\{ B(B-1) (1 - \sigma_T T_{AB}(b))^{2A} [1 - (1 - \sigma_S T_{AB}(b))^{B-2}]^A \right. \\ &\quad \left. - B(2B-1) (1 - \sigma_T T_{AB}(b))^A [1 - (1 - \sigma_S T_{AB}(b))^{B-1}] + B^2 [1 - (1 - \sigma_S T_{AB}(b))^B]^A \right\} \end{aligned} \quad (A-22)$$

$$\begin{aligned} \langle s \eta_B \rangle^c &= \frac{AB \sigma_S}{\sigma_{AB}^c} \int d^2b T_{AB}(b) \left[B(1 - (1 - \sigma_S T_{AB}(b))^B)^{A-1} - \right. \\ &\quad \left. (B-1) (1 - \sigma_S T_{AB}(b))^A (1 - (1 - \sigma_S T_{AB}(b))^{B-1})^{A-1} \right] \end{aligned} \quad (A-23)$$

$$\begin{aligned} \langle F \eta_B \rangle^c &= \frac{AB \sigma_F}{\sigma_{AB}^c} \int d^2b T_{AB}(b) \left\{ B(1 - (1 - \sigma_S T_{AB}(b))^B)^{A-1} - B(1 - \sigma_S T_{AB}(b))^{B-1} (1 - (1 - \sigma_S T_{AB}(b))^B)^{A-1} \right. \\ &\quad \left. - (B-1) (1 - \sigma_S T_{AB}(b))^A \left[(1 - (1 - \sigma_S T_{AB}(b))^{B-1})^{A-1} - (1 - \sigma_S T_{AB}(b))^{B-2} (1 - (1 - \sigma_S T_{AB}(b))^B)^{A-1} \right] \right\} \end{aligned} \quad (A-24)$$

$$\langle m \eta_B \rangle^c = \langle s \eta_B \rangle^c + \langle F \eta_B \rangle^c \quad (A-25)$$

$$\begin{aligned} \langle SF \rangle &= \frac{AB \sigma_S}{\sigma_{AB}^c} \int d^2b T_{AB}(b) \sigma_F T_{AB}(b) (1 - (1 - \sigma_S T_{AB}(b))^B)^{A-2} \\ &\quad \left\{ (AB-1) (1 - (1 - \sigma_S T_{AB}(b))^{B-1}) + (B-1) \sigma_S T_{AB}(b) (1 - \sigma_S T_{AB}(b))^{B-1} \right\} \end{aligned} \quad (A-26)$$

$$\begin{aligned} \langle S^2 \rangle &= \frac{AB \sigma_S}{\sigma_{AB}^c} \int d^2b T_{AB}(b) (1 - (1 - \sigma_S T_{AB}(b))^B)^{A-2} \\ &\quad \left\{ (1 - (1 - \sigma_S T_{AB}(b))^B) (1 + (B-1) \sigma_S T_{AB}(b)) + B(A-1) \sigma_S T_{AB}(b) \right\} \end{aligned} \quad (A-27)$$

$$\langle F^2 \rangle = \frac{AB \sigma_F}{\sigma_{AB}^c} \int d^2b T_{AB}(b) (1 - (1 - \sigma_S T_{AB}(b))^B)^{A-2}$$

$$\left\{ \sigma_F T_{AB}(b) \left[B(A-1) + (B-1)(1 - (1 - \sigma_S T_{AB}(b))^B) - 2B(A-1)(1 - \sigma_S T_{AB}(b))^{B-1} \right. \right. \\ \left. \left. + (1 - \sigma_S T_{AB}(b))^{B-2} (1 - (1 - \sigma_S T_{AB}(b))^B) + B(1 - \sigma_S T_{AB}(b))^{B-2} \right. \right. \\ \left. \left. (A(1 - \sigma_S T_{AB}(b))^B - 1) \right] + (1 - (1 - \sigma_S T_{AB}(b))^B)(1 - (1 - \sigma_S T_{AB}(b))^{B-1}) \right\} \quad (A-28)$$

$$\langle \eta^2 \rangle^c = \langle s^2 \rangle + \langle F^2 \rangle + 2 \langle sF \rangle \quad (A-29)$$

The formulae (A-19) to (A-29) together with the formulae (A-7) to (A-12) allow to compute all the averages appearing in our equations* for the rapidity densities and dispersions (see section 2).

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* The numerical differences in these average values between the optical approximation used here and the rigid projectile approximation are small. Even for heavy nuclei is less than 10 per cent.

A	B	$\langle n \rangle$	$\langle n_A \rangle$	$\langle n_B \rangle$	$\langle n \rangle^c$	$\langle n_B \rangle^c$
4	64	5.23	2.54	5.03	10.23	9.65
4	108	6.69	2.74	6.46	12.16	11.62
4	207	9.04	2.96	8.80	14.95	14.51
27	64	17.55	10.09	13.50	70.47	43.12
27	108	24.30	11.76	19.42	87.23	60.24
27	207	35.10	13.52	29.76	113.68	87.70
64	64	31.68	19.49	19.49	155.36	58.58
64	108	44.33	23.32	29.08	201.43	91.63
64	207	65.93	27.56	46.72	263.27	149.31
108	108	65.17	36.39	36.39	316.22	102.42
108	207	99.52	44.15	60.67	439.51	182.49
207	207	170.45	82.41	82.41	772.49	202.17

TABLE 1

A	B	E(TeV/N)	$N_{AB}^{(-)} \times 2$	N_{Exp}^{shower}	N(MCM)	N(CTM)	$\frac{dN_{AB}^{(-)}}{dy} \bigg _{y=0}$
11	108	.3	139	204	200	155	17.4
28	108	.3	343	517	420	345	42.1
40	207	.3	568	518	700	562	72.6
11	108	.5	174	242	235	174	38.4
27	108	.5	402	242	500	383	45.2
28	108	.5	415	515	510	389	46.7
6	108	1.5	128	175	200	160	12.6
9	108	1.5	193	175	300		19.0
11	108	1.5	237	175	340	220	23.4
11	108	1.7	243	193	350	228	23.8
12	108	3	299	215	450	282	28.2
28	108	3.6	684	1050	990		64.8
12	40	100	387	600	760		29.4

TABLE 2

A	B	E(TeV/N)	$\langle n \rangle^c$	$\langle n_B \rangle^c$	$\langle \bar{n} \rangle^c$	$\langle \bar{n}_B \rangle^c$
11	108	.3	34	30	55	34
28	108	.3	89	61	133	58
40	207	.3	165	114	251	100
11	108	.5	34	30	56	34
27	108	.5	87	60	130	57
28	108	.5	90	62	135	58
6	108	1.5	19	17	33	23
9	108	1.5	29	25	49	31
11	108	1.5	36	31	59	35
11	108	1.7	36	31	60	35
12	108	3	40	34	67	37
28	108	3.6	99	65	149	60
12	40	100	34	23	50	23

TABLE 3

Table captions

- 1 Average number of inelastic collisions $\langle n \rangle$, wounded nucleons of nucleus A, wounded nucleons of nucleus B, inelastic central collisions $\langle n \rangle^c$, wounded nucleons of nucleus B in central collision $\langle n_B \rangle^c$ for collisions of different nuclei A and B.
- 2 Results on the particle multiplicities in central collisions compared with cosmic ray data. N^{shower} stands for the experimental shower multiplicity, $N_{AB}^{(-)}$ is our negative particle multiplicity, $N(\text{MCM})$ is the charged multiplicity in the multichain model, $N(\text{CTM})$ is the charged multiplicity in the coherent tube model and $\left. \frac{dN_{AB}^{(-)}}{dy} \right|_{y=0}$ is the height of the plateau in the DPM.
- 3 Comparison of the average inelastic central collisions and the wounded nucleons of B in central collisions in the DPM ($\langle n \rangle^c$ and $\langle n_B \rangle^c$) and in the MCM ($\langle \bar{n} \rangle^c$ and $\langle \bar{n}_B \rangle^c$).

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Figure Captions

- 1 Rapidity densities (full lines) in the central region normalized to the nucleon-nucleon rapidity density of the multichain dual parton model compared to the predictions of the additive quark model.
- 2 Average of negative particle multiplicity in the DPM (full lines) and in the Brodsky model (broken lines).
- 3 Dispersion versus multiplicity of produced negative particles for different nuclei collisions at two different energies.
- 4 Multiplicity of negative particles for central collisions in the DPM (full lines) and in the Brodsky model (broken lines) for different nuclei at two different energies, $\sqrt{s} = 31$ GeV and $\sqrt{s} = 62$ GeV.
- 5 Rapidity densities for central collisions for different collisions of nuclei at two different energies, $\sqrt{s} = 31$ GeV and $\sqrt{s} = 62$ GeV. The straight line $dN^{(-)}/dy = 100$ gives rise to an energy density of 3 GeV/fm^3 .
- 6 Dispersion versus multiplicity for central collisions of different collisions of nuclei. The broken straight line is the straight line of fig. 3 which represents the dispersion versus multiplicity of total nucleus-nucleus collisions. The other straight lines are drawn to guide the eye and represent the dispersion versus multiplicity for different nuclei collisions with the same beam nucleus. Broken curves are drawn for fixed target nucleus. The full curve is the $D = \sqrt{N}$, Poisson curve.
- 7 Cosmic ray data for B+Ag(B_P) collisions (7a) and Ca+Pb collisions (7b) together the results of the DPM (full lines) and the MCM (broken lines). The data and MCM results are plotted in pseudorapidity and the DPM results in rapidity.
- 8 Cosmic ray events of reference 48 together the results of the DPM for two different degrees of centrality: more central (full line) less central (broken line).

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 6

FIG. 7A

FIG. 7B

FIG. 8A

FIG. 8B