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ALL-ORDER FACTORIZATION FOR DRELL-YAN CROSS SECTIONS*

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ABSTRACT

We outline all-order perturbative arguments, which show that factorization holds for Drell-Yan cross sections.

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1 INTRODUCTION

Despite their central importance [1] in the interpretation of experiment, factorization theorems in QCD have had an uneven history. The original all-order arguments for factorization in $e^+e^- \rightarrow h + X$ and hard hadron-hadron scattering [2] met with general, if uncritical, acceptance. However, the justifications given in the original papers are seriously incomplete. This incompleteness of proof is evident from the calculations of Ref. [3], which found infra-red divergences at non-leading twist, and from the recent work in Refs. [4-6]. See also Refs. [7] and [8].

What is missing in [2] is a proper treatment of the effects of low-momentum partons. (A "low-momentum parton" we define to be one for which all its momentum components are much smaller than the center-of-mass energy, as viewed in the center-of-mass frame.) Exchanges of low-momentum partons include (at least at a perturbative level) the effects of Pomeron exchanges. It was well-known [9], long before the advent of the QCD factorization theorems, that Pomeron exchange causes serious, but not insurmountable [10], difficulties in the justification of parton-model results. The same difficulties, plus some new ones, reappear when we try to justify factorization theorems within QCD.

In Ref. [11], we completed the proof of the factorization theorems in the case of e^+e^- annihilation. Moreover, the techniques developed for this proof enabled us to extend the theorems to treat correctly low-transverse-momentum phenomena [12]. This entailed a complete solution [13] to the long-standing problem of systematically calculating the Sudakov form factor [14]. As we explained in Refs. [11,12], we were unable to extend our results to processes with two incoming hadrons. The Drell-Yan process [9] (of dimuon production) is the best-studied example of such processes. The technical difficulty was to mesh our own methods [11,12] with the pre-QCD treatment of initial- and final-state interactions which was given in Ref. [10]. We have now resolved this difficulty. Our demonstration follows the lines that we have already suggested in Ref. [5].

We give below physical and technical arguments (refinements of those based on causality and analyticity in [5]) that should make it clear why and how factorization holds in QCD. It will be noted where more work is required to show that the forms we exhibit can be treated with the full renormalization

group program [15].

To be explicit, we shall consider Drell-Yan cross-sections. Similar arguments apply to any similar hard process.

2 FORMS OF FACTORIZATION

Drell-Yan experiments examine the production of a lepton pair (momentum Q^μ) in the collision of two hadrons (momenta p_A^μ and p_B^μ). The distribution of such pairs as a function of their transverse momenta Q_T is of particular interest, since the dependence on Q^2 of this distribution is a good indicator of the nature of the underlying field theory [15,16].

The proposed factorization theorem for the Q_T -integrated Drell-Yan cross-section is given in QCD by the expression [2,17]

$$\frac{d\sigma}{dQ^2 dx_F} = \sum_{a,b} \int_0^1 \frac{dt_A}{t_A} \frac{dt_B}{t_B} f_{a/A}(t_A, Q^2, m^2) C_{ab}(t_A, t_B, x_F, s, Q^2) f_{B/B}(t_B, Q^2, m^2). \quad (2.1)$$

Here x_F is $Q^+ p_A^+ - Q^- p_B^-$ (in light-plane coordinates), C is an ultraviolet (UV) dominated function, the f 's are appropriate parton densities, and we have suppressed dependence on the coupling $\alpha_s(Q^2)$. The sum is over parton species. Equation (2.1), which is successful phenomenologically, is the form most often thought of as a "factorization theorem". It is not the only such form, however. The bulk of the cross section is at low values of Q_T . To study this region we have proposed another form of factorization [5,15,18],

$$\frac{d\sigma}{dQ^2} = \frac{16\pi\alpha^2}{27Q^2} \sum_{q,q'} \int_q^2 |H(Q)|^2 \int d^2k_A d^2k_B \int \frac{d^2q}{2\pi} \delta^{(2)}(k_A^T + k_B^T - q^T) U(q^T) \Phi_{q/A}(Q^+/p_A^+, k_A^T, s) \Phi_{q'/B}(Q^-/p_B^-, k_B^T, s). \quad (2.2)$$

The functions in (2.2) are defined in [18,19], and the equation is supposed to be valid when $Q_T \ll Q$. The quark densities, Φ , are different from the densities f in the first form of factorization (2.1); they depend on the transverse momentum of the quark. In Refs. [11] and [12] we have proved formulae corresponding to eqs. (2.1) and (2.2) for the case of e^+e^- annihilation.

In the following, we describe the reasoning that demonstrates both eq. (2.1) and (2.2). We shall try to give a physical motivation for, as well as a brief technical description of, each major step in the argument. Complete details are left for future publications.

3 LEADING REGIONS

We begin by characterizing the set of all regions $\{R\}_G$ in momentum space that give leading contributions to Drell-Yan cross-sections for an arbitrary Feynman graph G , in a covariant gauge. This is possible using a combination of analyticity and power-counting arguments [2,20]. The result of this analysis is illustrated in Fig. 1. In each region, all the lines of G have either UV momentum (those in H), low momentum (those in U), or momentum collinear to an external particle (those in $\mathcal{E}_A, \mathcal{E}_B$). As defined in Sec. 1, the term "low momentum" includes finite as well as vanishingly small momenta, as $Q^2 \rightarrow \infty$. We assume our incoming lines are gauge singlet bound states. The solid lines attaching the jets to H represent quarks and antiquarks. The remaining lines attached to H are all longitudinally polarized gluons. All the lines attaching the low-momentum part U to the jets $\mathcal{P}_A$ and $\mathcal{P}_B$ are gluons. In the case of factorization (2.2) at low transverse momentum, the "hard part" H contains only virtual lines, as shown in Fig. 1. In the case of factorization (2.1) of the cross-section integrated over transverse momentum, H may also include lines for real quanta going into the final state [2].

Fig. 1 describes a process which is already appealingly close to the parton model. The only problems are the longitudinally polarized gluons attaching jets to hard parts, and the low-momentum lines that connect the two jets. Showing factorization is a matter of organizing the contributions associated with these graphical features.

To organize gluon attachments to the hard parts and jets we must be able to exploit Ward identities. Jet gluons attached to hard parts are longitudinally polarized in leading regions, and their treatment in Feynman gauge has been discussed in [19]. Low-momentum gluons attached to jets pose a more difficult problem. Consider a gluon of low momentum k^μ attached to one of the jets. Provided that all of the components of k^μ (in the center-of-mass frame) are comparable, the "soft" (or Grammer-Yennie) approximation [21] is

valid. Such gluons we will call "soft".

If the soft approximation is valid for all attachments of low-momentum gluons to the jets, then Ward identities can be used to bring the low-momentum part into the form of the factor J in eq. (2.2). The way to do this was explained by us in Refs. [11] and [12], in the case of axial gauge. For our present work, we are in Feynman gauge, and the Ward-identity argument follows the lines given in Ref. [19] for the attachment of jet gluons to the hard part.

The upshot of the above is that our desired result of factorization, eq. (2.2), follows from the structure of Fig. 1 for the leading regions, provided that the soft approximation is valid. However this is not always true; not all low-momentum gluons are soft, in our precise sense. The integration over loop momenta passes through regions where the soft approximation fails.

To be specific, let us consider a gluon of low momentum k^μ connected to jet A. It is soft only when

$$p_A^+ |k^-| \gg k_T^2, \quad (3.1a)$$

$$\text{and} \quad p_A^+ |k^-| \gg p_A^- |k^+|. \quad (3.1b)$$

Similar restrictions hold for connections to jet B. If (3.1a) is violated, then k^μ is said to be in the "Glauber" region [4]. If (3.1b) is violated, then k^μ is in part of the "infra-red" region. By "infra-red", we mean a low momentum with one or more components that are much less than M^2/Q , where M is a hadronic mass scale. We will show that such subregions either cancel or may be avoided by contour deformation.

We shall see that it is much easier to organize the integrals if we first sum over final-state cuts of each topologically distinct Drell-Yan graph. This leads to considerable simplification because, as we now observe, all final-state interactions cancel.

Note that below we will refer to some subgraph as being composed of "low-momentum gluons" or "soft gluons". We will not mean by this that the subgraph is composed entirely of gluon lines. Rather we will mean that we are considering a subgraph of the low-momentum part of Fig. 1. The word "gluon"

is used as a reminder that the low-momentum part connects to the jets only by gluon lines. In the interior of the low-momentum part the gluons are allowed to interact with each other; these interactions may include quark and/or ghost loops.

4 CANCELLATION OF FINAL-STATE INTERACTIONS

The cancellation of final-state interactions may be thought of as a consequence of causality and unitarity. Once the annihilation event has occurred, it is too late for the subsequent evolution of the system to change the simple probability of its occurrence.

The distinction between final and initial-state interactions is meaningful in time-ordered perturbation theory (TOPT) because the annihilation vertex specifies a unique event. (Note that the time-scale of the annihilation is $1/Q$, whereas the time-scale of typical collinear interactions is Q/Λ^2 . Here, times are measured in the center-of-mass frame.) The technical arguments here are similar to those which show that the total e^+e^- -cross-section is infra-red finite.

The cancellation of final-state interactions leads to a number of important intermediate results for leading regions, which we now describe. These results precisely eliminate those configurations where the soft approximation fails.

We first use the cancellation of final-state interactions to show that infra-red gluons cancel. The intuitive reason for this is simple. Initial-state interactions are associated with virtual states of the incoming hadrons, and these live for only the time allowed by the uncertainty principle. Thus, jet lines in the initial state must remain off-shell by some finite amount. Lines that approach the mass shell very closely must do so by propagating for long periods into the final state. But final-state interactions cancel, so the effect of these almost on-shell jet lines also cancels in the complete result. The technical implementation of these arguments is not difficult. We are thus assured that (3.1b) is satisfied in all non-cancelling low-momentum regions.

5 GLAUBER GLUONS

We now consider leading regions where low-momentum gluons attaching to the jets violate (3.1a). These gluons carry primarily transverse momenta. They are present in e^+e^- -annihilation diagrams as well as in Drell-Yan. In the former case, however, we have shown [11] that contour deformations can always be made to avoid these regions. The contour is moved to a region where the soft approximation is good, and Ward identities may be applied. In e^+e^- -annihilation, these deformations may be carried out in the Feynman amplitudes directly. This is not the case for Drell-Yan, where longitudinal-momentum contours are pinched between singularities from active and spectator lines. Near these pinches longitudinal momenta are negligible compared to transverse momenta. These are the "Glauber" regions [4], where partons from opposite jets scatter elastically. We now show, however, that after the sum over final states the situation is precisely analogous to e^+e^- -annihilation.

To see how this works, consider Fig. 2. We take p_A in the $+z$ direction. The k^- -contour is pinched in the Glauber region by singularities from the active line $l-k$ (UHP) and the spectator line p_A-l+k (LHP). To relate this observation to time orderings, we write propagators of the active and spectator line in terms of their Fourier transforms:

$$1/(q^2 - i\epsilon) = \frac{-i}{2|q|} \int_{-\infty}^{\infty} dt \{ \theta(t) e^{it(q^0 - |q| + i\epsilon)} + \theta(-t) e^{it(q^0 + |q| - i\epsilon)} \}. \quad (5.1)$$

Only the first term in (5.1) need be considered to leading power, when q^μ is a collinear momentum (in this case $p_{A-1}^\mu + k^\mu$ or $l^\mu - k^\mu$). We use

$$q^0 - |q| = 2^{1/2} [q^- - q_T^2 / (2q^+)]. \quad (5.2)$$

The contribution of the active and spectator lines to the k^- -integral is then approximately proportional to

$$I(k) = \int_0^\infty dt' e^{i\sqrt{2}(t-t')k^- - i\Delta(t,t')}, \quad (5.3)$$

where $\Delta(t,t')$ is independent of k^- . Here t is the parameter associated with

the spectator line $p_A - l + k$, and t' is the parameter associated with the active line $l-k$.

The time t should be understood as the time after the wave function at which the quark-gluon vertex occurs, while t' is the time at which the annihilation occurs. Thus the integration region $t < t'$ contributes to graphs in TOPT in which the gluon vertex is an initial-state interaction, while the region $t > t'$ contributes to TOPT graphs in which the gluon vertex is a final-state interaction. Now notice that the contribution to $I(k)$ from the initial-state region $t < t'$ is analytic in the lower half k^- -plane, but has a pole in the upper half k^- -plane. This pole is, in fact, one of the two poles that pinched the k^- -contour in the Glauber region. The other pole involved in the pinch lies in the lower half k^- -plane, and arises from the final-state integration region $t > t'$. Thus, if we omit the final-state integration region $t > t'$, the troublesome pinch disappears, and the k^- -contour may be deformed into the lower half plane. The result of Sec. 4 is that we may indeed omit the final-state region.

A similar argument applied to jet B allows us to deform the k^+ -contour. Note that these arguments are similar to those used to relate analyticity to causality in the derivation of the Kramers-Kronig relation given in Ref. [22].

The extension of this reasoning to arbitrary order is trivial. We conclude that after the sum over final states, Glauber regions may be avoided by contour deformation, just as in e^+e^- annihilation. Relation (3.1a) is thus satisfied.

The physics behind our arguments is that, after the cancellation of final-state interactions, the Glauber region corresponds to interactions in the initial state $[h]$, with a time-scale of at least $1/M$. But at such times the hadrons are still space-like separated, and we do not expect them to be able to communicate. This is our reason for working in the Feynman gauge, rather than, for example, in the Coulomb gauge, where the gluon propagator has a term representing an instantaneous potential. Such a term violates relativistic causality, graph-by-graph, and introduces singularities into the propagator that complicate the deformation argument given above [5].

6 FACTORIZATION

With the elimination of infra-red and Glauber gluons, we have verified that the soft approximation is good in every leading region for those gluons which attach U to the jets, as in Fig. 1. Factorization in each leading region is now a result of Ward identities, as we will show. We write the cross sections as a sum over cut Feynman graphs. Schematically, we have:

$$\sigma = \sum_{\Gamma} \sum_{\text{cuts}} \Gamma_C' \quad (6.1)$$

where Γ denotes an arbitrary uncut graph for the cross-section of eq. (2.2), and Γ_C' is any cut of the graph. For each graph there is a set of leading regions of the form of Fig. 1. If the soft approximation is valid, we can apply Ward identities to obtain the factorized form, eq. (2.2).

For individual cut graphs there are pinches which prevent low-momentum gluons from being treated as soft. Therefore, let us consider a cut graph Γ_C' in one of its leading regions, and let k^μ denote a generic momentum (such as k in Fig. 2) that is routed through both the hard part and the low-momentum part. We deform its $+$ and $-$ components away from the initial-state poles. In doing this, we must cross some final-state poles. Thus the original graph equals the graph with deformed contours plus some pole terms:

$$\Gamma_C' = \Gamma_C, \text{ deformed} + \text{pole terms.} \quad (6.2)$$

Our work in the previous sections shows that the pole terms cancel after we sum over all the cuts C [7]. This is essentially the result of Ref. [10]. The soft approximation may be applied in the first term on the right of eq. (6.2), just as it may in e^+e^- annihilation [11,12].

Using the results above, we conclude that every gluon exhibited in Fig. 1 is effectively longitudinally polarized in the corresponding leading region. Ward identities may now be used to obtain Fig. 3, where a set of longitudinally polarized collinear gluons is factorized from H , and the double lines represent eikonal propagators of the sort defined in [18,19]. In Fig. 3, the combination of an eikonal propagator and vertex is of the form

$$\frac{k \cdot n}{n \cdot k + i\epsilon} \quad (6.3)$$

Here n^μ is a space-like vector. Details are given in [18,19]. In the figure, $\tilde{H}$ is a hard part with only a single external line from each jet.

We emphasize that Fig. 3 holds even in covariant gauges, where the basic Ward identities have ghost terms. It depends on the fact that the original hard part H in Fig. 1 is a function only of the longitudinal momentum components of collinear lines, in the leading region. A similar analysis holds for soft gluons, with light-like rather than space-like eikonal lines.

The result is represented by Fig. 4, which should be compared to Fig. 1. The functions $\mathcal{F}_{a/H}$, $\tilde{H}$ and U are now disentangled graphically, and are linked only by group and momentum space sums. The double lines represent "eikonal" propagators defined in [18,19]. Summing over all different leading regions constructs the full functions of eq. (2.2). For eq. (2.1), $\tilde{H}$ and U are combined to form C .

To show that we have found useful forms of factorization, we must show that $U(q_T)$ is IR finite. Moreover, a treatment of the Sudakov effects that are present in the parton distributions $\mathcal{F}_{a/H}(x, k_T, s)$ must be constructed, along the lines of Refs. [12,13]. Finally, in order to give precise definitions to $\tilde{H}$ and U in such a way that these functions obey simple renormalization-group equations, it is desirable to go beyond a treatment of leading regions, and to discuss the overlaps between these regions. This can be done by developing a subtraction formalism [12] analogous to the forests of BPHZ renormalization [23]. We are currently pursuing this program.

REFERENCES

- [1] See for example, P. Petiau and M. Porneuf, eds., 21st International Conference on High Energy Physics (Editions de Physique, Les Ulis, France, 1982).
- [2] D. Amati, R. Petronzio and G. Veneziano, Nucl. Phys. B140 (1978) 54, B146 (1978) 29;
S. Libby and G. Sterman, Phys. Rev. D18 (1978) 3252; *Ibid* 4737;
A.H. Mueller, Phys. Rev. D18 (1978) 3705;
R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer and G.G. Ross, Nucl. Phys. B152 (1979) 285;
A.V. Efremov and A.V. Radyushkin, Theor. and Math. Phys. 44 (1981) 664; *Ibid* 774.
- [3] R. Doria, J. Frenkel and J.C. Taylor, Nucl. Phys. B168 (1980) 93;
C. DiLietto, S. Gendron, I.G. Halliday and C.T. Sachrajda, Nucl. Phys. B183 (1981) 223.
- [4] G.T. Bodwin, S.J. Brodsky and G.P. Lepage, Phys. Rev. Lett. 47 (1981) 1799.
- [5] J.C. Collins, D.E. Soper and G. Sterman, Phys. Lett. 109B (1982) 388.
- [6] W.W. Lindsay, D.A. Ross and C.T. Sachrajda, Nucl. Phys. B214 (1983) 61;
W.W. Lindsay, Southampton preprint SHEP 82/83-7 (1983).
- [7] A. Sen and G. Sterman, Fermilab-Pub 83/42-THY, to be published in Nucl. Phys.
- [8] J. Frenkel, J.C.M. Gatheral and J.C. Taylor, Sao Paulo Instituto de Fisica preprint IFUSP/P-405 (1983);
C.A. Nelson, Nucl. Phys. B223 (1983) 366.
- [9] S.D. Drell and T.M. Yan, Ann. Phys. 66 (1971) 578.
- [10] J.L. Cardy and G.A. Winbow, Phys. Lett. 52B (1974) 95;
C. De Tar, S.D. Ellis and P.V. Landshoff, Nucl. Phys. B87 (1975) 176.
- [11] J.C. Collins and G. Sterman, Nucl. Phys. B185 (1981) 172.

- [12] J.C. Collins and D.E. Soper, Nucl. Phys. B193 (1981) 381.
- [13] J.C. Collins, Phys. Rev. D22 (1980) 1478;
A.H. Mueller, Phys. Rev. D20 (1979) 2037;
A. Sen, Phys. Rev. D24 (1981) 3281.
- [14] V. Sudakov, Zh. Eksp. Teor. Fiz. 30 (1956) 87 [Sov. Phys.-JETP 3 (1956) 65];
R. Jackiw, Ann. Phys. (N.Y.) 58 (1968) 292.
- [15] J.C. Collins and D.E. Soper, in Tran Thanh Van, ed., Proc. Moriond Workshop on Lepton Pair Production (Editions Frontières, Dreux, 1981).
- [16] Yu. L. Dokshitzer, D.I. D'Yakanov and S.I. Troyan, Phys. Lett. 78B (1978) 290;
G. Parisi and R. Petronzio, Nucl.Phys. B154 (1979) 427;
S.D. Ellis and W.J. Stirling, Phys. Rev. D23 (1981) 214;
C. Curod, M. Greco and Y. Shrivastava, Nucl. Phys. B159 (1979) 451.
- [17] A.H. Mueller, Phys. Rep. 73 (1981) 237.
- [18] J.C. Collins, D.E. Soper and G. Sterman, Nucl. Phys. B223 (1983) 381.
- [19] J.C. Collins, D.E. Soper and G. Sterman, Phys. Lett. 126B (1983) 275;
and "Weak factorization implies strong factorization" (preprint, in preparation).
- [20] G. Sterman, Phys. Rev. D17 (1977) 2773.
- [21] G. Grammer and D.Yennie, Phys. Rev. D8 (1973) 4332;
S.B. Libby and G. Sterman, Ref. 2;
J.C. Collins and G. Sterman, Ref. 5;
J.C. Collins and D.E. Soper, Ref. 6.
- [22] J.D. Bjorken and S.D. Drell, Relativistic Quantum Fields (McGraw-Hill, 1965).
- [23] W. Zimmermann, in S. Deser, M. Grisaru and H. Pendleton, eds., Lectures on Elementary Particles and Quantum Field Theory (MIT Press, 1970).

FIGURE CAPTIONS

- Fig. 1. Partition of a diagram associated with a leading region.
- Fig. 2. Example for discussion of Glauber region.
- Fig. 3. Factorization identity.
- Fig. 4. Factorized form.

Fig. 3

Fig. 4

Fig. 1

Fig. 2