“Six-Quark” Component in the Deuteron from a Comparison of 
Electron and Neutrino/Antineutrino Structure Functions

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We discuss a way to measure the “six-quark” component in the deuteron from a comparison of the structure functions in $ep$ and $ed$ deep-inelastic scattering and the structure in $\nu p$ and $\bar{\nu} p$ scattering. Such a determination is obtained by looking at the deviation from 1 in the ratio $T = d(x)u(x)/u(x)d(x)$, where $u$ and $d$ are the quark distributions determined from $\nu p$ and $\bar{\nu} p$, and $\bar{u}$ and $\bar{d}$ are the effective quark distributions determined from $ep$ and $ed$ by neglect of coherent six-quark effects.

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Our present understanding of hadrons as extended objects containing colored quarks and gluons suggests that a nucleus might not always behave as a collection of nucleons. Even in the loosely bound deuteron there is a few percent probability that the nucleons are separated by a distance less than their radius. In such a situation it seems reasonable that instead of talking of two clusters of three quarks one should speak of a single six-quark system.\(^1,2\)

Of course, if we were to decompose the six-quark system into clusters they could be either color singlet or octet.\(^3,4\) A specific estimate of about 5% is obtained from models for the deuteron form factor.\(^5,6\) Boundary-condition models yield about 5% for the difference between $I$ and the integrated deuteron wave function squared from $I = 0$ to infinity.\(^7,8\)

Although one might consider fitting low-energy reactions and static deuteron properties in order to determine this probability, it seems to us that deep-inelastic scattering (DIS) is the tool likely to provide the least ambiguous answer.\(^9\) The quark distribution functions in a six-quark system are different from those of a bound proton-neutron system, whose intrinsic quark distributions suffer no polarization correction. One obvious difference is the structure function for $x > 1$ ($x = Q^2/2M_{NN}$ in the usual notation). For the deuteron the kinematically allowed range for $x$ is $0 \leq x \leq 2$. Although taking the momentum of the nucleons in the deuteron into account (the so-called smearing correction) yields structure functions which extend beyond $x = 1$, there will be no typical behavior near $x \approx 2$ as one would expect from quark counting rules. The high-$Q^2$ behavior of the deuteron form factor, however, seems to indicate that quark counting rules work quite well.\(^9\) The structure functions near $x = 2$ would definitely show the coherent six-quark effects that we are after,\(^12\) but it is doubtful that reliable results can be achieved experimentally.\(^13\)

For $x$ sufficiently large, say $x > 0.3$, we believe that it is not necessary to worry about the contributions of sea quarks. We then have (assuming isospin symmetry)

$$F_{2p}^D(x)/x = [4u(x) + d(x)]/9,$$

$$F_{2s}^D(x)/x = [u(x) + 4d(x)]/9,$$

where $u(x)$ and $d(x)$ are the up- and down-valence-quark distributions in the proton. Following the arguments given above we assume that in addition to the smearing correction, one should add a contribution to $F_{2D}^D(x)$ because of the probability of scattering coherently off six quarks (which are not restricted to be in color singlets),

$$F_{2D}^D(x)/x = \left(1 - \delta_6\right)\left[F_{2p}^D(x)/x + F_{2s}^D(x)/x\right] + \delta_6[4u^D(x) + d^D(x)]/9.$$

Here $u^D(x) = d^D(x) = n(x)$ are the up- and down-quark distributions in an isosinglet six-quark state (equal because of isospin symmetry); the index $s$ indicates that a smearing correction has been applied.\(^14\) The quantity $\delta_6$ measures the probability that the deuteron behaves like a system of six quarks.

In order to be able to learn something about $\delta_6$...

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we need to know the quark distributions in Eqs. (1)–(3). The functions \( u(x) \) and \( d(x) \) may be obtained from \( np \) and \( \bar{p}p \) scattering. The accuracy with which these functions are extracted, however, is not very high. Perhaps the most accurately known quantity is the ratio \( d(x)/u(x) \), which for \( x > 0.3 \) is obtained as the ratio \( F_{2}^p(x)/F_{2}^{\bar{p}}(x) \). Statistically much more accurate determinations of the quark distributions are usually obtained from \( ep \) and \( ed \) scattering—but they are not obtained by use of Eqs. (1)–(3). Rather, one customarily uses

\[
F_{2}^p(x)/x = [\tilde{u}(x) + \tilde{d}(x)]/9, \tag{4}
\]

\[
\tilde{F}_{2}^{u}(x)/x = [\tilde{u}(x) + 4\tilde{d}(x)]/9, \tag{5}
\]

\[
F_{2}^{d}(x)/x = F_{2}^{p}(x)/x + \tilde{F}_{2}^{u}(x)/x, \tag{6}
\]

where we put \( \tilde{u}(x) \), \( \tilde{d}(x) \), and \( \tilde{F}_{2}^{u} \) to indicate that these are effective distributions deduced from proton and deuteron data. Equating Eqs. (1) and (4), and Eqs. (3) and (6), and assuming a simple smearing correction\(^{14}\)

\[
S(x) = F_{2}^{p}(x)/F_{2}^{p}(x)
= \tilde{F}_{2}^{u}(x)/\tilde{F}_{2}^{u}(x) \tag{7}
\]

one finds the following expressions for the distribution functions \( \tilde{u} \) and \( \tilde{d} \), extracted from electron scattering (\( ep \) and \( ed \)) in terms of the correct distribution functions \( u \) and \( d \), extracted from (anti)neutrino scattering (\( np \) and \( \bar{p}p \)):

\[
\tilde{u}(x) = u(x) + \delta_{x}[u(x) + d(x) - S(x) n(x)]/3, \tag{8}
\]

\[
\tilde{d}(x) = d(x) - 4\delta_{x}[u(x) + d(x) - S(x) n(x)]/3. \tag{9}
\]

For the parametrization of the distribution functions we use the normalized \( \int_{0}^{1} dx q(x) = 1 \) function

\[
q(x,\alpha,\beta) = \frac{\Gamma(\alpha + \beta + 1)}{\Gamma(\alpha)\Gamma(\beta + 1)} x^{\alpha - 1}(1 - x)^{\beta}. \tag{10}
\]

We then have \( u(x) = 2q(x;\alpha_u,\beta_u) \), \( d(x) = q(x;\alpha_d,\beta_d) \), and \( n(x) = 1.5q(x/2;\alpha_n,\beta_n) \).

For the up- and down-quark distributions we have used the functions found from neutrino/antineutrino-hydrogen scattering in Parker et al.\(^{15}\). They are parametrized as \( u(x) = 2q(x;0.53,2.85) \), and \( d(x) = q(x;0.63,3.9) \). Quark counting rules, consistent with the Drell-Yan-West relation,\(^{11,16}\) indicate that for six quarks the coefficient \( \beta_n \) in Eq. (10) is equal to \( 2N_{\text{quarks}} - 3 = 9 \). Arguments from Regge theory indicate that the coefficient \( \alpha_n \) is of order 0.5, just as for the distribution functions in the proton. In Fig. 1 we have plotted the distribution functions \( xu(x) \), \( xd(x) \), and \( xn(x) \). For the last function a number of values of the parameters \( \alpha_n \) and \( \beta_n \) have been considered in order to check the sensitivity to them. For a 5% six-quark probability (\( \delta_n = 0.05 \)) the differences between \( xu(x) \) and \( x\tilde{u}(x) \) and between \( xd(x) \) and \( x\tilde{d}(x) \) are very small as one may check from Eqs. (8) and (9). To see the effect one would need to determine these functions to very great precision.

A much more useful quantity is the ratio

\[
T = \frac{d(x)/u(x)}{\tilde{d}(x)/\tilde{u}(x)}, \tag{11}
\]

which has the following features:

1) For \( \delta_n = 0 \) it is 1, irrespective of any corrections which are applied to relate the \( ed \) structure function to the \( ep \) and \( en \) structure functions, like the smearing correction, relativistic effects, shadowing, etc.\(^{14}\)

2) For \( \delta_n \neq 0 \) small changes in the way the above corrections are applied are an order of magnitude smaller than the effects of putting in the “six-quark” contribution itself. This is demonstrated in Fig. 2, where the effect for \( n(x) = 1.5q(x/2;0.5,9.0) \) including the smearing correction\(^{14}\) (solid line 1) is compared with the same choice for \( n(x) \) without any smearing (dashed line).

3) The ratio \( d(x)/u(x) \) is expected to be much less dependent on \( Q^2 \) than the quark distributions themselves.\(^{17}\)

4) The ratio \( d(x)/u(x) \) can be obtained more
FIG. 2. The calculated value for the ratio $T(x)$ [see Eq. (11)] for various choices for $x n(x)$ (solid lines 1–5; see Fig. 1 for parameters). The smearing correction is taken into account. Neglecting this correction for curve 1 gives the dashed line. The dot-dashed line shows how curve 1 is modified if we take $u(x) = 2q(x; 0.5, 3.0)$ and $d(x) = q(x; 0.6, 4.0)$. The dotted line shows the result for a scale change in the deuteron [see Eq. (12)].

FIG. 3. The comparison of some calculated values for $T(x)$ (solid lines 1–5 from Fig. 2) with the experimental values from Refs. 18 (triangles) and 15 (dots).

accurately from the neutrino data than the quark distributions itself.

(5) Unfortunately, there is a strong dependence on the form of $n(x)$, the nonstrange-quark distribution in a six-quark system. Although the value $\beta_6 = 9$ may be trusted near $x \approx 2$, the effective form for $n(x)$ in the relevant region $0.3 < x < 0.7$ may be better described with slightly different parameters. The effect of various choices for $n(x)$, and also for different forms for $u(x)$ and $d(x)$, are shown in Fig. 2.

Qualitatively we always find an enhancement of $T$ in the region $0.3 < x < 0.7$. For $\delta_6$ equal to 5% this enhancement is $(5-20)\%$. A quantitative determination of $\delta_6$ is not possible because of the sensitivity to the quark distribution functions. The most optimistic point of view is, of course, that a more accurate experimental determination of $T$ may teach us about both the magnitude of the six-quark contribution and about the distribution function $n(x)$. At this stage one is still far from this, as is shown in Fig. 3, where some of the results for $T$ (see Fig. 2) are compared with the experimentally determined ratio.\textsuperscript{15,18}

Recently, it has been conjectured that the difference in structure functions in nuclei as compared to those in the nucleon indicates a change of scale taking place.\textsuperscript{19} For the deuteron this means that in the

range $0.2 < x < 0.6$ one would have

$$F_2^d(x, Q^2) / x = \frac{F_2(x, Q^2)}{x},$$

where $\xi = \xi(Q^2)$ is proportional to the change of scale squared with a $Q^2$ dependence caused by the strong coupling constant. Using $F(x, Q^2) \sim \xi^{0.25} F_2(x, Q^2)$ (Ref. 9) we can again find $u$ and $d$ by comparing Eqs. (1), (2), and (12) with Eqs. (4)–(6). The result for $T$ for a rather arbitrarily chosen $\xi = 0.95$ is also shown in Fig. 2. In the region $0.3 < x < 0.7$ such a change of scale has the same qualitative effect on $T$ as a six-quark distribution as discussed by us. At any $Q^2$ the effect of a change in scale as in Eq. (12) can, of course, be considered as a six-quark contribution as in Eq. (3). Because of the $Q^2$ dependence of $\xi$, however, $T$ in this case has a much stronger $Q^2$ dependence.

Finally we would like to discuss what the effect in the deuteron implies for the "EMC effect," where the structure function $F_2^d$ for some nucleus is compared with $F_2^d$. We have compared $F_2^d$ with the idealized structure function "$F_2^d"$, which does not contain any six-quark effects, i.e., is given by Eqs. (4)–(6), but with the correct quark distributions $u$ and $d$ instead of the effective ones $u$ and $d$. The ratio $F_2^d / "F_2^d, which might be called the "deuteron EMC effect," is given in Fig. 4 for a set of reasonable parameters ($\delta_6 = 0.05, \alpha_6 = 0.5, \beta_6 = 9.0$) and is indeed small. From this we can conclude that the error made in analyzing the EMC effect in heavier nuclei\textsuperscript{21} (in a six-quark model) be-
cause of neglect of the same effect in the deuteron is not larger than a few percent, in agreement with results found by Bodek. 22 We have also plotted the effect when $F_2^D$ is given by Eq. (12) and come to the same conclusion. We note that in both cases the deviation from 1 in the ratio $F_2^D / F_2^D$ is about a factor of 6 smaller than the deviation from 1 in the ratio $T$. This makes $T$ much more suitable to extract the six-quark effects in the deuteron. For this reason we would very much like to have new high-precision neutrino and antineutrino measurements on hydrogen.

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