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QUARK MASSES, CHARGED CURRENT MIXING ANGLES AND CP VIOLATION

B. Stech *)
CERN -- Geneva

A B S T R A C T

A simple structure of the charged fermion mass matrix is suggested, motivated in part by grand unified theories. The relations obtained predict a very small $b \rightarrow u$ matrix element, a B meson lifetime $\tau_B \gtrsim 5 \times 10^{-13}$ sec, a range of values for the top quark mass and a CP violating phase angle $\sin \delta' \approx 1$.

*) Permanent address : Institut für Theoretische Physik,
Universität Heidelberg, Philosophenweg 16, D69 Heidelberg,
Germany.

In present theories, the quark and lepton masses as well as the charge current mixing angles appear as parameters to be determined by experiments. A fundamental understanding of these parameters seems not yet possible. In order to detect any regularity which could help solving the mass parameter and generation puzzle, one would like to know the fermion mass matrix in a basis where the currents are diagonal in the fermion fields. This matrix contains all information about masses and mixings ¹⁾.

Grand unified theories ²⁾ can give us some hints for the form of the mass matrix. Consider $SO(10)$, E_6 or any group which contains the fermions of one generation in an irreducible representation ["16" in $SO(10)$, "27" in E_6] and imbeds the $SU(5)$ model. The masses arise from vacuum expectation values of Higgs fields. At most three scalar fields are needed : in $SO(10)$ the representations "10", "126" and "120". The corresponding $SU(5)$ content which contributes to the masses of quarks and charged leptons are given in the Table.

$SO(10)$	10_S	126_S	120_{AS}
$SU(5)$			
up quarks	5_S	5_S	$45_{AS}, 45_{AS}$
down quarks and charged leptons	$\bar{5}_S$	45_S	$\bar{5}_{AS}, 45_{AS}$

Higgs representations contributing to quark and charged lepton masses.

The symmetry (S) with respect to $SO(10)$ indices implies symmetric Yukawa coupling in generation space ; the antisymmetric Higgs coupling (AS) implies antisymmetric matrices in generation space. An important property is the fact that the matrices in generation space which arise from one and the same Higgs representation must be proportional to each other ; for instance, the matrices corresponding to the first column of the Table are proportional. I can now state the first assumption on which this paper is based.

I. - The mass matrices belonging to "5" representations of $SU(5)$ are dominant and the symmetric "5" representations are proportional to each other. This assumption requires that the "5" from the "126" is either small [due to an approximate Pati-Salam $SU(4)$ symmetry, for example] or proportional to the main symmetric "5" (dynamically generated Higgs and Higgs couplings). The

various "5" parts of the Higgses presumably combine to a single SU(5) Higgs field of lowest mass. One obtains for the up (U), down (D) and charged lepton (E) mass matrices

$$\begin{aligned} U &= S, \quad D = \alpha S + A \\ rE &= \alpha S + A \end{aligned} \quad (1)$$

S stands for a symmetric, A for an antisymmetric 3x3 matrix (for three generations) and α and r denote proportionality constants. The symmetry and antisymmetry properties are supposed to hold for an appropriate choice of the phases of the fermion fields. Small "45" contributions will be added later on. They perturb the lepton matrix E stronger (by a relative factor three) than the down quark matrix.

It follows immediately from Eq. (1) that in the formal limit $A \rightarrow 0$ the up and down quark masses become proportional and all generalized Cabibbo angles vanish. One obtains a mass formula suggested by many authors ³⁾

$$\frac{m_d^0}{m_u^0} = \frac{m_s^0}{m_c^0} = \frac{m_b^0}{m_t^0} \quad (2)$$

This formula always results in SO(10), E6, ..., if solely one Higgs representation (the "27" of E6, for example) contributes to the Yukawa interaction. The requirement on the fermion mass matrix to give (2) in the limit of vanishing mixing angles was emphasized by Achiman ⁴⁾. Equation (1) fulfils it automatically.

The second assumption is the following.

II. - The up, down and charged lepton mass matrices are Hermitian :

$$U = U^\dagger, \quad D = D^\dagger, \quad E = E^\dagger \quad (3)$$

Equations (1) and (3) are supposed to hold simultaneously for an appropriate choice of phases. If right-handed charged currents are sufficiently suppressed at low energies, Eq. (3) by itself is not an assumption but can always be fulfilled using the freedom in defining the mass matrix. Equation (3) has to be seen in connection with (1). If right-handed currents do play a rôle, the assumption implies equal charge current mixings for left- and right-handed currents. Equation (3) defines a kind of "maximal CP violation" : there exists a current quark basis in which the diagonal elements are real (even under CP) and all off-diagonal elements purely imaginary (odd under CP). (Of course, CP conservation still holds in exceptional cases : for two generations, for three generations with one off-diagonal element equal to zero, etc.).

An orthogonal transformation brings the up quark matrix in a diagonal form. In this basis, the down quark matrix becomes

$$\hat{D} = \alpha \hat{U} + A, \quad \hat{U} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \quad (4)$$

The new matrix A is again antisymmetric and Hermitian, its determinant is zero (for three generations). One can finally diagonalize D and obtains

$$K \hat{D} K^+ = \alpha \hat{U} + A, \quad \hat{D} = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \quad (5)$$

The unitary matrix K is the weak interaction mixing matrix (Kobayashi-Maskawa matrix). Any phase change of the basis in which K is given is equivalent to a unitary phase transformation of the matrix A . The diagonal elements $\alpha \hat{U}$ remain unchanged. By such a phase change the matrix A remains Hermitian and without diagonal elements. Only the antisymmetry property is lost. Thus, for a specific parametrization of the weak mixing matrix K the requirement on A is to have no diagonal elements and (for three generations)

$$\det A = \operatorname{Re} (A_{13} A_{12}^* A_{23}^*) = 0 \quad (6)$$

If Eqs. (5) and (6) are fulfilled one can always find a phase transformation giving back the antisymmetry property of A .

The parametrization of K given by Maiani ⁵⁾ will be particularly convenient

$$K = \begin{pmatrix} c_\beta c_\theta & c_\beta s_\theta & s_\beta \\ -s_\theta c_\gamma - s_\beta s_\gamma c_\theta e^{i\delta'} & c_\gamma c_\theta - s_\beta s_\gamma s_\theta e^{i\delta'} & s_\gamma c_\beta e^{i\delta'} \\ -s_\beta c_\theta c_\gamma + s_\gamma s_\theta e^{-i\delta'} & -s_\beta s_\theta c_\gamma - s_\gamma c_\theta e^{-i\delta'} & c_\beta c_\gamma \end{pmatrix} \quad (7)$$

($s_\theta = \sin\theta$, $c_\theta = \cos\theta$, $s_\beta = \sin\beta$, etc.). The angles θ, β, γ are taken to lie in the first quadrant ($0 \leq \theta, \beta, \gamma \leq \pi/2$). The phase angle δ which describes CP violation ranges from $-\pi \leq \delta \leq \pi$.

The diagonal elements in Eq. (5) give three relations, one combination of them is simply the trace condition

$$\alpha = (m_d + m_s + m_b) / (m_u + m_c + m_t) \quad (8)$$

The remaining two relations are :

$$m_d + c_\beta^2 s_\theta^2 (m_s - m_d) + s_\beta^2 (m_b - m_d) = \alpha \cdot m_u \quad (9)$$

$$m_d + \left| c_\gamma c_\theta - s_\beta s_\gamma s_\theta e^{i\delta'} \right|^2 (m_s - m_d) + s_\gamma^2 c_\beta^2 (m_b - m_d) = \alpha \cdot m_c \quad (10)$$

At this place, it is worth pointing out that the mass values occurring here can have negative as well as positive values. Except for the freedom of an over-all sign (fixing $m_t > 0$) solutions of Eq. (5) with different signs for the masses give physically different answers. m_s/m_c should, however, have the same sign as m_b/m_t in order not to have a drastic change in the limit $A \rightarrow 0$. For the magnitude of the quark masses I use the values (best average values) obtained by Gasser and Leutwyler ⁶⁾ in a careful QCD analysis :

$$\begin{aligned} |m_d| &= 0.0089, \quad |m_s| = 0.175, \quad |m_b| = 5.3 \\ |m_u| &= 0.0051, \quad |m_c| = 1.35 \end{aligned} \quad (11)$$

All mass values are given in GeV. They are defined at the scale $\mu = 1$ GeV. Before these values can be used in the equations they have to be scaled up to the mass of the top quark $\mu = m_t(m_t)$ by the renormalization group formula ^{7),8)}.

In a model with only the first two generations, Eq. (9) reduces to

$$\begin{aligned} m_d + s_\theta^2 (m_s - m_d) &= \frac{m_s + m_d}{m_c + m_u} \cdot m_u \\ \text{i.e. } \tan^2 \theta &= \left(-\frac{m_d}{m_s} + \frac{m_u}{m_c} \right) / \left(1 - \frac{m_u \cdot m_d}{m_c \cdot m_s} \right) \end{aligned} \quad (12)$$

With $m_d/m_s = -|m_d/m_s|$, Eq. (12) is in agreement with the known value of the Cabibbo angle (for $m_u/m_c > 0$ one obtains $\sin \theta = 0.228$). In the following the relative sign of m_d and m_s will always be taken to be negative.

Turning now to three generations, Eq. (9) shows immediately that s_β has to be very small, at least $s_\beta \lesssim 0.04$, even if there is only an incomplete cancellation of the first two terms. With such a small value for the $b \rightarrow u$ transition element, Eq. (10) simplifies considerably. Thus, s_γ can be obtained as a function of the top quark mass :

$$\begin{aligned} (m_c - m_s + \beta_\theta^2 (m_s - m_d)) \beta_\gamma^2 = \\ \alpha (m_t) \cdot m_c - m_s + \beta_\theta^2 (m_s - m_d) \end{aligned} \quad (13)$$

or approximately

$$\beta_\gamma^2 \approx \left(\frac{m_c}{m_t} - \frac{m_s}{m_b} \right) / \left(1 - \frac{m_s \cdot m_c}{m_b \cdot m_t} \right) \quad (14)$$

The last expression is precisely the same as the one obtained from a two-generation model, this time for the second and third generations.

There are two possible solutions :

$$\text{i) } \frac{m_c}{m_t} > 0 \quad , \quad \frac{m_s}{m_b} > 0$$

$$\beta_\gamma^2 \approx \left| \frac{m_c}{m_t} \right| - \left| \frac{m_s}{m_b} \right| \quad (15)$$

and

$$\text{ii) } \frac{m_c}{m_t} < 0 \quad , \quad \frac{m_s}{m_b} < 0$$

$$\beta_\gamma^2 \approx \left| \frac{m_s}{m_b} \right| - \left| \frac{m_c}{m_t} \right| \leq \left| \frac{m_s}{m_b} \right| \quad (16)$$

The $b \rightarrow c$ matrix element s_γ is directly related to the lifetime τ_B of the B meson. Equation (13) connects, therefore, the top quark mass with τ_B . In Fig. 1, the two solutions for the top quark mass are plotted. m_{top} is the "pole-mass" $m_{\text{top}} = (1 + (4/3\pi)\alpha_s)m_t(m_t) = 1.1 m_t(m_t)$. For τ_B the quark decay formula of Ref. 9) with s_β from Eq. (17) below and $m_b^{\text{effective}} = 5 \text{ GeV}$ has been used. m_{top} depends rather sensitively on the quark mass ratio m_s/m_b and hence on the mass of the strange quark whose value is still uncertain. The dotted lines in Fig. 1 correspond to 10% smaller and 10% larger values for this mass ratio compared to the value obtained from (11). [The error bars given in Ref. 6) amount to an uncertainty in m_s/m_b of $\pm 30\%$.] A very clear statement can be made for the B meson lifetime : it is restricted to a region above $\tau_B = 1 \times 10^{-13} \text{ sec}$. The present upper limit ¹⁰⁾ is $\tau_B \lesssim 1.4 \times 10^{-12} \text{ sec}$.

From Eqs. (9) and (13), $\alpha(m_t)$ can be eliminated. Taking (12) literally one obtains

$$s_\beta = \sqrt{m_u/m_c} s_\gamma, \quad m_u/m_c > 0 \quad (17)$$

which is again a result independent of the top quark mass. This relation fulfils the experimental constraint ¹¹⁾ $s_\beta < 0.2 s_\gamma$ obtained from semi-leptonic decays of B mesons.

The results for s_β and s_γ do not depend on the CP violating phase δ' . The product $s_\beta s_\gamma s_\theta$ is too tiny to have any influence on Eq. (10). To get information on the CP violation predicted by Eq. (5), the off-diagonal elements of the down quark mass matrix have to be considered. One can readily obtain A in its dependence on the three angles and the phase angle δ' , and explore the condition (6). One obtains, in an approximation which is good for the values of β and γ found here,

$$\cos \delta' \approx \frac{s_\theta s_\gamma}{s_\beta} \frac{m_s - m_d}{m_b} - \frac{s_\beta s_\gamma}{s_\theta} \frac{m_b}{m_s - m_d} \approx 0 \quad (18)$$

i.e. $\sin \delta' \approx \pm 1$

This result originates, of course, from the assumption that A is - for a particular choice of fermion phases - an antisymmetric and Hermitian matrix ("maximal CP violation").

By the formulas derived the weak mixing matrix is completely determined once all quark masses including the top quark mass is used as an input. Two solutions exist (with $\sin \delta' \approx \pm 1$ in each case). It is reassuring that they fulfil all experimental constraints. I have no argument which favours one solution over the other. Although the expression for the small quantity s_β might not be completely reliable, it is tempting to use the four angles to calculate the CP violating ϵ_K parameter in $K^0 - \bar{K}^0$ decays. In fact, small angles β and γ together with $\sin \delta' \approx 1$ give a particular interesting scenario which was recently studied by Paschos, Türke and the author ¹²⁾. For the estimation of $|\epsilon_K|$ use can be made of the box diagram formula ¹³⁾ which - in a suitable approximation - is given by ¹²⁾

$$|\epsilon_K| \approx \frac{1}{\sqrt{2}} \frac{s_\beta s_\gamma}{s_\theta} \sin \delta' \cdot \left\{ -1 + \frac{0.4}{0.7} \ln \left(\frac{m_{top}}{m_{charm}} \right)^2 + \frac{0.6}{0.7} s_\gamma^2 \left(\frac{m_{top}}{m_{charm}} \right)^2 C(m_{top}/m_W) \left(1 - \frac{s_\beta}{s_\theta s_\gamma} \cos \delta' \right) \right\} \quad (19)$$

$c(m_{\text{top}}/m_W)$ is a factor which takes the finite mass of the vector boson $W^\pm$ into account. For the QCD correction factors ¹⁴⁾ $\Lambda_{\text{QCD}} = 0.1 \text{ GeV}$ has been used. In Fig. 2, $|\epsilon_K|$ is plotted versus the B meson lifetime. The "pole mass" values for m_{charm} and m_{top} have been used. A 10% change of m_s/m_b is not very significant for these curves, the corresponding changes are indicated by little arrows. A comparison with the experimental value $|\epsilon_K| \approx 2.3 \times 10^{-3}$ favours a B meson lifetime around $1 \times 10^{-12} \text{ sec}$. Of course, one has to allow for uncertainties in the evaluation of $|\epsilon_K|$ via the box diagrams. Nevertheless, if the observed CP violation is due to the proposed properties of the fermion mass matrix a B meson lifetime $\tau_B \geq 5 \times 10^{-13} \text{ sec}$ should be found. The best values for the top quark mass are 27 and 33 GeV for solutions i) and ii), respectively. The uncertainties due to possible errors in the ratio m_s/m_b forbid, however, a definite prediction.

The so far neglected additional interactions which are responsible for the difference of the down quark and charge lepton mass matrix must also be discussed. Do they invalidate the relations between quark masses and mixing angles derived here? The additional interactions come presumably from the "45" contributions ¹⁵⁾ listed in the Table. Let us assume that the corresponding vacuum expectation values lie again mainly in the symmetric (CP-conserving) part. The corresponding corrections leave the up quark matrix unchanged while the down quark and lepton matrices are modified :

$$\begin{aligned} D &= \alpha \hat{U} + A + P \\ 3 E &= \alpha \hat{U} + A - 3P \end{aligned} \quad (20)$$

Use has been made of the renormalization group factor $r \approx 3$ relating leptons and coloured particles ¹⁶⁾ and of the relative Clebsch-Gordon coefficient -3 of the "45". Equation (20) gives

$$\begin{aligned} K \hat{D} K^\dagger + K_E \hat{E} K_E^\dagger &\approx \frac{4}{3} (\alpha \hat{U} + A) \\ \hat{E} &= \begin{pmatrix} m_e & 0 \\ 0 & m_\mu \\ 0 & m_\tau \end{pmatrix}, \quad K_E \equiv K(\theta_E, \beta_E, \gamma_E, \delta'_E) \end{aligned} \quad (21)$$

K_E denotes the charged lepton mixing matrix in the basis where U is diagonal. In the limit $P \rightarrow 0$ one has, of course, $K_E \rightarrow K$ and $\hat{E} \rightarrow \frac{1}{3}\hat{D}$. To get an idea of the implications of (21) one can use for the lepton angles θ_E and γ_E the values obtainable from appropriate two generation models, since the corresponding results in the quark sector, Eqs. (12) and (14), were encouraging :

$$\tan^2 \theta_E \approx - \frac{m_e}{m_\mu} + \frac{m_u}{m_c}$$

$$\tan^2 \gamma_E \approx - \frac{m_\mu}{m_\tau} + \frac{m_c}{m_t} \quad (22)$$

Equation (21) used in this way gives only very small corrections to the plot in Fig. 1. β_E has to be about as small as β and one finally finds again $\sin \delta' \approx \pm 1$ independent of any value of δ'_E . Thus, the relations between quark masses and mixing angles given in this paper are not sensitive to the "45" perturbation, provided Eq. (22) is correct.

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FIGURE CAPTIONS

Fig. 1 The solutions i) and ii) for the top quark mass as a function of the B meson lifetime. The dotted lines correspond to a $\pm 10\%$ deviation of the quark mass ratio m_s/m_b from the values quoted in (11).

Fig. 2 $|\epsilon_K|$ as a function of the B meson lifetime. The curves i) and ii) refer to the two solutions plotted in Fig. 1. The little arrows indicate the changes caused by a $\pm 10\%$ modification of the ratio m_s/m_b .

- Fig. 1 -

- Fig. 2 -