Ref.TH.3707-CERN

Ref.TH.3724-CERN

Note Added

TH.3707: Properties of Susy Hadrons in a Bethe-Salpeter Model

A.N. Mitra and S. Ono.

TH.3724: How would the world look like if there were supersymmetric particles?

D.V. Nanopoulos, S. Ono and T. Yanagida

NOTE ADDED

For glueballon ($\tilde{g}\tilde{g}$ -state) we quoted the selection rule $(-1)^{S+l} = -1$ derived by Zug et al., Phys. Rev. D28 (1983) 1706. However, this will be wrong since the intrinsic parity should not come into the Fermi symmetry condition. The correct selection rule will be $(-1)^{S+l} = +1$. Correct potential for $\tilde{g}\tilde{g}$ -state should contain a factor two from exchange diagram, i.e., $V_{\tilde{g}\tilde{g}}^{\sim}(r) = V_{\tilde{g}\tilde{g}}^{\sim}(r) = V_{\tilde{g}\tilde{g}}(r) = (9/4)V_{qq}^{\sim}(r)$. Thus, the decay width for the lowest $\tilde{g}\tilde{g}$ will become, e.g.,

$$\Gamma_{\tilde{S}_0 \rightarrow gg} = 7 \times 13.5 \times (1-5) \text{ MeV} = (400-500) \text{ MeV}$$

One of the authors (S.O.) would like to thank H. Kühn for pointing out them. More detailed properties of $\tilde{g}\tilde{g}$ -state are studied in the following paper: "Production and decay of gluino-gluino bound state", by J.H. Kühn and S. Ono, Orsay preprint, LPTHE Orsay 84/2.

Note Added

January 1984

Ref.TH.3724-CERN

HOW WOULD THE WORLD LOOK IF THERE WERE
SUPERSYMMETRIC PARTICLES ?

D.V. Nanopoulos, Seiji Ono *)
CERN — Geneva

and

T. Yanagida **)
Max-Planck-Institut für Physik
und Astrophysik, Munich, FRG

ABSTRACT

We study the spectroscopy and decay of $\tilde{g}\tilde{g}$, $\tilde{g}q\bar{q}$, $\tilde{g}\tilde{g}$, $\tilde{q}\tilde{t}$, $\tilde{t}\tilde{t}$, and $\tilde{t}qq$ states ($\tilde{g}$ = gluino, $\tilde{t}$ = scalar partner of t quark). Many of these particles have quite a long lifetime. We discuss the possible experimental detection of these particles.

*) Present address : Laboratoire de Physique théorique, Bâtiment 211, F-91405 Orsay CEDEX, France.

**) On leave of absence from the Dept. of Physics, College of General Education, Tohoku University, Sendai, Japan.

Supersymmetry seems to play a rather fundamental rôle in solving several long-standing cumbersome hierarchy problems, e.g., gauge hierarchy and strong CP hierarchy. If so, all the "known" particles should have superpartners with masses not larger, and in many cases, much smaller than M_W , the electroweak scale. Unfortunately, no experimental evidence, as yet, exists for supersymmetry. It is thus imperative to try to find a way of looking for supersymmetric particles ¹⁾. In this paper we concentrate on hadrons which contain one or two supersymmetric particles, such as the gluino and squark.

1. Glueballino, $\tilde{g}g$

First of all, we use the MIT bag model ^{2),3)} to study glueballino spectroscopy; the calculation is very similar to that ⁴⁾ for the glueball, and since the gluino is heavy, we can neglect its motion. We assume that the gluino is staying at the centre and that the gluon is moving in a fixed spherical bag (cavity approximation). For each value of the total angular momentum J ($J \geq 1$), two types of states for gluon are found ⁴⁾; transverse electric (TE) with parity $(-1)^{J+1}$ and transverse magnetic (TM) with parity $(-1)^J$. Assuming that the radius of the bag is R , the boundary condition is given by

$$j_\ell(kR) = 0 \quad \text{for TM} \quad , \quad (1)$$

(the first zero is at $kR = 4.4934$ for $\ell = 1$)

$$\frac{d}{d\tau} [\tau j_\ell(kR)]_{\tau=R} = 0 \quad \text{for TE} \quad , \quad (2)$$

(the first zero is at $kR = 2.7437$ for $\ell = 1$).

Thus, the lowest energy level ($J=1$) for the gluon is ⁴⁾

$$\begin{aligned} (TE) &\sim 480 \text{ MeV} \\ (TM) &\sim 795 \text{ MeV} \end{aligned} \quad (3)$$

In Table 1a, we show several low-lying energy levels of the glueballino ($\tilde{g}g, \tilde{g}g\tilde{g}$). As seen from this Table, the lowest glueballinos are $\tilde{g}g_{TE}$, spin = $\frac{1}{2}^-$ and $\frac{3}{2}^-$, where we assume $J^P = \frac{1}{2}^-$ for the gluino. The first excited states are higher than the ground states only by 300 MeV, while $m_{\tilde{g}} \gtrsim$ a few GeV. This means that most of the energy of the glueballino is carried by the

gluino. A similar calculation ⁵⁾ for glueballino spectroscopy exists, in which bag parameters are determined to fit masses of the "glueball candidate" $\chi(1440)$ and $\Theta(1700)$. However, neither χ nor Θ are likely to be pure glueball states ^{6),7)}.

One must note that, if the B-S equation is used ^{8),9)}, we obtain a completely different glueballino spectrum which is also shown in Table 1b. As is well known, in the B-S equation the P state is always higher than the S state, including the case of $m_g = 0$. On the other hand, in the bag model with cavity approximation, the state with a TE mode which corresponds to the P state is higher than the state with the TM mode which corresponds to the S states. If glueball states are found experimentally together with hybrino ($\tilde{g}q\bar{q}$) states, one can check experimentally which predictions are correct.

2. Hybrino, $\tilde{g}q\bar{q}$

If we replace the gluon in the $\tilde{g}q$ state by a colour octet $q\bar{q}$, we find a colour singlet $\tilde{g}q\bar{q}$ (hybrino) state. One can find a reasonable estimate of the S wave hybrino states by adding the constituent particle masses and the spin-spin force, since such a simple model is very successful for baryons and mesons ¹⁰⁾⁻¹²⁾. Masses of ordinary mesons and baryons can be reproduced by :

$$M_{\text{baryon}} = M_b + \sum_i m_i + \frac{2}{3} \xi_b \sum \frac{\vec{S}_i \cdot \vec{S}_j}{m_i m_j} \quad (4)$$

$$M_{\text{meson}} = M_m + m_q + m_{\bar{q}} + \frac{4}{3} \xi \frac{\vec{S}_q \cdot \vec{S}_{\bar{q}}}{m_q m_{\bar{q}}}$$

$\xi \sim 0.54 \text{ GeV}^3$ [see Ref. 10].

For hybrino states, masses are given by :

$$M = A + m_{\tilde{g}} + m_q + m_{\bar{q}} + \frac{3}{2} \xi \frac{\vec{S}_{\tilde{g}} \cdot \vec{S}_q}{m_{\tilde{g}} m_q} + \frac{3}{2} \xi \frac{\vec{S}_{\tilde{g}} \cdot \vec{S}_{\bar{q}}}{m_{\tilde{g}} m_{\bar{q}}} - \frac{1}{6} \xi \frac{\vec{S}_q \cdot \vec{S}_{\bar{q}}}{m_q m_{\bar{q}}} \quad (5)$$

The spin-dependent force is given by :

$$\begin{aligned}
 & \frac{\xi}{8 m_q m_{\bar{q}}} \quad \text{for } |\vec{S}_{q\bar{q}}| = 0, \quad J = \frac{1}{2} \\
 & - \frac{\xi}{24 m_q m_{\bar{q}}} - \frac{3\xi}{4 m_{\tilde{g}}^2} \left(\frac{1}{m_q} + \frac{1}{m_{\bar{q}}} \right) \quad \text{for } |\vec{S}_{q\bar{q}}| = 1, \quad J = \frac{1}{2} \\
 & \frac{3\xi}{8 m_{\tilde{g}}^2} \left(\frac{1}{m_q} + \frac{1}{m_{\bar{q}}} \right) - \frac{\xi}{24 m_q m_{\bar{q}}} \quad \text{for } |\vec{S}_{q\bar{q}}| = 1, \quad J = \frac{3}{2}
 \end{aligned} \tag{6}$$

The value A in Eq. (5) is an arbitrary constant which cannot be computed in this model. The energy spectrum for hybrino states is shown in Fig. 1a, where we use the same notation as Ref. 9). Since the value A determines the mass difference of the lowest hybrino state and the lowest glueballino state, the value A can be determined by experiment.

Any model agrees that the lowest hybrino state is in the S state. Mass spectra for the hybrino obtained by various models (e.g., bag model, BS model, non-relativistic potential model) are not very different from the ones in Fig. 1a. On the other hand, the value A is quite model dependent.

$$\begin{aligned}
 A &\sim 100 \text{ MeV} && \text{for bag model } ^9) \\
 A &\sim -100 \text{ MeV} && \text{for BS model } ^9) \\
 A &\sim -700 \text{ MeV} && \text{for non-relativistic} \\
 & && \text{potential model } ^{13)}
 \end{aligned} \tag{7}$$

From Fig. 1a and Table 1, one can conclude that in any model many hybrino states $K_{\tilde{g}}, K_{\tilde{g}}^*, K_{\tilde{g}}^{**}, \eta_{\tilde{g}}, \omega_{\tilde{g}}, \omega_{\tilde{g}}^*, \dots$ are stable against strong decays.

3. Glueballon, $\tilde{g}\tilde{g}$

If one replaces both gluons in a glueball state by two gluinos, one gets a particle which is a colour singlet. Since the gluino is heavy, the most appropriate model to describe the $\tilde{g}\tilde{g}$ state is the non-relativistic potential model. The potential between $\tilde{g}$ and $\tilde{g}$ is given by

$$V_{\tilde{g}\tilde{g}}(\tau) = 2 \times \frac{9}{4} V_{q\bar{q}}(\tau) \quad (8)$$

where the factor two comes from the exchange diagram and $9/4$ comes from the colour factor.

In order to compute the glueballino spectroscopy, let us use a typical $q\bar{q}$ potential ¹⁴⁾

$$V_{q\bar{q}}(\tau) = - \frac{16\pi}{25} \cdot \frac{1}{\tau f(\tau)} \left[1 + \frac{28\epsilon + \frac{53}{73}}{f(\tau)} - \frac{462\ln f(\tau)}{625 f(\tau)} \right] + a\sqrt{\tau} + C$$

$$f(\tau) = \ln \left((\Lambda_{\overline{MS}} \tau)^{-2} + b \right) \quad (9)$$

$$\Lambda_{\overline{MS}} = 140 \text{ MeV}, \quad a = 0.66 \text{ GeV}^{3/2}, \quad b = 20, \quad C = -1.45 \text{ GeV}$$

$$(m_u, m_s, m_c, m_b) = (0.336, 0.62, 1.9, 5.27 \text{ GeV})$$

This potential approaches the asymptotic behaviour predicted by a perturbative two-loop approximation ¹⁵⁾, and reproduces $c\bar{c}$, $b\bar{b}$ and other quarkonium spectra. The $\tilde{g}\tilde{g}$ spectrum is plotted as a function of the $m_{\tilde{g}}$ mass in Fig. 1b. Due to the factor $9/2$ in Eq. (8), energy levels are more sparsely populated than that of $t\bar{t}$ (or $\tilde{t}\tilde{t}$) states, which are plotted in Fig. 2. The binding energy of the $\tilde{g}\tilde{g}$ state becomes extraordinarily large. For $m_{\tilde{g}} = 9 \text{ GeV}$, the binding energy is as much as 8.5 GeV , i.e., $m(\tilde{g}\tilde{g}) \sim \sim 9.5 \text{ GeV}$. Thus the process $Y \rightarrow \gamma + \tilde{g}\tilde{g}$ is energetically allowed for $m(\tilde{g}) \lesssim 9 \text{ GeV}$. However, the branching ratio for this process is small, as will be shown later.

Since the gluino is a Majorana particle, allowed states are governed by the following selection rule ¹⁶⁾

$$(-1)^{S+l} = -1 \quad \text{i.e. } C\text{-parity of } \tilde{g}\tilde{g} = +1 \quad (10)$$

where $\vec{S} = \vec{S}_1 + \vec{S}_2$, and l is the angular momentum. This relation is derived from the fact that the intrinsic parity of the gluino is purely imaginary.

The ground state is 3S_1 , $J^{PC} = 1^{-+}$, and this decays into hadrons through ggg . By using Eq. (6), one numerically finds $|\psi_{\tilde{g}\tilde{g}}(0)|^2 \sim 20 |\psi_{Q\bar{Q}}(0)|^2$. The theoretical estimate ¹⁴⁾ for the toponium state is $\Gamma_{^3S_1 \rightarrow ggg} = 7-30 \text{ keV}$ for the $1S$ state, with $m_Q = 20-50 \text{ GeV}$. Thus, for $\tilde{g}\tilde{g}$ states,

$$\begin{aligned}\Gamma_{3S_1 \rightarrow ggg} &= 20 \times 13.5 \times (7-30) \text{ keV} = \\ &= (2-8) \text{ MeV}\end{aligned}\tag{11}$$

where 13.5 comes from a colour factor. It might be possible that the $gq\bar{q}$ branching ratio is larger than ggg , but we have not yet computed the ratio.

The P state allowed by Eq. (10) is 1P_1 (i.e., 1^{++}). The 1P_1 state decays into hadrons via $gq\bar{q}$ or ggg . By comparison with the theoretical estimate ¹⁴⁾ of the $gq\bar{q}$ decay width of the toponium 3P_1 states, i.e., (1-10 keV), one concludes that $\Gamma_{^1P_1 \rightarrow gq\bar{q}}$ for $\tilde{g}\tilde{g}$ is of the order of (1-10) MeV. As for $\Gamma_{^1P_1 \rightarrow ggg}$ we do not yet have any estimate.

4. Lifetime of the gluino

Wherever the gluino is trapped, it decays spontaneously into $\tilde{y}q\bar{q}$ (Fig. 3a) or $\tilde{\gamma}g$ (Fig. 3b), leaving other particles as spectators. Decays of metastable $\tilde{g}g$, $\tilde{g}q\bar{q}$ and $\tilde{g}\tilde{g}$ states are dominated by these free gluino decays. Decay rates are given by ¹⁷⁾

$$\begin{aligned}\Gamma_{\tilde{g}}^{(3a)} &= \sum_q \frac{\alpha_s \alpha e_q^2 M_{\tilde{g}}^5}{96\pi} \left(\frac{1}{\tilde{m}_L^4} + \frac{1}{\tilde{m}_R^4} \right) \\ \frac{\Gamma_{\tilde{g}}^{(3b)}}{\Gamma_{\tilde{g}}^{(3a)}} &= \sum_{\text{flavours}} \frac{\alpha_s}{4} \frac{(\tilde{m}_R^2 - \tilde{m}_L^2)^2}{\tilde{m}_L^4 + \tilde{m}_R^4}\end{aligned}\tag{12}$$

$\sim 1-5\%$ [in some models ^{17), 18)}].

The $\tilde{y}q\bar{q}$ decay produces two quark jets and the $\tilde{\gamma}g$ decay produces one gluon jet, while $\tilde{\gamma}$ escapes detection. Spectator particles do not have enough energy to produce jets. In the case of $\tilde{\gamma}g$ decay, nearly half of the total energy is carried away by the photino. Thus this process is rather easy to identify by measuring large missing transverse momentum.

Due to the extraordinarily large binding energy, the gluino inside $\tilde{g}g$ cannot always decay in this way. A possible decay process is

$$\tilde{g}\tilde{g} \rightarrow \tilde{g}g + \tilde{\gamma} \quad \text{or} \quad \tilde{g}\tilde{g} \rightarrow \tilde{g}q\bar{q} + \tilde{\gamma} \quad (13)$$

However, from Fig. 1b, one finds that this process is allowed only if $m_{\tilde{g}} \gtrsim 9$ GeV for $m_{\tilde{g}} = 0$, and the inverse processes $\tilde{g}g$ or $\tilde{g}q\bar{q} \rightarrow \tilde{g}\tilde{g} + \tilde{\gamma}$ are energetically allowed if $m_{\tilde{g}} \lesssim 9$ GeV. If the photino is massive, e.g., $m_{\tilde{\gamma}} = 5$ GeV, the processes (13) are allowed for $m_{\tilde{g}} \gtrsim 15$ GeV and the inverse processes are not allowed.

5. Decays of excited $\tilde{g}\tilde{g}$, $\tilde{g}g$, $\tilde{g}q\bar{q}$

i) $\tilde{g}\tilde{g}$: If the $\tilde{g}\tilde{g}$ state is above the threshold of $\tilde{g}g + \tilde{g}g$, the glueballon state decays very quickly ($\sim 10^{-23}$ sec) into $\tilde{g}g + \tilde{g}g$. Below threshold, if the $\tilde{g}\tilde{g}$ state has a large angular momentum, it is difficult for it to decay into two gluons. In this case, the excited glueballon state emits mesons (e.g., 2π) and falls into a lower glueballon state on one of the gluino decays, according to Eq. (12). The former is the OZI forbidden decay and the typical width is $O(1)$ keV for the toponium case and $O(10)$ keV for the $\tilde{g}\tilde{g}$ case.

ii) $\tilde{g}g$ and $\tilde{g}q\bar{q}$: Excited hybrino states fall down to lower states by emitting ordinary mesons π , K , etc. This is the OZI allowed transition, and the decay width is of the order of 100 MeV. Some of the glueballinos and the hybrinos have the same quantum number and mix with each other. If the energy levels are close to each other, they mix strongly. We must expect that many of the glueballino states contain substantial $\tilde{g}q\bar{q}$ components. If there exists an excited pure glueballino state, the decay width is small (~ 10 keV), since meson emission is an OZI forbidden process. The excited mixture state of $\tilde{g}q\bar{q}$ and $\tilde{g}g$ can fall down to the lowest state by emitting mesons quickly.

6. $\tilde{t}\tilde{t}$ and $\tilde{Q}\tilde{t}$ ($\tilde{t}$: scalar partner of the top quark)

If we neglect the spin-dependent forces, one finds $V_{\tilde{t}\tilde{t}}(r) = V_{\tilde{t}\tilde{t}}(r) = V_{\tilde{Q}\tilde{t}}(r)$; thus the spectra for these systems are similar. In Figs. 4a and 4b, we show the $\tilde{t}\tilde{t}$ and $\tilde{Q}\tilde{t}$ spectra where the previous potential, Eq. (9), is used to compute them. Since $m(\tilde{t}\tilde{t}) \cong 2m(\tilde{t})$ is the threshold for $\tilde{t}\tilde{t} \rightarrow \tilde{t}q + \tilde{t}\bar{q}$ (see Figs. 4a,b), there are many $\tilde{t}\tilde{t}$ states below threshold. The quantum number for the l excited $\tilde{t}\tilde{t}$ state is given by $J = l$, $P = (-1)^l$.

If $m_{\tilde{t}} < m_t$, t and $\bar{t}$ inside $t\bar{t}$ quickly decay into $\tilde{t}\tilde{\gamma}$ and $t\tilde{\gamma}$ respectively ¹⁹⁾. In this case, $\tilde{t}$ has a relatively long lifetime. A free $\tilde{t}$ can decay, for example, in the following way :

$$\begin{aligned}\tilde{t} &\rightarrow \tilde{b} \text{ (virtual)} + \mu \nu \\ &\quad \downarrow \\ &\quad b + \tilde{\gamma}\end{aligned}\quad (14)$$

$$\begin{aligned}\tilde{t} &\rightarrow t \text{ (virtual)} + \tilde{\gamma} \\ &\quad \downarrow \\ &\quad b \mu \nu\end{aligned}$$

Due to the relatively long lifetime of $\tilde{t}$, it is likely that the following two kinds of processes follow before $\tilde{t}$ decay [Eq. (14)] takes place :

i) $\tilde{t}$ remains in the system and the $\tilde{t}\bar{t}$ state is created. It falls into the ground state by emitting mesons or photons. Eventually, $\tilde{t}\bar{t}$ annihilates into $\tilde{\gamma}\gamma$ or $\tilde{g}g$ or $\tilde{\gamma}g$ or $\gamma\tilde{g}$. It is also possible to make the transition into $\tilde{t}\bar{t}$ state by emitting $\tilde{\gamma}$. The S state $\tilde{t}\bar{t}$ ($J^P = 0^+$) decays into gg , $\gamma\gamma$, $\tilde{g}g$, $\tilde{\gamma}\gamma$, etc. The P state ($J^P = 1^-$) decays into $\ell^+\ell^-$, ggg , $\gamma g\tilde{g}$, $\tilde{g}g\tilde{g}$, etc., and lives longer than the S state. The higher orbital excited state ($\ell \geq 2$) lives even longer. For these excited states, the electric dipole transition becomes important [$\Gamma \sim O(10 \text{ keV})$].

ii) The scalar quark $\tilde{t}$ escapes from the bound system by picking up a light quark if it has enough energy. In this case, the states such as $\tilde{u}\tilde{t}$, $\tilde{d}\tilde{t}$, $\tilde{s}\tilde{t}$, ... will be produced. If they are in the ground states, there will be almost no way to decay except free $\tilde{t}$ decay [Eq. (14)]. Let us assume that $\tilde{t}q$ ($q=u$ or d) lives such a long time that the produced $\tilde{t}\bar{q}$ can hit the proton target. In this case, the following reactions take place :

$$\begin{aligned}\tilde{t}\bar{q} + qqq &\rightarrow \tilde{t}qq + \gamma \\ &\quad \xrightarrow{oz} \tilde{t}qq + \text{mesons}\end{aligned}\quad (15)$$

We use Eq. (4) to compute the energy spectrum of the $\tilde{t}qq$ states. Results are shown in Table 2, where our notation is very similar to that for charmed baryons ($\Lambda_c, \Sigma_c, \dots$) ²⁰⁾.

7. Detection of SUSY hadrons

The $\tilde{g}\tilde{g}$ state can be produced via toponium decay, $^3S_1 \rightarrow g+\tilde{g}\tilde{g}$ or $\gamma+\tilde{g}\tilde{g}$. Keung²¹⁾ has calculated the ratio

$$\Gamma(^3S_1 \rightarrow g\tilde{g}\tilde{g})/\Gamma(^3S_1 \rightarrow ggg)$$

We compute $\text{BR}(^3S_1 \rightarrow ggg)$ by using the model described in Ref. 14). Results are shown in Fig. 5. Combining Fig. 5 and results by Keung, one finds that the branching ratio $\text{BR}(^3S_1 \rightarrow \tilde{g}\tilde{g}g)$ is as much as 10 % for $m(\text{scalar quark}) \lesssim m_Q \lesssim 35$ GeV. From a potential model estimate¹⁴⁾, one finds

$$\text{B.R.}(^3S_1 \rightarrow g\tilde{g}\tilde{g}) \sim 0.2 \text{ B.R.}(^3S_1 \rightarrow ggg)$$

A promising way to find $\tilde{g}\tilde{g}$ states is to find peaks in the γ spectrum in $^3S_1 \rightarrow \gamma X$, since the branching ratio is well within the experimental limit.

The following process provides us with a possibility to find glueballinos

$$^3S_1 \text{ quarkonium} \rightarrow \tilde{g}g + \tilde{\gamma} \quad (16)$$

The ratio $\Gamma(^3S_1 \rightarrow g\tilde{g}\tilde{\gamma})/\Gamma(^3S_1 \rightarrow gg\gamma)$ is computed by Keung²¹⁾. We show the branching ratio $^3S_1 \rightarrow gg\gamma$ as a function of m_t in Fig. 5. By combining results by Keung and Fig. 5, we find that the branching ratio of $^3S_1 \rightarrow g\tilde{g}\tilde{\gamma}$ is of the order of one per cent if $m(\tilde{t}) \lesssim m_t \lesssim 35$ GeV and $m_t \lesssim m_{\tilde{g}} \lesssim 1.5 m_t$. As for the bontonium, the branching ratio $\Upsilon \rightarrow g\tilde{g}+\gamma$ is not very large. If $m(\tilde{b})$ is 20 GeV, the branching ratio is less than 10^{-5} and decreases by a factor 10 for every 5 GeV increase of $m(\tilde{b})$.

If the produced glueballino is in an excited state, it can emit mesons and falls into the hybrino state. Main decays of the ground glueballino state or hybrino state are free gluino decay (Fig. 3a,b). The glueballino and hybrino states can mix with $\tilde{q}\tilde{q}$ states (Fig. 6a,b). The produced $\tilde{q}$ is unstable and decays into $q+\tilde{\gamma}$, but if $m_{\tilde{\gamma}} < m_t$, $\tilde{t}$ can survive for a rather long time. Even if $m_{\tilde{\gamma}} > m_t$, $\tilde{t}$ is not very unstable if $m_{\tilde{\gamma}}$ is not much larger than m_t (e.g., $\Gamma \lesssim 1$ MeV if $m_t = 30$ GeV, $|m_{\tilde{\gamma}} - m_t| < 2$ GeV). In Table 3, we list various decay modes for various particles.

In summary, we have found the following results :

- i) the $\tilde{g}\tilde{g}$ state has extraordinarily large binding energy, and the wave function at the origin becomes extremely large;

- ii) a promising way to find the $\tilde{g}\tilde{g}$ state is to look for peaks in toponium $\rightarrow \gamma + X$;
- iii) the lowest $\tilde{g}\tilde{g}$ and $\tilde{g}q\bar{q}$ states have the same parity in the BS model and opposite parity in the bag model;
- iv) if $m(\tilde{t}) < m(t)$, the toponium state becomes broad while $\tilde{t}\tilde{t}$ becomes narrow, and states such as $\tilde{t}u$ have quite a long lifetime. Decay modes of these states are shown.

ACKNOWLEDGEMENTS

One of us (S.O.) would like to thank J. Ellis, M.K. Gaillard, R.R. Horgan, A. Martin, A.N. Mitra and G. Schierholz for discussions.

		Mass	J^P	I
(a) Bag model prediction ($\tilde{g}g, \tilde{g}gg$)				
	$\tilde{g}g_{TE}$	$m_{\tilde{g}} + 480 \text{ MeV}$	$\frac{1}{2}^-, \frac{3}{2}^-$	0
	$\tilde{g}g_{TM}$	$m_{\tilde{g}} + 795 \text{ MeV}$	$\frac{1}{2}^+, \frac{3}{2}^+$	0
	$\tilde{g}g_{TE}g_{TE}$	$m_{\tilde{g}} + 960 \text{ MeV}$	$\frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+$	0
	$\tilde{g}g_{TE}g_{TM}$	$m_{\tilde{g}} + 1290 \text{ MeV}$	$\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$	0
	$\tilde{g}g_{TM}g_{TM}$	$m_{\tilde{g}} + 1590 \text{ MeV}$	$\frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+$	0
(b) BS model prediction ($\tilde{g}g$)				
	S state	$m_{\tilde{g}} + 300 \text{ MeV}$	$\frac{1}{2}^+, \frac{3}{2}^+$	0
	P state	$m_{\tilde{g}} + 900 \text{ MeV}$	$\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$	0

TABLE I - Predictions of glueballino mass spectra by the bag and BS models.

	Content	Mass	J^P	I
$\Sigma_{\tilde{t}}$	$\tilde{t}uu, \tilde{t}ud, \tilde{t}dd$	$m_{\tilde{t}} + 798 \text{ MeV}$	1^+	1
$\Lambda_{\tilde{t}}$	$\tilde{t}ud$	$m_{\tilde{t}} + 603 \text{ MeV}$	0^+	0
$A_{\tilde{t}}$	$\tilde{t}su, \tilde{t}sd$	$m_{\tilde{t}} + 826 \text{ MeV}$	0^+	$\frac{1}{2}$
$S_{\tilde{t}}$	$\tilde{t}su, \tilde{t}sd$	$m_{\tilde{t}} + 955 \text{ MeV}$	1^+	$\frac{1}{2}$
$\Omega_{\tilde{t}}$	$\tilde{t}ss$	$m_{\tilde{t}} + 1118 \text{ MeV}$	1^+	0

TABLE II - Mass spectrum of the $\tilde{t}qq$ state.

Particle	Decay mode	Remarks
$g\tilde{g}$ (glueballino)	FGD (free gluino decay) $\begin{cases} \tilde{g} \rightarrow q\bar{q} + \tilde{\gamma} \\ \tilde{g} \rightarrow g + \tilde{\gamma} \end{cases}$ $\tilde{g}\tilde{g} + \tilde{\gamma}$ $\tilde{g}q\bar{q} + \pi \text{ or } \gamma$ $\tilde{\gamma} + \gamma$	Usually the main decay mode, $\Gamma \propto m_{\tilde{g}}^5$ 1-5% of $q\bar{q} + \tilde{\gamma}$ Allowed for small $m(\tilde{g})$ ($\lesssim 9$ GeV) Allowed in some models Significant only for small $m_{\tilde{g}}$
$\tilde{g}q\bar{q}$ (hybrino)	FGD $\begin{cases} \tilde{g} \rightarrow q\bar{q} + \tilde{\gamma} \\ \tilde{g} \rightarrow g + \tilde{\gamma} \end{cases}$ $\tilde{g}\tilde{g} + \tilde{\gamma}$ $\tilde{g}g + \pi \text{ or } \gamma$ $\tilde{\gamma} + \gamma$	Usually the main decay mode, $\Gamma \propto m_{\tilde{g}}^5$ 1-5% of $q\bar{q} + \tilde{\gamma}$ Allowed for small $m(\tilde{g})$ ($\lesssim 9$ GeV) Energetically forbidden in most models Significant only for small $m_{\tilde{g}}$
$\tilde{g}\tilde{g}$ (glueballon)	FGD $\begin{cases} \tilde{g} \rightarrow q\bar{q} + \tilde{\gamma} \\ \tilde{g} \rightarrow g + \tilde{\gamma} \end{cases}$ $^3S_1 \rightarrow 3g$ $\rightarrow gq\bar{q}$ $^1P_1 \rightarrow gq\bar{q}$ $\rightarrow 3g$ $^3D_1 \rightarrow 3g, gq\bar{q}$ $^3D_{2,3} \rightarrow 2g$	Dominant for large $m_{\tilde{g}}$ ($>> 10$ GeV) Not allowed for $m_{\tilde{g}} \lesssim 9$ GeV (2-8) MeV Allowed (1-10) MeV Allowed Allowed Allowed
$\tilde{t}$ (if $m(\tilde{t}) < m(t)$)	$t^*\tilde{\gamma} \rightarrow b\bar{t}\nu + \tilde{\gamma}$ $\tilde{b}^*\bar{t}\nu \rightarrow b\tilde{\gamma}\bar{t}\nu$ $\tilde{W}^*b$ $\rightarrow \tilde{t}\nu, \bar{t}\bar{\nu}$ $\rightarrow q\bar{q}\tilde{\gamma}$	Allowed Allowed Allowed if $\tilde{t}$ or $\bar{t}$ are light enough Allowed
$t\bar{t}$ (if $m(\tilde{t}) < m(t)$)	$\tilde{t}\bar{t} + \tilde{\gamma}, t\bar{t} + \tilde{\gamma}$ $t\bar{t} + \gamma, \tilde{g}\tilde{g}$ $g\bar{g}, g\bar{g}g, \text{etc.}$	Allowed if $\tilde{\gamma}$ is light enough Allowed Allowed

TABLE III - Possible decay modes for $g\tilde{g}$, $\tilde{g}q\bar{q}$, $\tilde{g}\tilde{g}$, $\tilde{t}$ and $t\bar{t}$ states

R E F E R E N C E S

- 1) Proceedings of the CERN Workshop on SUSY versus Experiment, organized by D.V. Nanopoulos, A. Savoy-Navarro and Ch. Tao, CERN preprint TH-3311/EP.82/63-CERN (1982) - to appear soon as a Physics Reports.
- 2) A. Chodos et al., Phys.Rev. D9 (1974) 3471; Phys.Rev. D11 (1974) 2599.
- 3) T. DeGrand et al., Phys.Rev. D12 (1975) 2060.
- 4) R.L. Jaffe and K. Johnson, Phys.Letters 60B (1976) 201.
- 5) M. Chanowitz and S. Sharpe, Phys.Letters 126B (1983) 225.
- 6) S. Ono and O. Pène, Phys.Letters 109B (1982) 101; LPTHE Orsay 82/36, Z.f.Phys. C, to be published.
- 7) C. Heusch, private communication.
- 8) A.N. Mitra and D.S. Kulshreshtha, Phys.Rev. D26 (1982) 3123;
A.N. Mitra and I. Santhanam, Zeit.f.Phys. C8 (1981) 33.
- 9) A.N. Mitra and S. Ono, CERN preprint TH.3707 (1983).
- 10) S. Ono, Phys.Rev. D17 (1978) 888.
- 11) S.L. Glashow, Physica 96A (1979) 27.
- 12) A.D. Sakharov, SLAC TRANS-0191 (1980).
- 13) S. Ono and F. Schöberl, Phys.Letters 118B (1982) 419;
S. Ono, Lectures presented at the XXIII Cracow School of Theoretical Physics, Zakopane (May 1983), CERN preprint TH.3679 (1983).
- 14) H. Kühn and S. Ono, Zeit.f.Phys. C, to be published.
- 15) W. Buchmüller and S.-H. Tye, Phys.Rev. D24 (1981) 132.
- 16) J. Zuk, G.C. Joshi and J.W.G. Wignall, Phys.Rev. D, to be published.
- 17) H.E. Haber and G.L. Kane, University of Michigan preprint, UM TH83-18 (1983).
- 18) C. Kounnas, A.B. Lahanas, D.V. Nanopoulos and M. Quiros, CERN preprint TH.3657 (1983).
- 19) J. Ellis and S. Rudaz, Phys.Letters 128B (1983) 248.
- 20) M.K. Gaillard, B.W. Lee and J.L. Rosner, Rev.Mod.Phys. 47 (1975) 277.
- 21) W.-Y. Keung, Phys.Rev. D28 (1983) 1129.

FIGURE CAPTIONS

- Figure 1 : a) The mass spectrum of the S wave hybrino states. The notation is ⁹⁾, e.g., $\rho_{\tilde{g}} = \{[\bar{q}q]_{S=1}\tilde{g}\}_{J=\frac{1}{2}}$, $\rho_{\tilde{g}}^* = \{[\bar{q}q]_{S=1}\tilde{g}\}_{J=\frac{3}{2}}$, $\pi_{\tilde{g}} = \{[\bar{q}q]_{S=0}\tilde{g}\}_{J=\frac{1}{2}}$.
- b) The mass spectrum of the glueballino state. Thresholds for $\tilde{g}\tilde{g} \rightarrow \tilde{g}\tilde{g} + \tilde{g}\tilde{g}$, $\tilde{g}\tilde{g} + \gamma$ and $\tilde{g}q\bar{q} + \gamma$.
- Figure 2 : The mass spectrum of the $\tilde{t}\tilde{t}$ states. The toponium spectrum is identical to this if the spin-dependent force and the spin and parity are ignored.
- Figure 3 : a),b) Decays of the free gluino state.
- Figure 4 : The mass spectrum of the $\tilde{t}\tilde{q}$ state.
- Figure 5 : Branching ratios $BR(^3S_1 \rightarrow ggg)$ and $BR(^3S_1 \rightarrow \gamma gg)$ of toponium states.
- Figure 6 : a),b) Glueballino and hybrino mixing with $\tilde{g}\tilde{q}$.

Fig.1a

Fig.1b

Fig. 2

Fig. 4

Fig.3a

Fig.3b

Fig. 5

Fig. 6a

Fig. 6b