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SUPER-HIGGS EFFECT WITH LOCAL R SYMMETRY
AND VANISHING COSMOLOGICAL CONSTANT

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ABSTRACT

We consider the gravitino mass growth in theories with a Fayet-Iliopoulos term. We exhibit simple models with positive definite potential, vanishing cosmological constant and spontaneously broken R symmetry.

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Recent applications of supersymmetry to particle physics use models where the $N = 1$ supergravity is spontaneously broken at an intermediate energy scale $M_S \approx O(\sqrt{M_W M_P})$ ¹⁾. The "low energy" effects of supergravity couplings, in the limit $M_P \rightarrow \infty$ and $m_{3/2}$ fixed, result in soft breaking terms²⁾ of global supersymmetry induced from the hidden sector to the observable sector of the theory. The observable sector is defined as that part of the Lagrangian with non-gravitational interactions while the hidden sector is responsible for the super-Higgs mechanism and the consequent gravitino mass generation.

In this letter we consider a super-Higgs sector implemented by local R symmetry³⁾. This is the generalization to local supergravity of the Fayet-Iliopoulos mechanism for spontaneous supersymmetry breaking⁴⁾. We consider a class of "minimal" models in which the super-Higgs sector is due to a spin $(0^{\pm}, 1/2)$ chiral multiplet with non-trivial R charge and a spin $(1, 1/2)$ multiplet gauging R symmetry. This is the minimal supermultiplet content for a super-Higgs sector with local R symmetry, if one demands the vanishing of the cosmological constant with positive definite potential.

We shall show that with a single scalar multiplet, it is possible to break spontaneously both supersymmetry and local R symmetry. In fact, we exhibit a simple model in which the supersymmetry breaking receives equal contributions from the D term due to the vector multiplet and from the f term due to the chiral multiplet. The final mass spectrum contains a massive real scalar, a chiral spinor and a massive vector boson. It turns out that all of them have the same mass, $m = 2m_{3/2}$.

Two low-energy limits are possible when $M_P \rightarrow \infty$ with $m_{3/2}$ fixed depending on the behaviour of the R gauge coupling constant g . In the case when $g \rightarrow 0$ with $M_P \rightarrow \infty$, the Lagrangian for the super-Higgs sector just reduces to a free Lagrangian with mass terms of the several particles involved. This situation is analogous to the case of a super-Higgs sector without gauged R symmetry. Since in this case the behaviour of the gauge coupling constant g depends on M_P , we call it "gravitational R interaction". However, it is interesting enough that another limit is possible when the gauge coupling constant g remains finite when $M_P \rightarrow \infty$. In that case, the gauged R symmetry is of "non-gravitational" type, and the hidden sector has non-trivial interactions left after the limit $M_P \rightarrow \infty$ which are proportional to powers of g . The resulting $N = 1$ supergravity Lagrangian in this second limit will be explicitly presented below.

In an R symmetric theory with a single chiral multiplet coupled to $N = 1$ supergravity, the most general form of the scalar potential is⁵⁾

$$V(z, z^+) = V(z z^+) = e^G [G_z G_{zz^+}^{-1} G_{z^+} - 3] + \frac{1}{2} g^2 D^2 \quad (1)$$

with $G = G(z, z^+)$ and $D = z(\partial/\partial z)G \equiv z G_z$; g is the R gauge coupling constant and we have set

$$\frac{M_P}{\sqrt{8\pi}} = \frac{1}{k} = 1.$$

Note that the $U(1)$ invariance forbids in the Lagrangian a term of the form $f(z)F_{\mu\nu}^2$, and therefore there will be no direct "gaugino" masses. The only reparametrizations of z allowed by preserving the form of $U(1)$, $z \rightarrow e^{ia}z$ are

$$z \rightarrow \lambda z'^\beta \quad (2)$$

Under this change the Lagrangian keeps the same form provided we use $\tilde{G}(z', z'^+) = G(z, z^+)$ and $g' = g/\beta$. If, for instance, there is a singularity in G_{zz^+} at $z = 0$ of the type $(zz^+)^\gamma$, we can always remove it by interchanging the variable $z \rightarrow \lambda z'^\beta$ with $\beta\gamma/1-\beta = 1$.

Using the identities

$$G_z = z^+ G'(\rho) \quad , \quad G_{z^+} = z G'(\rho) \quad (3a)$$

$$G_{zz^+} = G'(\rho) + \rho G''(\rho) = D'(\rho) = \frac{1}{\rho} D'(\omega) \quad (3b)$$

$$D = \rho G'(\rho) = G'(\omega) \quad (3c)$$

where $\rho \equiv z z^+$ and $\omega = \log \rho$, we can rewrite Eq. (1) as follows

$$V(\omega) = e^{G(\omega)} \left[\frac{D^2(\omega)}{D'(\omega)} - 3 \right] + \frac{1}{2} g^2 D^2(\omega) \quad (4)$$

Because of the particular form of V in Eq. (4) it is convenient to choose D as a variable and define

$$\varphi(D) \equiv e^G > 0 \quad (5)$$

and re-express everything in terms of the function $\varphi(D)$, its derivative and D through

$$D = \frac{d}{d\omega} G(\omega) = \frac{\varphi'(D)}{\varphi(D)} \cdot \frac{dD}{d\omega} \quad (6)$$

Note that, given the function $\varphi(D)$, one can perform the change of variable $D \leftrightarrow \omega$ using Eq. (6)

$$\omega = \int \frac{\varphi'(D)}{\varphi(D)} \frac{dD}{D} \quad (7)$$

In terms of the function $\varphi(D)$ the scalar potential takes the simple form

$$V = \left(D \frac{d}{dD} - 3 \right) \varphi(D) + \frac{1}{2} g^2 D^2. \quad (8)$$

The kinetic term G_{zz^+} is written as

$$\int G_{zz^+} = \frac{D \varphi(D)}{\varphi'(D)} \quad (9)$$

which implies that the domain of the corresponding manifold is defined by

$$D \varphi'(D) > 0. \quad (10)$$

We can now choose $\varphi(D)$ such that in this domain the potential V is positive definite. A particularly interesting case corresponds to the Einstein manifold defined by

$$R_{zz^+} = \frac{2}{n} G_{zz^+} \quad (11a)$$

where

$$R_{zz^+} = \partial_z \partial_{z^+} \log G_{zz^+}. \quad (11b)$$

In terms of φ and D , Eq. (11b) is easily re-expressed as

$$R_{zz^+} = \frac{d^2}{dD^2} \left(\frac{D \varphi}{\varphi'} \right) \cdot G_{zz^+}. \quad (12)$$

Therefore, the Einstein manifolds correspond to

$$\frac{D\varphi}{\varphi'} = \frac{1}{n} (D - D_1) \cdot (D - D_2) \quad (13a)$$

or

$$\varphi(D) = c \frac{(D - D_1)^{n(\gamma+1)}}{(D - D_2)^{n\gamma}} \quad (13b)$$

with

$$\gamma = \frac{D_2}{D_1 - D_2} \quad (13c)$$

and D_1, D_2 integration constants.

The change of variable from D to φ can be easily found using Eq. (7)

$$\log \frac{D - D_1}{D - D_2} = \frac{1}{n} (D_1 - D_2) \log \frac{\rho}{\rho_0}. \quad (14)$$

We can then analyze the various cases depending on D_1, D_2 and the parameter n . Here we will concentrate on the simple case where the potential has the form $V \sim (D - \xi)^2$. The only solution in that case is $D_2 = 0, D_1 = \xi/3$ and $n = 1$:

$$\varphi(D) = \frac{g^2 \xi}{2} \cdot \left(D - \frac{\xi}{3} \right) \quad (15a)$$

$$V(D) = \frac{1}{2} g^2 (D - \xi)^2 \quad (15b)$$

$$D = \frac{\xi}{3} \cdot \left[1 - \left(\frac{\rho}{\rho_0} \right)^{1/3} \right]^{-1}. \quad (15c)$$

At the minimum $D = \xi$, the gravitino mass is given as

$$m_{3/2}^2 = \varphi(\xi) \quad (16a)$$

and, making use of the normalization

$$G_{zz^*}(\rho_{min}) = 1, \quad (16b)$$

we get

$$V(\rho) = \frac{2}{3} m_{3/2}^2 \frac{\left[1 - \left(\frac{\rho}{\rho_{min}}\right)^{5/3}\right]^2}{\left[1 - \frac{2}{3} \left(\frac{\rho}{\rho_{min}}\right)^{5/3}\right]^2} \quad (17a)$$

with

$$\rho_{min} = \frac{2 m_{3/2}^2}{g^2} = \frac{2}{3} \xi^2 \quad (17b)$$

and

$$\left(\frac{\rho_{min}}{\rho_0}\right)^{5/3} = \frac{2}{3}$$

The metric $G_{zz}(\rho)$ is

$$G_{zz} = \frac{1}{9} \frac{\left(\frac{\rho}{\rho_{min}}\right)^{5/3-1}}{\left[1 - \frac{2}{3} \left(\frac{\rho}{\rho_{min}}\right)^{5/3}\right]^2} \quad (18)$$

The potential becomes singular at the boundary of the domain

$$\frac{\rho}{\rho_{min}} = \left(\frac{3}{2}\right)^{3/5} \quad (\rho = \rho_0) \quad (19)$$

The kinetic term is singular at $\rho = 0$ except for $\xi = 3$, but, as we have already mentioned, we can always remove this singularity by a proper redefinition of the variable z

$$z = z'^{3/5} \quad \text{and} \quad g = \frac{3g'}{\xi} \quad (20)$$

Note that at the minimum one has the relation

$$e^G G_z G_{zz}^{-1} G_z = \frac{1}{2} g^2 D^2 = \frac{1}{2} g^2 \xi^2 = \frac{3}{2} m_{3/2}^2. \quad (21)$$

This means that the gravitino mass gets equal contributions from the "D breaking" and the "f breaking" terms. As we will see below, this also means that the would-be Goldstino is an equal mixture of the χ and λ spinors of the scalar and vector multiplets.

It is now straightforward to compute the particle masses of the theory. The scalar square mass matrix is

$$M_B^2 = g^2 f_{\min} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \text{ i.e. } m_1^2 = 0, \quad m_2^2 = 2g^2 f_{\min} = 4m_{3/2}^2. \quad (22)$$

The massless mode is the would-be Goldstone boson of the spontaneously broken R symmetry, and the physical massive mode has a mass equal to $m_2 = 2m_{3/2}$. The vector boson mass is

$$m_V^2 = 2g^2 f_{\min} = 4m_{3/2}^2. \quad (23)$$

For the fermion mass matrix, one notices the fact that the $\chi\chi$ term of the $N = 1$ supergravity Lagrangian⁶⁾

$$- \bar{\chi}_L \chi_L \left[G_{zz} + G_z G_z - G_z G_{zz} G_z + G_z^{-1} \right] e^{G/2} \quad (24)$$

is absent due to the vanishing of the expression in brackets in our model. The remaining term of the spin 1/2 mass matrix is

$$(\bar{\lambda}_L \chi_L (2igz^+) + \text{h.c.}) e^{G/2} \quad (25)$$

The Goldstino mode is defined by the term coupled to the spin 3/2 gauge field in the supergravity Lagrangian⁶⁾

$$\bar{\psi}_R \cdot \gamma \eta_L + \text{h.c.}, \quad (26a)$$

$$\begin{aligned} \eta_L &= -m_{3/2} G_z \chi_L + \frac{i}{2} g z G_z \lambda_L = \\ &= i \sqrt{\frac{3}{2}} \cdot m_{3/2} \cdot (i\sqrt{2} \chi_L + \lambda_L) / \sqrt{2}. \end{aligned} \quad (26b)$$

Therefore Eq. (25) can be rewritten as

$$\bar{\lambda}_L (i\sqrt{2} \chi_L) \sqrt{2} g z^+ + h.c. , \quad (27)$$

with $\sqrt{2} g z_{\min}^+ = 2m_{3/2}$. Equation (27) shows that the orthogonal combination to the would-be Goldstone fermion

$$\psi = (i\sqrt{2} \chi_L - \lambda_L) / \sqrt{2} \quad (28)$$

has mass $m_\psi = 2m_{3/2}$. Therefore, the physical spectrum consists of a massless spin 2 graviton, a massive spin 3/2 gravitino with mass $m_{3/2}$ and a scalar, a spinor and a vector all with the same mass

$$m_s = m_v = m_\psi = 2 m_{3/2} . \quad (29)$$

The square mass sum rule gives in this case

$$\text{Str } \mathcal{M}^2 = \sum_{J=0}^{3/2} (-1)^{2J} \mathcal{M}_J^2 = 4 m_{3/2}^2 . \quad (30)$$

To understand this result better, we have to apply the general mass formula in $N = 1$ supergravity coupled to matter in the presence of non-vanishing Kähler curvature and a non-vanishing D term⁷⁾

$$\text{Str } \mathcal{M}^2 = 2 \left[g^2 D + g^2 z \Gamma_z D \right] - 2 R_{zz^+} \frac{G_z G_{z^+}}{(G_{zz^+})^2} e^G \quad (31)$$

where Γ_z is the Kähler connection

$$z \Gamma_z = z \partial_z \log G_{zz^+} . \quad (32)$$

We can re-express this supertrace in terms of φ , φ' and D as

$$\text{Str } \mathcal{M}^2 = 2 g^2 D \frac{d}{dD} \left(\frac{D\varphi}{\varphi'} \right) - 2 \frac{d^2}{dD^2} \left(\frac{D\varphi}{\varphi'} \right) \cdot D\varphi' \quad (33)$$

At the minimum, using $V = V' = 0$, the normalization condition $G_{zz^+} = ((1/\rho)(\varphi D/\varphi'))_{\min} = 1$ and defining

$$\frac{2}{n_0} = \frac{d^2}{dD^2} \left(\frac{D\varphi}{\varphi'} \right) \Big|_{\text{minimum}}$$

$$\varphi_0 = m_{3/2}^2 \Big|_{\text{minimum}} \quad (34)$$

$$x = \left(g^2 D^2 / \varphi \right) \Big|_{\text{minimum}}$$

we get

$$St_e M^2 = 2 m_{3/2}^2 \left[x - \frac{x}{3 - \frac{x}{2}} + \frac{x^2}{\left(3 - \frac{x}{2}\right)^2} - \frac{2}{n_0} \left(3 - \frac{x}{2}\right) \right] \quad (35)$$

The particular case studied in this paper corresponds to

$$St_e M^2 \Big|_{\substack{n_0 = 1 \\ x = 3}} = 4 m_{3/2}^2 \quad (36)$$

which agrees with the left-hand side given by Eq. (30).

We can now consider the potential given by Eq. (17a) in the low-energy limit $M_P \rightarrow \infty$ with $m_{3/2}$ fixed. By means of Eq. (27b), we can express the dimensionless variable ξ as

$$\xi = \frac{m_{3/2} \sqrt{24 \pi}}{M_P \cdot g}, \quad (37)$$

so that two different limits are possible. If $g \rightarrow 0$ with $M_P \rightarrow \infty$, then it is easy to see that

$$V \Big|_{\substack{M_P \rightarrow \infty \\ g \rightarrow 0}} = 2 m_{3/2}^2 \phi^2 + O\left(\frac{1}{M_P}\right) \quad (38)$$

where ϕ is the physical scalar degree of freedom. The vector, spinor and scalar degrees of freedom decouple in this limit and one gets a free theory of massive particles of spin 0, 1/2 and 1, respectively. This situation is analogous to the normal super-Higgs effect without local chiral symmetry when the scalar field of the hidden sector just decouples in the limit $M \rightarrow \infty$, $m_{3/2}$ fixed.

However, in the presence of a gauge R symmetry, a second non-trivial limit exists for the supergravity Lagrangian with $M_P \rightarrow \infty$, $m_{3/2}$ fixed and g fixed. In this case, the gauge R interaction is non-gravitational in the sense that g does not vanish with $M_P \rightarrow \infty$.

In this limit, the bosonic part of the Lagrangian becomes

$$\mathcal{L}_B \underset{M_P \rightarrow \infty, g \text{ fixed}}{=} -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} R_{\text{LIN}} + \frac{1}{2} (\partial_\mu \phi)^2 / \left[1 + \frac{g\phi}{2m_{3/2}} \right] + \frac{1}{2} (2m_{3/2})^2 A_\mu^2 - \frac{8m_{3/2}^2}{g^2} \log^2 \left(1 + \frac{g\phi}{2m_{3/2}} \right) + O(1/M_P). \quad (39)$$

The scalar, spinor and vector particles remain interacting in this limit. Equation (39) describes the boson interactions. The other interaction terms in the complete supergravity Lagrangian in the super-unitary gauge ($\eta_L = 0$) are:

$$\mathcal{L}_F = \frac{1}{2} m_{3/2} \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu + \frac{2m_{3/2}}{|1 + g\phi/(2m_{3/2})|} \bar{\psi}_L \psi_L + \frac{1}{4} \left(1 + \frac{1}{1 + g\phi/(2m_{3/2})} \right) \bar{\psi} \not{D} \psi + h.c. \quad (40)$$

where ψ_μ is the massive gravitino field and ψ_L is the physical massive fermion.

Let us emphasize that we could also have taken this limit starting with a non-singular kinetic term at $\rho = 0$ ($\xi = 3$) by redefining, before the limit, $z = z^{\xi/3}$ and $g = g^{\xi/3}$. The limit will be taken with $g \rightarrow 0$ but g' fixed which will be the effective low-energy gauge coupling ($\xi = m_{3/2} \sqrt{24\pi}/M_P g'$). Note that for $g \rightarrow 0$, the interaction terms given by Eqs. (39) and (40) have as a limit the mass term for the ϕ and ψ_L particle with a common mass $m = 2m_{3/2}$.

In both limits we get explicit supersymmetry breaking terms as in the normal super-Higgs mechanism.

Before we close this letter, let us mention that if we want to apply the proposed D breaking in particle physics, we must couple the matter multiplets to the supergravity. In that case a problem related to the positivity of the total scalar potential may, in general, arise. This in fact implies some restrictions on the total G function.

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