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QCD AT THE COLLIDER

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INTRODUCTION

Although QCD¹ imposes itself as the only theory of strong interactions within reach of the weapons of conventional quantum field theory, yet it is still the least established sector of the standard model. Testing QCD is, in fact, more difficult than testing the electroweak sector. In the latter domain the theory is more explicit because perturbation theory can always be applied. Besides that, the leptons and the weak gauge bosons are at the same time the fields in the Lagrangian and the particles in our detectors. On the other hand, QCD is the theory of quarks and gluons while only hadrons are observable; also, perturbation theory can only be applied in that particular domain of the strong interactions where approximate freedom, which is only asymptotic, can be reached.

This difficulty of testing QCD is the reason why substantial evidence in support of the theory has only been gathered during the last few years from a collection of converging indications from many different processes, while no single feasible test is by itself decisive.

We can actually identify three broad classes of QCD tests:

a) Purely topological tests. Asymptotic freedom implies that a perturbative expansion is valid in the deep inelastic region. This fact induces a hierarchy of levels that one tries to expose by a suitable analysis. For example, at zero order in e^+e^- annihilation the naïve parton model is obtained. The final state consists of two jets and shows a collinear topology. At order α_s a fraction of three-jet events is also produced with a planar topology.

The transverse momentum increases in a plane, while $(p_T)_{out}$, the transverse momentum out of the event plane, stays fixed. In a three-jet event the slim jet is the same as the jets in a two-jet event (at a somewhat smaller energy). At order α_s^2 four jets appear, $(p_T)_{out}$ and the acoplanarity start increasing, the slim jet becomes fatter and so on. This group of experiments has led to spectacular successes for QCD, not only in e^+e^- , but also in deep inelastic scattering (DIS) (where the final state analysis is essential as a cross examination of the QCD explanation of scaling violations).

b) Correct estimates of rates and cross-sections. For many processes, definite predictions are obtained at the leading approximation in α_s , which cannot be made completely quantitative, because of ignorance, either of next-to-leading corrections, or of some non-perturbative aspects. This kind of prediction is still significant because the answer often changes by orders of magnitude or is qualitatively different in alternative approaches. For example: jets in $\gamma\gamma$ processes, large p_T photons, charm production in DIS and photoproduction, slopes of p_T versus $\sqrt{S}$ (at fixed scaling variables), etc.

c) Fully quantitative tests. The typical exponents of this class are $R_{e^+e^-}$, the scaling violations in DIS, the γ structure functions and, to some extent also, the three-jet distributions in e^+e^- , the widths of quarkonium decays, the Drell-Yan cross-section, etc. These are predictions that also include the complete evaluation of next-to-leading corrections, so that every quantity which appears in the leading term is well defined. In principle one can use these results in order to measure α_s , to fix the gluon spin, and to test the gauge nature of the strong interactions. For example, we report in Fig. 1 a number of measurements of α_s , in the $\overline{MS}$ definition. We see a plot of $\alpha_s(Q^2)$ for different values of $\Lambda_{\overline{MS}}$ (with next-to-leading accuracy). We observe that even a rough determination of α_s at low Q^2 is transformed into a rather restrictive prediction at large Q^2 . Thus, the mere observation of precocious scaling, which is interpreted to imply $\alpha_s \lesssim 1$ at $Q^2 \sim 1 \text{ GeV}^2$, leads to a strong constraint on the value of α_s at the largest available Q^2 , if QCD is right. This prediction is in fact in agreement with the existing measurements in spite of many uncertainties and ambiguities in the extraction of α_s from the data. Note that the sensitivity of α_s on $\Lambda_{\overline{MS}}$ decreases with Q^2 , so that at $Q^2 \sim 10^3 \div 10^4 \text{ GeV}^2$ an order of magnitude in Λ corresponds to less than a factor 1.5 on α_s .

The importance for QCD of experiments at the SPS collider rests on the possibility they offer of testing parton dynamics in a new and highly non-trivial configuration. For example, hadron-hadron interactions in the deep inelastic, large p_T , region are non-linear in parton densities. The relevant predictions cannot be derived

Fig. 1 α_s as a function of $\Lambda_{\overline{MS}}$ to next-to-leading accuracy $\alpha_s = \alpha_0 [1 - b' \alpha_0 \ln \ln Q^2 / \Lambda_{\overline{MS}}^2]$ with $\alpha_0 = 1/(b \ln Q^2 / \Lambda_{\overline{MS}}^2)$, $b = (33 - 2f)/12\pi$, $b' = (153 - 19f)/[2\pi(33 - 2f)]$, for $f = 4$. The experimental points are from: ● T decays, ■ CDHS (deep inelastic scattering), ● TASSO (e^+e^-), △ MARK J (e^+e^-), ▽ CELLO (e^+e^-), ☒ JADE (e^+e^-).

by less committed formulations than the explicit QCD improved parton model, such as, for example, light cone dominance and operator expansion. This complexity, which is important for providing qualitatively new testing grounds is, however, paid for by a loss of precision in predictive power. Thus experiments at the SPS collider give a very valuable contribution to classes a) and b) of QCD tests as listed above.

In addition to that, $p\bar{p}$ collisions are also important as jet sources with an energy scale comparable to that of an e^+e^- ring with beam energy of about 50 GeV and more. As partons are studied through their jets and there are many kinds of partons, one needs different jet factories in order to be able to disentangle quark flavours and gluons. In particular, the problem of distinguishing gluon jets is of special relevance. As we shall see, most of the jets produced at large p_T and high energies in $p\bar{p}$ collisions are

predicted to be gluon jets. This fact adds further interest to the jet studies that will be carried out in the future at the SPS collider.

INCLUSIVE JET YIELD

The clear observation of large p_T jets of the SPS collider² is in itself a distinct success of the QCD parton picture of deep inelastic processes. Once more, the early apparent failure of a parton prediction has been demonstrated to be merely due to lack of sufficient energy. Not only that, but the observed cross-section for inclusive jet production, as a function of p_T and $\sqrt{s}$, agrees remarkably well with the QCD estimates³⁻⁶ (see Fig. 2). This is particularly significant in that the very steep dependence from energy and transverse momentum greatly facilitates the appearance of large factors, such as, for example, in the comparison³ of jet yields at ISR⁷ and at SPS energies.

Fig. 2 The jet p_T distribution (as measured by UA2) compared with the QCD predictions³⁻⁶. The shaded band is obtained from different forms of parton densities and different values of Λ as explained in Ref. 6.

The leading term of the inclusive jet yield is computed by a sum of convolutions of products of two given parton densities with the corresponding cross-section into any two final partons, evaluated in lowest order QCD:

$$\frac{Ed\sigma}{d^3p} = \sum_{A,B} \int dx_1 dx_2 p_A(x_1, p_T^2) p_B(x_2, p_T^2) \delta(s+t+u) \alpha_s^2(p_T^2) \sum_f \frac{|M|_{AB \rightarrow f}^2}{s} \quad (1a)$$

where s, t and u are the Mandelstam variables for the (massless) parton subprocesses and $p_{A,B}$ are the parton densities in the colliding hadrons. In the Table we report the relevant parton processes and their associated $|M|^2$. Also shown is a figure of merit F_M , namely the value of $|M|^2$ at $\theta = 90^\circ$ in the parton centre of mass. In this frame,

$$s = 4\epsilon^2; \quad t, u = -2\epsilon^2(1 \pm \cos \theta) \quad (1b)$$

so that at $\theta = \pi/2$, $s:t:u = 4:-2:-2$. The importance of qg and gg subprocesses at large angles is immediately seen.

Table
Parton cross-sections: $d\sigma/dt = (\pi\alpha_s^2/s^2) |M|^2$ [averaged (summed) over initial (final) colours and spins]. s, t, u refer to the parton processes. F_M is the value of $|M|^2$ at c.m. angle $\theta = 90^\circ$ ($s:t:u = 4:-2:-2$).

PARTON PROCESS	$ M ^2$	F_M
$q\bar{q}' \rightarrow q\bar{q}'$ $q\bar{q}' \rightarrow q\bar{q}'$	$\frac{4}{9} \frac{s^2+u^2}{t^2}$	2.22
$q\bar{q} \rightarrow q\bar{q}$	$\frac{4}{9} \left(\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} \right) - \frac{8}{27} \frac{u^2}{st}$	3.26
$q\bar{q} \rightarrow q'\bar{q}'$	$\frac{4}{9} \frac{t^2+u^2}{s^2}$	0.22
$q\bar{q} \rightarrow q\bar{q}$	$\frac{4}{9} \left(\frac{s^2+u^2}{t^2} + \frac{t^2+u^2}{s^2} \right) - \frac{8}{27} \frac{u^2}{st}$	2.59
$q\bar{q} \rightarrow gg$	$\frac{32}{27} \frac{u^2+t^2}{ut} - \frac{8}{3} \frac{u^2+t^2}{s^2}$	1.04
$gg \rightarrow q\bar{q}$	$\frac{1}{6} \frac{u^2+t^2}{ut} - \frac{3}{8} \frac{u^2+t^2}{s^2}$	0.15
$qg \rightarrow qg$	$-\frac{4}{9} \frac{u^2+s^2}{us} + \frac{u^2+s^2}{t^2}$	6.11
$gg \rightarrow gg$	$\frac{9}{2} \left(3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2} \right)$	30.4

Precise trigger requirements are demanded in order for Eq. (1a) to apply. An inclusive trigger is necessary for us to be able to identify the $\vec{p}_T$ of each final parton with the measured $\vec{p}_T$ of the corresponding jet (defined as the sum of the $\vec{p}_T$ of the hadrons in the jet). An inclusive trigger in principle allows us to do without fragmentation functions, thus simplifying the expression of the cross-section. On the other hand, the presence of jets must be defined by demanding energy clustering in a limited region of phase space. The allowed cones must be sufficiently small to define precisely the jet momentum and sufficiently large to include most of the jet fragments.

For central production, i.e., for almost zero rapidity, which is the dominant configuration, each incoming parton has an energy $\epsilon = xE$ where E is the P or $\bar{P}$ c.m. energy. That is, the energy fraction of the two incoming partons are nearly the same, $x_1 \approx x_2 \approx x$. Then the average energy per parton is given by $\bar{E} = (\sqrt{S}/2)x$, where $\sqrt{S} = 540$ GeV at the collider and x is the average x of a given kind of partons in the nucleon. For valence $\bar{x} \approx 0.25 \div 0.30$, which corresponds to $\bar{\epsilon} \approx 70 \div 80$ GeV. The final parton transverse momentum is $p_T = \epsilon \sin \theta$. The perturbative region is at large angle θ , where p_T is almost as large as $\bar{\epsilon}$, i.e., $\bar{x}_T \approx 2\bar{p}_T/\sqrt{S} \lesssim \bar{x}$. At the collider, this means $p_T \gtrsim 10 \div 20$ GeV. At such small values of x_T , the production of gluon jets is seen to be dominant, both because there are many gluons in the nucleon at small x and because the colour factors are favourable. At fixed x_T , $d\sigma/dp_T^2$ is predicted to fall as p_T^{-4} , apart from logarithms:

$$\frac{d\sigma}{dp_T^2} = \frac{f(x_T, \alpha(p_T^2))}{p_T^4} \quad (2)$$

At fixed S a change in p_T implies a proportional variation of x_T . The steeper dependence on p_T which is observed in this case is well reproduced by the decrease of parton densities as x_T is increased.

There are a number of ambiguities and uncertainties that affect both the predicted cross-section and the data. First the measurements are affected by systematic errors, which are quoted to be of 60% for UA1 and of 40% for UA2 (in addition to the errors shown in the plots such as that in Fig. 1). From the point of view of theory there is, first of all, a scale ambiguity, as in all leading logarithmic calculations. In Eq. (1a), the parton densities $p(x, Q^2)$ and the coupling $\alpha_s(Q^2)$ all depend on the scale Q^2 . It is only by a computation of next-to-leading terms that in principle one can distinguish a priori equivalent possibilities, such as $Q^2 = p_T^2$, $\frac{1}{2} p_T^2$, $2stu/(s^2+t^2+u^2)$, etc. Note that the QCD evolution must be applied to the parton densities as measured in DIS

at $Q_0^2 \sim 5 \div 20 \text{ GeV}^2$, to boost them up to $Q^2 \approx 10^3 \div 10^4 \text{ GeV}^2$. It makes a quite sizeable difference if one stops at $Q^2 = p_T^2$ or $\frac{1}{2} p_T^2$ or $2p_T^2$. Scaling violations influence the shape of the p_T distribution at fixed energy by a very significant amount, which is somewhat different for different choices of the scale. Also note that the rate is proportional to $\alpha_s^2(Q^2)$ which doubles its sensitivity to first-order variations of α_s . A related ambiguity is the choice of Λ . For α_s this is exactly the same as the scale ambiguity, because α_s only depends on the ratio Q^2/Λ^2 . For the evolution equations the situation is different, because the solutions only depend on Q^2/Λ^2 provided the initial conditions are also scaled in terms of a fixed Q_0^2/Λ^2 ratio.

On the other hand, the calculation of non-leading terms is quite difficult because there are many subprocesses. The only existing⁸ fragment of this calculation refers to $q_i + q_j \rightarrow q_k + x$ where i, j, k indicate different flavours. In this case the "correct" scale (the one which minimizes the next-to-leading corrections) was found to be $Q^2 = \frac{1}{2} p_T^2$. Also in Ref. 4) the large π^2 terms in the corrections to all subprocesses were computed.

The calculation of next-to-leading terms is not only very involved, but also it is not clear that it would be completely worthwhile in view of the fact that the gluon density in the nucleon is poorly known. In fact, the exact definition of the gluon (or any other) density is to be specified in terms of some measurement. Different definitions change $g(x, Q^2)$ by terms of order α_s . In DIS this is not very important because the gluons only enter at order α_s in the structure functions, so that a variation of order α_s in $g(x, Q^2)$ only affects the structure functions at order α_s^2 . But in the case of jet production in $p\bar{p}$, the gluons already enter in lowest order, so that, if the next-to-leading corrections to the production rate are to be meaningfully treated, one must know the gluon density to a resolution which is comparable to the terms of order α_s . This is certainly not the case because the gluon density is only roughly measured from the scaling violations in DIS, which are themselves a small effect (order α_s). Fortunately, the sensitivity of the different empirical forms of the gluon density appears⁵ to be incidentally reduced in the interval of x values involved in the calculation of the jet yield at the relevant energies. In fact, in this particular range of x values the various determinations of the gluon density show a limited dispersion.

These ambiguities do not allow a greater accuracy than a factor of two or so. In spite of that the observed agreement with the data is still highly significant because of the very steep dependence of the jet yield on $\sqrt{s}$ and p_T .

At small angles, i.e., for $\Lambda^2 \ll p_T^2 \ll \epsilon^2$, the situation becomes more complicated. Terms of order $\log \epsilon^2/p_T^2 \sim \log \theta^2$ are no longer negligible in comparison to $\log p_T^2/\Lambda^2$. There are two scales in the parton processes that make all the scale ambiguities more acute. In addition to that, it also becomes more difficult to disentangle spectator jet effects. Further work is needed in order to extend the theory at small angles (as well as to different triggering conditions).

ANGULAR DISTRIBUTION OF JETS

The study of the angular distribution of jets offers a valuable double check of the underlying QCD dynamics by testing the vector nature of gluons. From the table, for each subprocess, the angular distributions in $\cos \theta$ are simply derived (averaged over $\cos \theta \rightarrow -\cos \theta$ because one cannot tell which jet is which), θ being the scattering angle of the subprocess in the parton centre of mass. This angle can be identified with the jet-beam angle in the large p_T jets rest frame, provided that one neglects the errors introduced by the intrinsic momentum spread of partons in the nucleon and by fragmentation effects. For example, for $gg \rightarrow gg$ one finds:

$$\frac{dn}{d\cos \theta} \sim 3 - \frac{\sin^2 \theta}{4} + \frac{4(1+3\cos^2 \theta)}{\sin^4 \theta} \quad (3)$$

The Coulombic singularity $\sin^{-4} \theta$ is typical of gauge vector boson exchange. In the subprocesses $qg \rightarrow qg$ and $gg \rightarrow gg$, it arises from the three-gluon vertex, while in $q\bar{q} \rightarrow q\bar{q}$ it would be present in the Abelian case as well. It is thus of particular interest to study experimentally the jet angular distribution near the forward direction. A word of caution is, however, necessary in that we already remarked that other contributions than the Born parton diagrams become important at small angles. The latter presumably are important at angles such that $\alpha_s/\pi \ln \theta^2 \sim 1$, so that one expects that the much stronger Coulombic singularity is still unaffected at not too small θ values, where it should be already clearly visible.

The comparison with the data is shown in Fig. 3. The larger acceptance at small angles of UA1 is crucial in order to probe the interesting angular region. The results are encouraging but more statistics at small angles are needed.

Fig. 3 The jet angular distribution (data from UA1)

GLUON JETS VERSUS QUARK JETS

A lot of the jets produced at large p_T at the collider are predicted to be gluon jets. It would be important to find an objective difference between the jets produced in $p\bar{p}$ collisions (mostly gluons) and in e^+e^- (mostly quarks). Up to now, the difference did not show up, although maybe the matter has not yet been attacked with adequate means. The longitudinal momentum distributions of particles in jets of comparable energies look the same at $p\bar{p}$ and in e^+e^- . The same is true for the p_T distributions of jet fragments (transverse with respect to the jet axis in this case). We recall that some indications have actually been reported by the JADE collaboration⁹ at PETRA that in a three-jet event in e^+e^- the p_T spread of fragments is more pronounced for the less energetic jet (which has the largest probability of being a gluon). I also recall that there are hints from several sources (DIS, quarkonium decay final states versus continuum, etc.) that baryons are more abundant in gluon than in quark jets. Perhaps the most interesting analysis is the attempt¹⁰ at comparing the multiplicity in the jets at $p\bar{p}$ and e^+e^- . The idea is that the multiplicity should be larger in a gluon jet than in a quark jet. In fact, at the level of parton multiplicities (i.e., as if the partons were directly observable) the ratio of 9 to 4 is asymptotically predicted¹¹ for gluon and quark jets respectively. However,

some assumptions on fragmentation are needed in order to translate the parton multiplicity into a hadron multiplicity. The main experimental ambiguity which up to now makes the result still inconclusive, has to do with the criterion for attributing to one jet a given particle in the central region of $p\bar{p}$ where the four jets merge.

In conclusion, there is no evidence as yet of a difference between gluon jets and quark jets. However, the collider appears to be in a good position for adding much to this issue in the near future.

THREE JETS AT LARGE p_T

The next level in topological complexity is given by events with three jets at large p_T . In principle, all the experience gained from the analysis of three-jet events in e^+e^- can be carried to $p\bar{p}$. On the theoretical side, the very simple, well-known formula for the three-jet distribution¹ in e^+e^- is replaced in this case by a much more complicated expression, which is obtained by considering the whole plethora of $2 \rightarrow 3$ parton subprocesses. This calculation has been completed¹² as far as the distribution away from the collinear and soft singularities is concerned, while the more involved one-loop corrections to the $2 \rightarrow 2$ subprocesses have not been yet attacked (the addition of the virtual corrections is needed to compute the next-to-leading corrections to the two-jet production rate).

The result can be cast in a relatively simple form by using a method by Berends et al.¹³ which is a generalization of the well-known factorization of infra-red singularities valid in the soft limit. They argue that in the massless theory, when no two four momenta are parallel, the $2 \rightarrow 3$ cross-section can be factorized as:

$$\sigma(1+2 \rightarrow 3+4 + g) = |M|^2 I \quad (4)$$

where $|M|^2$ and I in the limit $k \rightarrow 0$ (k being the g four momentum) approach the non-radiative cross-section and the soft infra-red factor respectively. The correct form of the two factors, checked by comparison with the exact result, has been obtained for the relevant subprocesses.

These calculations allow us to derive a whole set of testable predictions to be compared with the data. In particular, a $2 \rightarrow 2$ process is planar, while a $2 \rightarrow 3$ process is not. Thus, an acoplanarity and a momentum component p_{out} (out of the "event plane") can be defined and their distributions studied. A preliminary comparison of the p_{out} distribution with the data shows a remarkable agreement (see Fig. 4).

Fig. 4 The distribution in p_{out} , the momentum perpendicular to the plane defined by the trigger jet and the beam momentum.

MULTIPLICITIES

A wealth of very interesting data on multiplicity distributions have been collected at the SPS collider¹⁴. Experiment has shown that $dn/dy|_{y=0}$, i.e., the total charged multiplicity per unit rapidity y at $y = 0$, increases from ISR to collider energy at a rate compatible with $dn/dy|_{y=0} \sim \ln S$. This implies that Feynman scaling is violated (which is no news) and suggests that the total average multiplicity $\bar{n}$ increases with energy with a law compatible with $\bar{n} \sim \ln^2 S$, as confirmed by direct measurements of $\bar{n}$. The bulk of the multiplicity is determined by soft processes which lie far away from the domain of perturbative QCD. This is why we shall be very brief here. On the other hand, QCD leads to important predictions for the scale dependence of multiplicities within hard jets, such as, for example, those produced at large p_T^2 in $p\bar{p}$ or in e^+e^- annihilation at large Q^2 . In hard jets, the parton multiplicity (i.e., as if partons could be directly observed) is predicted¹⁵ to evolve in Q^2 with the law:

$$\bar{n}(Q^2) \approx C \left(\ln \frac{Q^2}{\Lambda^2} \right)^{k_1} \exp(k_0 \sqrt{\ln Q^2 / \Lambda^2}) \quad (5)$$

where

$$k_0 = \sqrt{\frac{72}{33-2f}} \sim 1.7 \quad (6)$$

$$k_1 = (\frac{111}{4} - \frac{11}{18}f)/(33-2f) \sim 1 \quad (7)$$

(f being the number of excited flavours). The non-leading term with exponent k_1 has been recently derived¹⁶. Thus if one assumes that the hadronization mechanism does not significantly depend on Q^2 , the same behaviour can be expected to hold for the measurable hadronic multiplicity in the jet. Extending this prediction to the total multiplicity in $p\bar{p}$ collisions is not justified, because the mechanisms at work in soft processes might be different. However, it is interesting to observe that $\bar{n} \sim \ln^2 S$ and $\bar{n} \sim \exp\{k_0 \sqrt{\ln S/S_0}\}$ are both compatible¹⁷ with the available data.

Another interesting experimental result is the approximate validity of Koba, Nielsen, Olesen (KNO) scaling. As is well known, Feynman scaling implies KNO scaling. Since the former is violated the question is whether there are arguments for the latter to be respected. The question can be discussed in QCD. Again one must distinguish the case of multiplicity distributions in hard jets and in soft processes. In the first case, one can study the distribution of colour singlet clusters of gluons in a jet^{15,18}. This distribution shows KNO scaling broken by logarithms. Probably this conclusion is likely to be true in general. On the other hand, the shape of the KNO scaling function appears to be more model-dependent.

W and Z PRODUCTION

The discovery¹⁹ of $W^\pm$ and Z^0 at the expected masses is so important at a very fundamental level that the QCD aspects of W and Z physics are relatively secondary. They are still very interesting, though. Indeed, the production mechanism is a Drell-Yan type process with $\sqrt{S} \approx 540$ GeV and $Q \approx 90$ GeV. All the very non-trivial QCD improved parton model predictions for this sort of process¹ can be confronted with experiment in a domain of energies far deeper inside the deep inelastic region than previously attained. In the context of W and Z physics, one interesting challenge to QCD theory from experiment is the measured shape of the transverse momentum distribution, (in addition to the determination of the production cross-section). The QCD theoretical analysis of p_T distributions in Drell-Yan processes not only allows a very severe and demanding test of the validity of our views on the underlying, quite complex parton dynamics, but is also interesting in itself because it involves many new issues not present in the calculation of the total production cross-section, such as, for example, the

presence at intermediate p_T of two different scales and the related appearance of double logs.

The p_T distribution is built up from three components. The first component is the wave function spread of partons in hadrons, the so-called intrinsic p_T . This is a bound state effect, and is not within reach of theory at present. Thus, it is a model-dependent term in the distribution. However, we know empirically that the intrinsic p_T is small, roughly of the order of some units of Λ , as expected. Thus we expect that, as $(p_T)_{MAX}$ increases, this uncertainty will at most affect a thin slice of the p_T spectrum at small p_T . The second component is the "hard" component that determines the distribution at $p_T \sim Q$. At large values of p_T , $\alpha_s(p_T)$ becomes small and the lowest order contribution, where the heavy lepton pair is produced in association with a massless parton, is dominant. This term is completely under control. Finally, there is a "soft" component arising at $\Lambda \ll p_T \ll Q$ from multiple gluon emission which is enhanced by logs, in the sense that the effective coupling becomes $\alpha(p_T) \ln Q^2/p_T^2$. One is able to take into account these logarithmically enhanced terms by resumming them to all orders in perturbation theory²⁰.

It is important to stress that while a complete determination of the p_T distribution requires the knowledge of all three sources of p_T as listed above, on the other hand, some particular quantities can be more directly obtained. The most important example is perhaps the slope of the average p_T ($\langle p_T^n \rangle$) versus $\sqrt{S}(S^{n/2})$ at fixed $\tau = Q^2/S$:

$$\langle p_T \rangle = \alpha_s(Q^2) [1 + O(\alpha_s(Q^2))] \sqrt{S} f(\tau, \ln Q^2) + \text{const.} \quad (8)$$

While the constant terms (apart from logs) also contain non-perturbative contributions, the leading term is completely determined by the perturbative result. Actually, the experimental verification of the size with energy of p_T at fixed τ and the rough check of the rise of the slope in $\sqrt{S}$ is one of the striking semi-quantitative successes of QCD. The uncertainties which are left over by a lowest order evaluation have to do once more with a scale ambiguity, with the poor knowledge of the gluon density and with the presence of large corrective terms in the total cross-section (K factor²¹). In the quark-antiquark channel a calculation of the next-to-leading terms in the p_T distribution was completed²². This allows us to remove some important uncertainties, while those connected with the rather large size of the resulting corrections are still present.

Going back to the general problem of a calculation of the whole p_T distribution, the existing²³ theoretical p_T distributions for W and Z are still not completely satisfactory in that not all the available information is used, and, in some cases, useless and

and questionable assumptions were made. What it is important to stress is that a reasonable precise prediction of the p_T distribution is indeed possible, and it will be extremely important to test it in all its complicated features both in Drell-Yan processes at moderate energies, where statistics are large, and in W and Z production (or in Drell-Yan processes at collider energies in the future), where the energy is much larger.

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