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WEAK INTERACTIONS ON THE LATTICE

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ABSTRACT

We show that lattice QCD can be used to evaluate the matrix elements of four fermion operators which are relevant for weak decays. A first comparison between the results obtained on the lattice and other determinations is also presented.

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1. - Introduction -

With the advent of more powerful computers, which allow the study of gauge theories on large lattices ($10^3 \times 20$ or larger), lattice QCD is becoming a powerful tool in giving quantitative predictions for hadronic physics at low energy.

In this paper we show that lattice QCD can be used to compute the value of matrix elements of four fermion operators which are relevant for weak interactions. The methods developed here can be applied to the matrix elements of four fermion operators between bosonic states using the same quark propagators used for hadronic spectroscopy. If the propagators are available the computation can be carried out using a small amount of computer time.

As a first application of our methods we report an evaluation of the matrix elements, between pseudoscalar states, of operators of the form:

$$\begin{aligned} O_{LL} &= \left[\bar{\psi}_1 \gamma^\mu \frac{(1-\gamma_5)}{2} \psi_2 \right] \left[\bar{\psi}_3 \gamma_\mu \frac{(1-\gamma_5)}{2} \psi_4 \right] \\ O_{LR} &= \left[\bar{\psi}_1 \gamma^\mu \frac{(1-\gamma_5)}{2} \psi_2 \right] \left[\bar{\psi}_3 \gamma_\mu \frac{(1+\gamma_5)}{2} \psi_4 \right] \end{aligned} \quad (1)$$

The effective weak Hamiltonian including short distance gluon effects, can be expressed as a combination of operators of this form /1-5/.

In the next Section we outline the method of computation. In section 3 we describe the numerical results based on 14 link configurations in a $10^3 \times 20$ lattice, for which the Wilson quark propagators were already available /6/. This analysis is encouraging in that the level of statistical fluctuations has resulted to be fully acceptable even with a small number of configurations.

In Section 4 we discuss the relation between four fermion operators and their continuum limit.

Finally, in Section 5 we use the results of this simulation for a first evaluation of matrix elements relevant for $K-\bar{K}$ oscillations and $\Delta T = 1/2$ and $\Delta T = 3/2$ K decays.

2. - General Methods -

Consider the propagator of bosonic operators B^i at vanishing space momentum

$$G^{ab}(t) = \sum_{\vec{x}} \langle 0 | T \{ B^a(\vec{x}, t) B^b(0, 0) \} | 0 \rangle \quad (2)$$

On general grounds, on a Euclidean lattice of length T in time and periodic boundary conditions, G will have the form

$$G^{ab}(t) = \sum_n Z_n^{ab} \cosh[m_n(t - T/2)] \quad (3)$$

where the sum is over all allowed intermediate states^(*).

A perturbation of the Hamiltonian

$$H + \lambda O \quad (4)$$

affects the values of both Z_n and m_n

$$\begin{aligned} \frac{dG^{ab}}{d\lambda} = \sum_n \left\{ \left[\frac{d}{d\lambda} Z_n^{ab} \right] \cosh[m_n(t - T/2)] + \right. \\ \left. \left[\frac{dm_n}{d\lambda} Z_n^{ab} \right] (t - T/2) \sinh[m_n(t - T/2)] \right\} \end{aligned} \quad (5)$$

(*) This functional form applies when B^a and B^b are tensors of the same order, (e.g. two scalars, etc.) or when their orders differ by an even number. When the tensor orders of B^a and B^b differ by an odd number (e.g. a pseudoscalar and the time component of an axial vector) Eq. (3) should be substituted by

$$\tilde{G}^{ab}(t) = \sum_n \tilde{Z}_n^{ab} \sinh[m_n(t - T/2)]$$

where

$$\frac{dm_n}{d\lambda} = -\langle n|0|n\rangle$$

The diagonal matrix elements of 0 can then be obtained from a fit of G^{ab} and $dG^{ab}/d\lambda$ to their functional forms in Eqs. 3 and 5. With a suitable re-diagonalization this method can yield non diagonal matrix elements between degenerate states (e.g. K^0 , $\bar{K}^0$).

Matrix elements between non degenerate states (e.g. K , π) can be estimated by neglecting the mass difference or approximately evaluating its effects.

We note that $dG^{ab}/d\lambda$ can be expressed as:

$$\frac{dG^{ab}}{d\lambda} = -\sum_{\vec{x}, \vec{z}, t_z} \langle 0|T\{B^a(\vec{x}, t) O(\vec{z}, t_z) B^b(0, 0)\}|0\rangle \quad (6)$$

which reduces to the evaluation of diagrams of the kind in Fig. 1. This would seem to require the computation of fermion propagators between any pair of arbitrary points in the lattice, a computation which would exceed by a very large factor (the number of lattice points) the amount of computer time needed for usual hadron spectroscopy, which uses only fermion propagators from a fixed point to any other point of the lattice.

However, by making use of translational invariance, we can rewrite Eq. (6) as:

$$\frac{dG^{ab}}{d\lambda} = -\sum_{\vec{x}, \vec{z}, t_z} \langle 0|T\{B^a(\vec{x}-\vec{z}, t-t_z) O(0, 0) B^b(-\vec{z}, -t_z)\}|0\rangle \quad (7)$$

which corresponds to the graphs in Fig. 2 and makes use only of propagators stemming from the origin.

Unfortunately this trick does not work for baryon states where, as shown in Fig. 3, we cannot identify a single point from which all fermion lines emerge.

In the following we will consider the physically interesting case of pseudoscalar mesons, and of the two operators in Eq. (1). If we rewrite the rhs of Eq. (7) in terms of quark propagators we find an expression

$$\pm \sum_{y,z} \langle \text{tr} \{ S(-z) S^\dagger(-z) \gamma_{L,R}^\mu S(y) S^\dagger(y) \gamma_L^\mu \} \rangle_U \delta(y_0 + z_0 - t) \quad (8a)$$

for diagram 2a, and

$$\mp \sum_{y,z} \langle \text{tr} \{ S(-z) S^\dagger(-z) \gamma_{L,R}^\mu \} \text{tr} \{ S(y) S^\dagger(y) \gamma_L^\mu \} \rangle_U \delta(y_0 + z_0 - t) \quad (8b)$$

for diagram 2b, where $S(y) = S(0, y)$ and we use the identity, valid for any configuration of the gluon fields:

$$S(y, 0) = \gamma^5 S^\dagger(0, y) \gamma^5$$

The $\pm$ ($\mp$) sign applies to the L-L and L-R cases respectively.

In the above expressions $\langle X \rangle_U$ indicates the functional average:

$$\langle X \rangle_U = \frac{\int \mathcal{D}[U] \exp[-S_{\text{eff}}(U)] X(U)}{\int \mathcal{D}[U] \exp[-S_{\text{eff}}(U)]}$$

In the quenched approximation $S_{\text{eff}}(U)$ reduces to the gluonic action. In a Monte Carlo simulation $\langle X \rangle_U$ is evaluated by averaging $X(U)$ over a set of thermalized configurations.

3. - Numerical Results -

In this Section we discuss the numerical results for the analysis of $dG/d\lambda$ computed with the expression in Eqs. (8a) and (8b). We had available fermion propagators for 14 configurations in a $10^3 \times 20$ lattice at $\beta_W = 6$, and $k = 0.150$, and 0.155 , the latter value being very close to the estimated critical value $k_c \cong 0.157$.

If the lattice time dimensions were very large, compared with the inverse mass difference between the lowest and the first excited state,

$$(m^* - m)T \gg 1$$

at sufficiently large values of t the Eqs. (3) and (5) would be dominated by the contribution of the lowest state.

One could then use the simplified expressions:

$$G(t) = Z \cosh[m(t - T/2)] \quad (9)$$

$$\frac{dG}{d\lambda}(t) = \frac{dZ}{d\lambda} \cosh[m(t - T/2)] + Z \frac{dm}{d\lambda}(t - T/2) \sinh[m(t - T/2)] \quad (10)$$

In order to verify the applicability of these simplified expressions to our case we have done the following fits:

- 1) a fit of $G(t)$ to the sum of two terms with masses m and m^* for $5 \leq t \leq 15$.
- 2) a fit of $G(t)$ to a single term (Eq. 9) for $7 \leq t \leq 13$.

The results are displayed in Table 1. In order to estimate the statistical errors reported in this paper, we have subdivided the sample in two clusters of seven configurations, the fit being done separately on each cluster. The reported values are the average over the clusters, and the quoted errors are estimated from the dispersion.

A comparison between the values of m and Z obtained in the two cases, as well as the smallness of Z^*/Z for the first case indicate that Eq. (9) is a good approximation in the central region /7/.

$$7 \leq t \leq 13$$

Before proceeding to $dG(t)/d\lambda$ we have computed for our sample the mixed propagator between a pseudoscalar and an axial operator. This has been fitted (see footnote in Section 2) to the two term expression:

$$\tilde{G}(t) = Z \sinh[m(t - T/2)] + Z^* \sinh[m^*(t - T/2)] \quad (11)$$

The results for m and Z from this fit are also reported in Table 1.

For the case of $dG/d\lambda$ we have made two different fits:

- 1) A fit of $dG/d\lambda$ to the expression

$$\frac{dG(t)}{d\lambda} = a G(t) + \frac{dm}{d\lambda} Z(t - T_{1/2}) \sinh[m(t - T_{1/2})] \quad (12)$$

with m constrained to the value obtained from the fit to $G(t)$ for the same cluster.

- 2) A fit of $dG/d\lambda$ to the expression

$$\frac{dG(t)}{d\lambda} = a G(t) + b(t - T_{1/2}) \tilde{G}(t) \quad (13)$$

The two expressions should be equivalent at large t , where the propagators are dominated by the lowest mass state, and when averaged over a very large number of configurations.

We expect that Eq. (12) or Eq. (13) will give a good description of the data configuration by configuration, as they should incorporate to a good approximation the effects of gluon field fluctuations.

Chirality arguments suggest that in the continuum limit $dm_{LL}/d\lambda$ vanishes as m^2 as the quark mass goes to 0, i.e. $K \rightarrow K_c$ while $dm_{LR}/d\lambda$ in the same limit goes to a constant. Therefore in Table 2 we report the values of $-1/m^2 (dm_{LL}/d\lambda)$ and of $-dm_{LR}/d\lambda$ for different values of K .

We note that the difference between methods 1 and 2 can be ascribed to systematic effects coming from excited states, and gives a rough estimate of the systematic error.

We also note that the quality of the results for $K = 0.155$, which is very close to K_c is worse than for the other value. Not only for $K = .155$ statistical errors are much larger, but the fit of the data to any of the two expressions is very poor; in some cases the fitting procedure has failed to converge, while for the higher value of K this never happens.

4. - Relation between lattice operators and their continuum limit

In order to translate values of $dm/d\lambda$ as obtained on the lattice to physical values in the continuum limit one has to take into account certain corrections.

In this section we report briefly on a one loop calculation of the relation of the continuum operators to their lattice counterparts. A more detailed account of this calculation and of its results will be presented elsewhere /8/.

The general form of this relation is

$$O_i^{\text{CONT}} = \left(\delta_{ij} + \frac{g^2}{16\pi^2} Z_{ij} \right) O_j^{\text{LATT}} \quad (14)$$

The values of Z_{ij} depends critically on r , a parameter which appears in the Wilson form of the lattice fermion action:

$$S_\psi = \sum_x \left\{ \frac{-1}{2a} \sum_\mu \left[\bar{\psi}(x)(z - \gamma_\mu) U_\mu(x) \psi(x + \hat{\mu}) + \bar{\psi}(x + \hat{\mu})(z + \gamma_\mu) U_\mu^\dagger(x) \psi(x) \right] + \left[m + \frac{4z}{a} \right] \bar{\psi}(x) \psi(x) \right\}$$

We recall that r represents an explicit breaking of chiral invariance. In the $r \rightarrow 0$ limit one would recover the Kogut-Susskind formulation which is explicitly chiral invariant for massless quarks.

For $r \neq 0$ the LL and LR operators will mix with operators of different chirality. As an example we have:

$$\begin{aligned} (O_\pm^{\text{LL}})^{\text{CONT}} = & \left[1 + \frac{g^2}{16\pi^2} Z_\pm(z) \right] (O_\pm^{\text{LL}})^{\text{LATT}} \\ & + \frac{g^2}{16\pi^2} z^2 Z^*(z) \left[O_\pm^{\text{STP}} + O_\pm^{\text{VA}} + O_\pm^{\text{SP}} \right]^{\text{LATT}} \end{aligned} \quad (15)$$

where the sign $\pm$ refers to Fierz symmetric and antisymmetric combinations, e.g.

$$O_{\pm}^{LL} = \frac{1}{2} \left\{ [\bar{\Psi}_1 \gamma_{\mu}^L \Psi_2] [\bar{\Psi}_3 \gamma_{\mu}^L \Psi_4] \pm (2 \leftrightarrow 4) \right\} \quad (16)$$

and:

$$\begin{aligned} O_{\pm}^{STP} &= \pm \left[\frac{N+1}{16N} \right] \left\{ [\bar{\Psi}_1 \Psi_2] [\bar{\Psi}_3 \Psi_4] + [\bar{\Psi}_1 \gamma^5 \Psi_2] [\bar{\Psi}_3 \gamma^5 \Psi_4] \right. \\ &\quad \left. + [\bar{\Psi}_1 \sigma^{\mu\nu} \Psi_2] [\bar{\Psi}_3 \sigma_{\mu\nu} \Psi_4] \pm (2 \leftrightarrow 4) \right\} \\ O_{\pm}^{VA} &= - \left(\frac{N^2 \pm N - 1}{32N} \right) \left\{ [\bar{\Psi}_1 \gamma^{\mu} \Psi_2] [\bar{\Psi}_3 \gamma_{\mu} \Psi_4] \right. \\ &\quad \left. - [\bar{\Psi}_1 \gamma^{\mu} \gamma^5 \Psi_2] [\bar{\Psi}_3 \gamma_{\mu} \gamma^5 \Psi_4] \pm (2 \leftrightarrow 4) \right\} \end{aligned} \quad (17)$$

$$\begin{aligned} O_{\pm}^{SP} &= \frac{1}{16N} \left\{ [\bar{\Psi}_1 \Psi_2] [\bar{\Psi}_3 \Psi_4] - [\bar{\Psi}_1 \gamma^5 \Psi_2] [\bar{\Psi}_3 \gamma^5 \Psi_4] \right. \\ &\quad \left. \pm (2 \leftrightarrow 4) \right\} \end{aligned}$$

$$\sigma^{\mu\nu} = \frac{1}{2} [\gamma^{\mu}, \gamma^{\nu}]$$

A similar expression holds for O^{LR} :

$$\begin{aligned} (O_{1,2}^{LR})^{CONT} &= \left[1 + \frac{g^2}{16\pi^2} Z_{1,2}(\tau) \right] (O_{1,2}^{LR})^{LATT} \\ &\quad + \frac{g^2}{16\pi^2} \tau^2 Z^*(\tau) \left[O_{1,2}^{STP} + O_{1,2}^{VA} + O_{1,2}^{SP} \right]^{LATT} \end{aligned} \quad (18)$$

$$O_1^{LR} = [\bar{\psi}_1 t^A \gamma_L^\mu \psi_2] [\bar{\psi}_3 t^A \gamma_\mu^R \psi_4] - \left(\frac{N^2-1}{2N}\right) [\bar{\psi}_1 \gamma_L^\mu \psi_2] [\bar{\psi}_3 \gamma_\mu^R \psi_4]$$

$$O_2^{LR} = [\bar{\psi}_1 t^A \gamma_L^\mu \psi_2] [\bar{\psi}_3 t^A \gamma_\mu^R \psi_4] + \frac{1}{2N} [\bar{\psi}_1 \gamma_L^\mu \psi_2] [\bar{\psi}_3 \gamma_\mu^R \psi_4] \quad (19)$$

and

$$\begin{aligned} O_1^{STP} &= \left(\frac{N^2-1}{16N^2}\right) (S \times S + P \times P - T \times T) - \frac{1}{8N} (t^A S \times t^A S + t^A P \times t^A P - t^A T \times t^A T) \\ O_2^{STP} &= \left(\frac{N^2-1}{16N^2}\right) (S \times S + P \times P - T \times T) + \left(\frac{N^2-1}{8N}\right) (t^A S \times t^A S + t^A P \times t^A P - t^A T \times t^A T) \\ O_1^{VA} &= \frac{(N^2-1)^2}{32N^2} (V \times V - A \times A) + \frac{1}{16N} (t^A V \times t^A V - t^A A \times t^A A) \\ O_2^{VA} &= -\frac{(N^2-1)}{32N^2} (V \times V - A \times A) + \frac{1}{16N} (t^A V \times t^A V - t^A A \times t^A A) \end{aligned} \quad (20)$$

$$O_1^{SP} = \left(\frac{N^2-1}{16N^2}\right) (S \times S - P \times P) - \frac{(N^2+1)}{8N} (t^A S \times t^A S - t^A P \times t^A P)$$

$$O_2^{SP} = \left(\frac{N^2-1}{16N^2}\right) (S \times S - P \times P) - \frac{1}{8N} (t^A S \times t^A S - t^A P \times t^A P)$$

where $t^A V \times t^A V = (\bar{\psi}_1 t^A \gamma^\mu \psi_2) (\bar{\psi}_3 t^A \gamma_\mu \psi_4)$ and analogously for the other operators in Eqs. (20).

In the evaluation of the matrix elements of O^{LL} and O^{LR} we included the perturbative corrections by evaluating the matrix elements of the combination of operators given in Eqs. (15) and (18).

$$t^A \text{ are normalized to } \text{Tr}(t^A t^B) = \frac{1}{2} \delta_{AB}.$$

In Fig. 4 we display the results for $r^2 Z^*(r)$. We note that apart from this term, which reflects the explicit chiral symmetry breaking, the eigenvectors of Z_{ij} coincide with those of the anomalous dimension matrix γ_{ij} .

An explicit evaluation in our case ($r = 1$) of the matrix elements of Z gives:

$$\begin{aligned} Z_+ &= -47.6 & Z_1 &= -47.3 \\ Z_- &= -45.9 & Z_2 &= -44.8 \\ Z^* &= 9.59 & & \end{aligned} \quad (21)$$

These numbers, as well as the results in Fig. 4 have been computed using for the continuum case the dimensional reduction $\overline{MS}$, defined in Ref. /9/.

Since for $\beta_w = 6$ we have $g^2 = 1$ the diagonal corrections are in all cases $\sim 30\%$. The non diagonal corrections are potentially very serious for the case of O^{LL} when $K \rightarrow K_C$. In fact the matrix elements of $(O^{LL})_{CONT}$ is expected to decrease proportionally to m^2 , while the elements of wrong chirality operators O^{STP} , O^{VA} , O^{SP} are expected to have a small dependence on m^2 .

Our computation indicates that at $\beta_w = 6$ these corrections are $\sim 10\%$ at $K = .150$ and $\gtrsim 30\%$ at $K = .155$. Non-diagonal corrections for O^{LR} were found, as expected, to be small ($\sim 10\%$) and almost independent of K ; the particular combinations of operators appearing in Eqs. (15-17) have however matrix elements much smaller than the single operators O^{STP} , O^{VA} or O^{SP} as we checked by an explicit Monte Carlo computation. This result is confirmed by evaluating the correction in the vacuum saturation approximation.

5. - Physics - a first evaluation -

In this section we compare the results of our analysis with results obtained by well-known approximations and with some experimental values. The values we will quote originate from an average using methods 1 and 2 at $K = 0.150$, including one loop corrections.

Although we quote only statistical errors we are aware of many possible sources of systematic error:

a) Following Ref. /7/ we assume $a^{-1} = 2$ GeV; a modification of this value will be reflected in our results as explicitly shown in the following.

b) At $K = 0.150$ the "pion" mass is 1.2 GeV.

A correct evaluation of physical results would require an extrapolation to the physical value of K . We do not consider the presently available data at $K = .155$ to be reliable enough for such an extrapolation.

Lacking a correct extrapolation to the physical values of the quark masses, we base our results on the assumption that the mass dependence of different operators is the one suggested by simple chiral symmetry arguments:

$$\frac{1}{m^2} \frac{dm_{LL}}{d\lambda} \approx \text{constant}$$

$$\frac{dm_{LR}}{d\lambda} \approx \text{constant}$$

c) We have not evaluated the possible effects of neglecting the contribution of excited states.

Given these uncertainties, we cannot expect more than a rough agreement with other determinations in order to obtain a

first check of our methods. In this respect our results are very encouraging.

In Table 2 we report the matrix elements between a K^+ and a π^+ of four different operators:

$$\begin{aligned} & (\bar{u} \gamma_L^\mu u) (\bar{s} \gamma_\mu^L d) \\ & (\bar{s} \gamma_L^\mu u) (\bar{u} \gamma_\mu^L d) \\ & (\bar{s} \gamma_L^\mu d) (\bar{u} \gamma_\mu^R u) \\ & (\bar{s} t^A \gamma_L^\mu d) (\bar{u} t^A \gamma_\mu^R u) \end{aligned}$$

Suitable combinations of these matrix elements give the $K - \pi$ matrix elements of the $\Delta T = 1/2$ and $\Delta T = 3/2$ operators

$$O_1 = (\bar{s} \gamma_L^\mu d) (\bar{u} \gamma_\mu^L u) - (\bar{s} \gamma_L^\mu u) (\bar{u} \gamma_\mu^L d) \quad \Delta T = 1/2$$

$$O_4 = (\bar{s} \gamma_L^\mu d) (\bar{u} \gamma_\mu^L u) + (\bar{s} \gamma_L^\mu u) (\bar{u} \gamma_\mu^L d) - (\bar{s} \gamma_L^\mu d) (\bar{d} \gamma_\mu^L d) \quad \Delta T = \frac{3}{2}$$

$$O_5 = (\bar{s} t^A \gamma_L^\mu d) [(\bar{u} t^A \gamma_\mu^R u) + (\bar{d} t^A \gamma_\mu^R d) + (\bar{s} t^A \gamma_\mu^R s)] \quad \Delta T = 1/2$$

$$O_6 = (\bar{s} \gamma_L^\mu d) [(\bar{u} \gamma_\mu^R u) + (\bar{d} \gamma_\mu^R d) + (\bar{s} \gamma_\mu^R s)] \quad \Delta T = \pi/2$$

The notation for these operators is that used in Ref. /3/.

Our results are:

$$\frac{\langle \pi^+ | (\bar{u} \gamma_L^\mu u) (\bar{s} \gamma_\mu^L d) | K^+ \rangle}{m^2} = (4a^2) (3.7 \pm 0.3) 10^{-3} \quad [2.4 \cdot 10^{-3}]$$

$$\frac{\langle \pi^+ | (\bar{s} \gamma_L^\mu u) (\bar{u} \gamma_\mu^L d) | K^+ \rangle}{m^2} = (4a^2) (8.8 \pm 0.7) 10^{-3} \quad [7.2 \cdot 10^{-3}]$$

$$\langle \pi^+ | (\bar{s} \gamma_L^\mu d) (\bar{u} \gamma_\mu^R u) | K^+ \rangle = (16a^4) (-4.2 \pm 0.9) 10^{-2} \quad [-4.0 \cdot 10^{-2}]$$

$$\langle \pi^+ | (\bar{s} t^A \gamma_L^\mu d) (\bar{u} t^A \gamma_\mu^R u) | K^+ \rangle = (16a^4) (-6.8 \pm 0.8) 10^{-2} \quad [-21.5 \cdot 10^{-2}]$$

where the values in brackets are those given in Ref. /10/ on the basis of vacuum saturation and - in the case of O_5 and O_6 - of an estimate of the contribution of intermediate scalar meson states (σ).

There is a general agreement between our results and those of Ref. /10/. In particular we seem to confirm the enhancement of "penguin" operators O_5 and O_6 with respect to the O_1 and O_4 , which the authors of Ref./10/ proposes on the basis of the $\Delta T = 1/2$ rule in K decays. On the contrary we do not reproduce the large ratio $O_5 / O_6 = 16/3$.

As a second test we have evaluated the $K_0 - \bar{K}_0$ matrix element:

$$\langle K^0 | (\bar{s} \gamma^\mu (1 - \gamma_5) d) (\bar{s} \gamma_\mu (1 - \gamma_5) d) | \bar{K}^0 \rangle = (4a^2) (10 \pm 1) 10^{-2}$$

$$[\sim 7.7 \cdot 10^{-2}]$$

This result is in good agreement in sign and magnitude with the vacuum insertion value of Ref. /10/, but about 3 times larger than the estimate of Ref. /11/.

The fact that our results are close to those obtained by a well-accepted approximation scheme is a clear indication of the validity of the method. We are confident that our method can lead to an accurate determination of matrix elements which are essential for the theoretical predictions of important physical quantities, such as $\Delta T = 1/2$ and $\Delta T = 3/2$ rates for K meson decays, and K - $\bar{K}$ mixing, including CP violation.

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K	Two term fit to G(t) $5 \leq t \leq 15$				Single term fit to G(t) $7 \leq t \leq 13$		Two term fit to $\tilde{G}(t)$ $5 \leq t \leq 15$	
	m	m*	Z $\times 10^4$	Z* $\times 10^7$	m	Z $\times 10^4$	m	(-1) $\times Z \times 10^4$
0.150	0.59 $\pm$ 0.02	1.57 $\pm$ 0.10	2.3 $\pm$ 0.2	5 $\pm$ 3	0.60 $\pm$ 0.02	2.3 $\pm$ 0.2	0.54 $\pm$ 0.04	1.1 $\pm$ 0.1
0.155	0.34 $\pm$ 0.02	-	34 $\pm$ 4	-	0.35 $\pm$ 0.02	34 $\pm$ 5		

- TABLE 1 -

K	$-\frac{1}{m^2} \frac{dm_{LL}^1}{d\lambda} \times 10^4$	$-\frac{1}{m^2} \frac{dm_{LL}^2}{d\lambda} \times 10^4$	$\frac{dm_{LR}^1}{d\lambda} \times 10^3$	$\frac{dm_{LR}^2}{d\lambda} \times 10^3$
0.150	9.4 $\pm$ 0.8 11.0 $\pm$ 1.0	22.0 $\pm$ 2.0 25.0 $\pm$ 3.0	-2.7 $\pm$ 0.6 -2.9 $\pm$ 0.4	-4.2 $\pm$ 0.8 -4.5 $\pm$ 0.4
0.155	15.0 $\pm$ 2.0	23.0 $\pm$ 7.0	-4.9 $\pm$ 0.5	-7.6 $\pm$ 1.3

- TABLE 2 -

- Table Captions -

Table 1: We report the values of m , m^* , Z and Z^* obtained from a two term fit to the pseudoscalar propagator at $K = 0.150$ and $K = 0.155$. The numbers are in dimensionless units i.e. $m = ma$, $Z = Za^4 m$ and Z from a single term fit to $G(t)$ is also given together with the results from a two term fit to $G(t)$.

Table 2: The values of $-\frac{1}{m^2} \frac{dm_{LL}}{d\lambda}$ and $-\frac{dm_{LR}}{d\lambda}$ are given:

- a) m_{LL}^1 refers to the matrix element of the operator $(\bar{u} \gamma_\mu^L u) (\bar{s} \gamma_\mu^R d)$ between K^+ and π^+
- b) m_{LL}^2 refers to $(\bar{s} \gamma_\mu^L u) (\bar{u} \gamma_\mu^R d)$ between the same states;
- c) m_{LR}^1 refers to $(\bar{s} \gamma_\mu^L d) (\bar{u} \gamma_\mu^R u)$;
- d) m_{LR}^2 refers to $(\bar{s} t^A \gamma_\mu^L d) (\bar{u} t^A \gamma_\mu^R u)$.

The upper and lower values in the boxes at $K = 0.150$ are obtained using the methods 1) and 2) described in Section 3.

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- Figure Captions -

Fig. 1a and 1b: Diagrams contributing to a Boson propagator after the insertion of a four fermion operator like those defined in Eqs. 1 of the text.

Fig. 2a and 2b: Diagrams as in Fig. 1. after a translation $y \rightarrow y-z$. All the quark propagators stem from a common origin where the four fermion operator has been fixed.

Fig. 3: Diagram contributing to a baryon propagator after the insertion of a four fermion operator.

Fig. 4: $r^2 Z^*(r)$ (Eqs. (15) and (18)) as a function of r is shown.

fig.1a

fig.1b

fig.2a

fig.2b

fig.3

fig. 4