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NON-PERTURBATIVE BREAKDOWN OF THE NON-RENORMALIZATION THEOREM
IN SUPERSYMMETRIC QCD

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A B S T R A C T

Reliable, small-size instanton effects and cluster decomposition imply that the non-renormalization theorem in supersymmetric QCD is broken non-perturbatively both in the massive and in the massless case. The resulting picture agrees with the one obtained in the effective Lagrangian approach and with Witten index considerations.

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1. The properties of the vacuum in supersymmetric (SUSY) gauge theories have been the object of much recent interest. SUSY perturbative vacua can be searched for easily by the general method of Ref. 1, i.e. by setting to zero the F and D terms occurring in the scalar field potential. For the case of SUSY QCD (SQCD), these scalars are the lowest components of the chiral matter superfields

$$q_{r,i}(x,\theta) = \phi_{r,i}(x) + \theta \psi_{r,i}(x) + \dots \quad (1a)$$

$$\tilde{q}_i^r(x,\theta) = \tilde{\phi}_i^r(x) + \theta \tilde{\psi}_i^r(x) + \dots \quad (1b)$$

$r = 1, \dots, N$ colour SU(N) index
 $i = 1, \dots, M$ flavour SU(M) index

which together with the gauge superfield

$$W^a(x,\theta) = \lambda^a(x) + \theta(F_{\mu\nu}^a(x) \sigma_{\mu\nu} + D(x)) + \dots; \quad a = 1, \dots, N^2 - 1 \quad (2)$$

and their complex (antichiral) conjugates $\bar{q}$, $\bar{\tilde{q}}$, $\bar{W}$ exhaust the fundamental fields of the theory. Adding to the standard gauge interaction terms the mass term

$$\mathcal{L}_m = - \sum_{i=1}^M \int d^2\theta \, m_i q_i \tilde{q}_i + \text{h.c.}, \quad (3)$$

it can be shown¹⁾ that for $m_i \neq 0$, the only SUSY vacuum (SUSY minimum of the scalar effective potential) is characterized by

$$\langle \phi_i \rangle = \langle \tilde{\phi}_i \rangle = 0. \quad (4)$$

If some $m_i = 0$, a class of degenerate vacua, labelled by one or more parameters, can be found, solving the equations

$$\sum_{i \neq j} (\langle \phi_i^{s*} \rangle (T^a)_s^z \langle \phi_{r,i} \rangle - \langle \tilde{\phi}_{s,i}^* \rangle (T^a)_s^z \langle \tilde{\phi}_i^z \rangle) = 0, \quad \langle \phi_j \rangle = 0 \quad (5)$$

$$m_i = 0, \quad m_j \neq 0.$$

What is typical of a SUSY theory is that the non-renormalization theorems²⁾ ensure that, to any order of perturbation theory, these expectation values still give the SUSY minima of the effective potential.

The question we wish to address here is whether non-perturbative phenomena can either shift the minimum of the potential away from $\langle \phi_i \rangle = \langle \tilde{\phi}_i \rangle = 0$ in the massive case or remove the degeneracy of the massless situation^{*)}. We find that the answer to both issues is affirmative.

In the massive case the minimum is shifted to a non-vanishing value of $\langle \phi_i \tilde{\phi}_i \rangle$ for each i . This implies spontaneous breaking of the axial part of the approximate ($m_i \ll \Lambda$) $SU(M) \times SU(M) \times U_V(1) \times \hat{U}_A(1)$ symmetry of SQCD which is preserved by the vacuum to each finite order in perturbation theory [Eq. (4)]. Here the $\hat{U}_A(1)$ factor is the unique anomaly-free combination of the two axial $U(1)$ classical symmetries.

In the massless case the combination of all the constraints existing for $\langle \phi_i \tilde{\phi}_i \rangle$ is found to be incompatible with the full perturbative degeneracy of the minima, as obtained from Eq. (5). Both results agree, however, with Witten's index considerations³⁾ and with the effective Lagrangian approach⁴⁻⁶⁾.

2. The way to arrive to the stated results is as follows. We assume the existence of SUSY vacua also at the non-perturbative level^{**)}. In such a vacuum we look at particular Green's functions consisting of vacuum expectation values (v.e.v.'s) of time-ordered products of lowest components of chiral superfields. Such Green's functions have the peculiarity of being x -independent, since

$$\begin{aligned} \bar{D}_{\alpha\dot{\alpha}}^{\mu} D_{\mu}^{x_i} G(x_1, \dots, x_m) &\equiv D_{\alpha\dot{\alpha}}^{x_i} T \langle \phi_1(x_1) \dots \phi_i(x_i) \dots \phi_m(x_m) \rangle = \\ &= T \langle \phi_1(x_1) \dots \{ \bar{Q}_{\dot{\alpha}}, \psi_{\alpha}(x_i) \} \dots \phi_m(x_m) \rangle = 0, \end{aligned} \quad (6)$$

if $\bar{Q}|0\rangle = 0$ and the ϕ_i commute at equal times (recall also that $[\bar{Q}, \phi_i] = 0$).

Green's functions of this type have recently been considered in the interesting series of papers quoted in Ref. 7. It was noticed there that the property (6) allows one to compute G at short distances ($|x_i - x_j| \ll \Lambda^{-1}$), where semiclassical instanton calculations are trustworthy. If a non-zero constant is found at short

*) Suggestions that this might be the case were made to one of us (G.V.) some time ago by M. Peskin.

**) If that assumption leads to inconsistencies, breaking of the non-renormalization theorems *a fortiori* follows.

distances, G will also now be zero at large $|x_i - x_j|$ separations because of Eq. (6).

In the case of Super-Yang-Mills (i.e. $M = 0$ in our notations) with an $SU(2)$ gauge group, it was shown⁷⁾ that ($\alpha = g^2/4\pi$):

$$G_{N=2}^{M=0}(x) \equiv T \left\langle \frac{\alpha}{8\pi} \lambda \lambda(x) \frac{\alpha}{8\pi} \lambda \lambda(0) \right\rangle = c \Lambda^6, \quad (7)$$

with c a non-vanishing finite constant.

As noticed in Ref. 7, eq. (7) can hardly be understood within a semiclassical approximation in view of the vanishing one instanton contribution to $\langle \lambda \lambda \rangle$. This last result is anyway forced by the $Z_{2N} = Z_4$ symmetry preserved by instantons.

The above situation is not so different from, say, two flavour QCD, where $\langle \bar{\psi}_{1R} \psi_{1L}(x) \bar{\psi}_{2R} \psi_{2L}(0) \rangle \xrightarrow{|x| \rightarrow \infty} \langle \bar{\psi}_{1R} \psi_{1L} \rangle \langle \bar{\psi}_{2R} \psi_{2L} \rangle$ which is non-zero in the Goldstone realization of chiral symmetry. Here a $Z_{2M} = Z_4$ symmetry prevents $\nu = 1$ instantons from contributing to $\langle \bar{\psi}_R \psi_L \rangle$ while the four-fermion amplitude is given by the $\nu = 1$ sector. Clearly, contributions with $\nu \neq 1$ are needed to have consistency with clustering.

With unbroken SUSY, eq. (7) can be similarly interpreted as $\langle \lambda \lambda \rangle \neq 0$, hence as spontaneous symmetry breaking of Z_4 to Z_2 . A possible mechanism yielding a non-vanishing $\langle \lambda \lambda \rangle$ has recently been proposed in Ref. 8, where non-perturbative field configurations other than instantons ("torons") have been considered.

One cannot, however, exclude that, in a SUSY vacuum, other unknown non-perturbative contributions wash out the constant (7) giving a vanishing $\langle \lambda \lambda \rangle$ with unbroken Z_4 . Finally, if there was no SUSY vacuum, there would be no reason to trust the constant instanton contribution.

3. In SQCD we shall follow a very similar reasoning. We compute certain Green's functions in the small-distance regime and, using SUSY, we boost the result to large distances. We then use cluster decomposition arguments as discussed above, and find that certain v.e.v.'s are non-zero. Combining this result with other known

properties of v.e.v.'s in SUSY vacua we arrive directly at the conclusions stated at the beginning.

Let us start with the case of an SU(2) gauge group for which SQCD is asymptotically free for $M < 6$ and consider the simplest generalization of Ref. 7, namely $M = 1$. In this case a good candidate for a single instanton effect is

$$G_2^1(x) \equiv G_{N=2}^{M=1}(x_1, x_2) = T \left\langle \frac{\alpha}{8\pi} \lambda \lambda(x_1) \phi \tilde{\phi}(x_2) \right\rangle \quad (8)$$

since it satisfies both the usual instanton-type selection rules and those coming from the (non-anomalous) $\hat{U}_A(1)$ symmetry⁵⁾.

Since $\lambda\lambda$ and $\phi\tilde{\phi}$ are lowest components of chiral superfields, then $G_2^1(x)$, as explained above, must actually be a constant (possibly zero). The instanton contribution to $G_2^1(x)$ can be computed following the techniques^{*)} used in the second paper of Ref. 7 for the extended ($n = 2$) SUSY case. With the appropriate modifications due to the appearance of matter fields also in the fundamental representation, we find at $m = 0$

$$G_2^1(x) = c \alpha^2 \left(\frac{2\pi}{\alpha} \right)^4 \mu^5 \exp \left(- \frac{2\pi}{\alpha(\mu)} \right) \times$$

$$\times \int d^4x_0 \frac{d\rho}{\rho^5} \rho^8 \frac{\rho^4 (x_1 - x_2)^2}{[(x_1 - x_0)^2 + \rho^2]^4 [(x_2 - x_0)^2 + \rho^2]^3} \equiv c_2^1 \Lambda^5, \quad (9)$$

where c is a known constant; Λ is the one-loop renormalization group invariant scale parameter; and c_2^1 contains some power dependence on α which should account for two-loop contributions to the β -function as well as for anomalous dimensions. At small but not zero m/Λ , we expect Eq. (9) to get corrections of order $O(m/\Lambda)$ while remaining a constant, as required by SUSY.

Two observations are in order here. The first is that the mass dimension of $G_2^1(x)$ is fully accounted for by the μ^5 factor in Eq. (9), where the exponent 5 is just the one-loop β -function coefficient, $b_1 = 3N - M = 5$. This is not a

*) We have, however, worked directly in components.

coincidence. In fact, for the lowest components of chiral superfields, the chiralities χ_i associated with the $U_A(1)$ axial current belonging to the super-current multiplet are precisely their naïve mass dimensions. Since the chirality selection rule for this current reads

$$(3N - M)\nu = \sum_i \chi_i, \quad (10)$$

Eq. (10) says that in the single-instanton case ($\nu = 1$) the total dimension,

$$d = \sum_i d_i = \sum_i \chi_i,$$

of the product of the field operators appearing in G_2^1 must be precisely $d = 3N - M = 5$.

We want to stress that Eq. (9) cannot be valid when higher components of chiral superfields are present or if chiral and antichiral components appear together because now $\chi_i \neq d_i$.

Our second observation is that the integration over the ρ collective coordinate in Eq. (9) is (unlike the QCD case) fully convergent with the most important contributions coming from instanton sizes $\rho \sim |x_1 - x_2|$. This gives us some confidence in the calculation and leads us to the crucial assumption of this approach, that the instanton contribution is not washed out by other unknown non-perturbative effects.

Following the general discussion given above, we conclude that Eq. (9) signals spontaneous breaking of the $\hat{U}_A(1)$ symmetry. In each one of the two degenerate vacua we must have, by clustering,

$$\left\langle \frac{\alpha}{8\pi} \lambda \lambda \right\rangle \left\langle \phi \tilde{\phi} \right\rangle = c_2^1 \wedge^5. \quad (11)$$

Equation (11) is not sufficient to give $\langle \phi \tilde{\phi} \rangle$ and, actually, it could suggest a whole set of possible values for such an order parameter. However, it has recently been shown in Ref. 9 that in a SUSY vacuum one must have, for each flavour, the

relation^{*)} ($i = 1, \dots, M$),

$$\langle m_i \phi_i \tilde{\phi}_i \rangle = \langle \frac{\alpha}{8\pi} \lambda \lambda \rangle, \quad m_i \neq 0 \quad (12a)$$

and

$$\langle \lambda \lambda \rangle = 0, \quad m_i = 0 \quad (\text{some } i). \quad (12b)$$

We thus see that for $m \neq 0$ we predict

$$m \langle \phi \tilde{\phi} \rangle^2 = c_2^1 \Lambda^5 \Rightarrow \langle \phi \tilde{\phi} \rangle = \sqrt{c_2^1} m^{-1/2} \Lambda^{5/2}, \quad (13)$$

in agreement with Ref. 5 and with more general Ward identity arguments⁶⁾, which could not exclude, however, the possibility that $c_2^1 = 0$. For $m = 0$, Eq. (12b) is clearly inconsistent with Eq. (11) for any finite value of $\langle \phi \tilde{\phi} \rangle$, strongly suggesting that the effective potential attains its minimum at infinite values of the scalar fields^{5,6)}.

Consider now the case $M = 2$. The relevant Green's function is now

$$G_{N=2}^{M=2}(x) = T \langle \phi_1 \tilde{\phi}_1(x) \phi_2 \tilde{\phi}_2(0) \rangle. \quad (14)$$

By a calculation similar to the one performed for G_2^1 , we find that G_2^2 equals $c_2^2 \Lambda^4$ with finite, non-zero c_2^2 . Since now by Eq. (12a)

$$\langle \phi_1 \tilde{\phi}_1 \rangle = \frac{1}{m_1} \langle \frac{\alpha}{8\pi} \lambda \lambda \rangle = \frac{m_2}{m_1} \langle \phi_2 \tilde{\phi}_2 \rangle, \quad (15)$$

we get

$$\langle \phi_1 \tilde{\phi}_1 \rangle = \sqrt{c_2^2} \sqrt{\frac{m_2}{m_1}} \Lambda^2, \quad \langle \phi_2 \tilde{\phi}_2 \rangle = \sqrt{c_2^2} \sqrt{\frac{m_1}{m_2}} \Lambda^2 \quad (16)$$

again in agreement with effective Lagrangian approaches⁵⁾.

*) This relation, valid in the effective Lagrangian approach⁵⁾, can also be derived from (anomalous) Ward identities (G. Shore, private communication). It is also implicitly contained in Ref. 10.

To extend these considerations to $SU(N)$, the object to be considered is, for $N \geq M$, the N -point function

$$G_N^M(x_1, \dots, x_{N-M}, y_1, \dots, y_M) =$$

$$= T \left\langle \frac{\alpha}{8\pi} \lambda \lambda(x_1) \dots \frac{\alpha}{8\pi} \lambda \lambda(x_{N-M}) \phi_1 \tilde{\phi}_1(y_1) \dots \phi_M \tilde{\phi}_M(y_M) \right\rangle. \quad (17)$$

The instanton measure that is adequate to this case reads, in our normalization,

$$[d\Sigma]_{\text{INST}} = d^4x_0 \frac{d\rho}{\rho^5} \mu^{3N-M} \left(\frac{2\pi}{\alpha}\right)^{2N} \rho^{4N} \quad (18)$$

up to a (for our purposes) irrelevant constant.

The expected generalization of Eq. (9) is $G_N^M = c_N^M \Lambda^{3N-M}$, implying a non-vanishing v.e.v. for $\lambda\lambda$ and $\phi_i \tilde{\phi}_i$. Using again Eq. (12a), we get, as in Ref. 5,

$$\langle \phi_i \tilde{\phi}_i \rangle = \Lambda^3 m_i^{-1} \prod_{j=1}^M \left(\frac{m_j}{\Lambda} \right)^{1/N} \quad (19)$$

$$\left\langle \frac{\alpha}{8\pi} \lambda \lambda \right\rangle = \Lambda^3 \prod_{j=1}^M \left(\frac{m_j}{\Lambda} \right)^{1/N} \quad (20)$$

The functions G_N^M are not the only ones that satisfy the various chirality selection rules. For instance, the $(N+M)$ -point function

$$\tilde{G}_N^M(x_1, \dots, x_N, x_{N+1}, \dots, x_{N+M}) =$$

$$= T \left\langle \frac{\alpha}{8\pi} \lambda \lambda(x_1) \dots \frac{\alpha}{8\pi} \lambda \lambda(x_N) \psi_1 \tilde{\psi}_1(x_{N+1}) \dots \psi_M \tilde{\psi}_M(x_{N+M}) \right\rangle \quad (21)$$

should also receive a non-zero contribution from the one-instanton sector. This can be checked by an explicit calculation (reliable for small space-time separations) giving, for instance (at $m = 0$),

$$\tilde{G}_2^1(x_i - x_j) = c \left(\frac{\alpha}{8\pi} \right)^4 \left(\frac{2\pi}{\alpha} \right)^4 \mu^5 \exp \left(-\frac{2\pi}{\alpha(\mu)} \right) \int d^4x_0 \frac{d\rho}{\rho^5} \rho^8 x$$

$$\times \frac{\rho^6 (x_1 - x_2)^2}{[(x_1 - x_0)^2 + \rho^2]^4 [(x_2 - x_0)^2 + \rho^2]^4} \frac{\rho^2}{[(x_3 - x_0)^2 + \rho^2]^3} \sim \frac{\Lambda^5}{|x_i - x_j|^4} \quad (22)$$

Unlike the case of G_N^M , this time one cannot prove from general SUSY arguments that $\tilde{G}_N^M$ is independent of $|x_i - x_j|$ (because $\psi_i \tilde{\psi}_i$ is not the lowest component of a chiral supermultiplet), and indeed Eq. (22) shows that this does not seem to be the case. The behaviour indicated by Eq. (22) is consistent with a decreasing correlation at large separations, hinting at the usual statement that in a SUSY vacuum

$$\langle \psi_i \tilde{\psi}_i \rangle = 0 \quad \text{at } m_i = 0. \quad (23)$$

In the case $M > N$ the situation looks more complicated because one cannot find reasonable Green's functions made only of lowest components of chiral superfields and satisfying all the required selection rules. Configurations other than single instantons (like, perhaps, those of Ref. 8), seem to be needed to check whether the effective Lagrangian results, which change drastically at $M = N$, are reproduced. We plan to study this case in the near future.

We would like to comment at this point on the relation between our work and the recent paper of Ref. 11. These authors were able to study the behaviour of the scalar effective potential of the theory at large fields in the special case $N = M + 1$ and zero mass by working with the complicated "constrained instanton method", whilst, by standard instanton techniques, we can cover a larger range of possibilities and study the occurrence of SUSY minima at all values of scalar fields. The limitation $N = M + 1$ is due to the authors' (in our opinion unjustified) insistence on eliminating the $\lambda\lambda$ degree of freedom in terms of $\phi\tilde{\phi}$. This is only possible in their scheme when G_N^M involves a single $\lambda\lambda$ field, i.e. for $N = M + 1$.

In conclusion, the results of this paper show (modulo the assumption that the instanton contribution is dominant at short distances) that both in the massive and in the massless situation the properties of the SUSY minima are drastically different from those predicted in perturbation theory, thus leading us to the conclusion that the non-renormalization theorem has to break down non-perturbatively.

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REFERENCES

- 1) F. Buccella, J.P. Derendinger, S. Ferrara and C.A. Savoy, Phys. Lett. 115B (1982) 375.
- 2) J. Wess and B. Zumino, Phys. Lett. 49B (1974) 52.
J. Iliopoulos and B. Zumino, Nucl. Phys. B76 (1974) 310.
S. Ferrara, J. Iliopoulos and B. Zumino, Nucl. Phys. B77 (1977) 413.
M.T. Grisaru, M. Roček and W. Siegel, Nucl. Phys. B159 (1979) 429.
- 3) E. Witten, Nucl. Phys. B188 (1981) 513; *ibid.* B202 (1982) 253.
S. Cecotti and L. Girardello, Phys. Lett. 110B (1982) 39.
E. Cohen and C. Gomez, Harvard preprint HUPT-83/A007 (1983).
- 4) G. Veneziano and S. Yankielowicz, Phys. Lett. 113B (1982) 321.
- 5) T.R. Taylor, G. Veneziano and S. Yankielowicz, Nucl. Phys. B218 (1983) 493.
- 6) G. Veneziano, Phys. Lett. 124B (1983) 357.
- 7) V.A. Novikov, M.A. Shifman, A.I. Vainshtein, M.B. Voloshin and V.I. Zakharov, ITEP preprint 76 (1983).
V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, ITEP preprints 77 and 91 (1983).
- 8) E. Cohen and C. Gomez, Harvard preprint HUTP-83/A061 (1983).
- 9) K. Konishi, CERN preprint TH-3732 (1983).
- 10) T.E. Clark, O. Piguet and K. Siebold, Nucl. Phys. 159B (1979) 1.
- 11) I. Affleck, M. Dine and N. Seiberg, Supersymmetry breaking by instantons, Princeton preprint, July 1983.