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BOUNDS ON THE SQUARK/GLUINO MASS RATIO

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ABSTRACT

From the absence of radiative $SU(3)_C$ and $U(1)_{EM}$ breaking through squark VEVs we derive a lower bound on a particular combination of squark and Higgs masses in terms of the gluino mass. A more stringent lower bound on the squark to gluino mass ratio in the context of low energy supergravity is pointed out.

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In view of the present interest in supersymmetric theories and the corresponding large number of models proposed recently¹⁾ it would be interesting to gain some knowledge about the spectrum of the new particles in a rather model-independent way. If supersymmetry is supposed to keep the hierarchy $M_W/M_{GUT} \ll 1$ stable under radiative corrections, the mass splittings among the superpartners cannot be much larger than M_W . Hence we expect scalar partners of the known fermions, squarks and sleptons, to have masses of at most a few hundred GeV.

Clearly, if these scalar particles exist, their VEV has to vanish; in the case of squarks, e.g., it would spontaneously break the $SU(3)_C$ and $U(1)_{EM}$ gauge symmetries. After looking at the one loop effective potential²⁾ this condition leads to constraints on the parameters of the theory.

In the context of the standard model corresponding constraints on the Higgs mass have been derived by Linde and Weinberg³⁾ from the presence of a Higgs VEV.

In order to proceed in our case we have to constrain the class of models somehow; however, our results will turn out to be quite general. For simplicity, let us assume at the beginning the absence of new particles with a mass below $\sim 10^5$ GeV except for the superpartners of the known gauge bosons, fermions and two Higgs doublets. Actually, a wide class of theories has such a structure (minimal particle content and soft supersymmetry breaking) at low energies; apart from theories with soft supersymmetry breaking "by hand"⁴⁾, this class contains theories with spontaneous supersymmetry breaking in a "hidden" sector. "Hidden" means connected with our world only via a number of small Yukawa coupling constants⁵⁾, very heavy particles⁶⁾, or gravity^{1), 7)}.

Fortunately we have to make only modest assumptions about the parameters of a minimal supersymmetric low energy theory in order to obtain our results. Let us first specify the supersymmetric part of the interactions involving scalar fields of the theories under consideration. The superpotential f is given by

$$f = \sum_{ij} \ell_1 U_R^i (H Q_L^j) + \sum_{ij} \ell_2 D_R^i (H' Q_L^j) + \sum_{ij} \ell_3 E_R^i (H' L_L^j) + m' H H' \quad (1)$$

where summation over the generation indices i, j and $SU(2)$ indices is understood. ℓ_{ijk} denote dimensionless Yukawa coupling constants. The scalar potential resulting from the D-terms is given by

$$\begin{aligned}
 V_D = & \frac{1}{2} g_1^2 \left[\sum_i \left(\frac{1}{6} |Q_L^i|^2 - \frac{2}{3} |U_R^i|^2 + \frac{1}{3} |D_R^i|^2 - \frac{1}{2} |L_L^i|^2 + |E_R^i|^2 \right. \right. \\
 & \left. \left. + \frac{1}{2} |H|^2 - \frac{1}{2} |H'|^2 \right) \right]^2 \\
 & + \frac{1}{2} g_2^2 \left[\sum_i (Q_L^{i\dagger} \vec{\tau} Q_L^i + L_L^{i\dagger} \vec{\tau} L_L^i + H^\dagger \vec{\tau} H + H'^\dagger \vec{\tau} H') \right]^2 \\
 & + \frac{1}{2} g_3^2 \left[\sum_i (Q_L^{i\dagger} \vec{T} Q_L^i - D_R^{i\dagger} \vec{T} D_R^i - U_R^{i\dagger} \vec{T} U_R^i) \right]^2
 \end{aligned} \tag{2}$$

where g_1 , g_2 and g_3 denote the gauge coupling constants of $U(1)$, $SU(2)$ and $SU(3)$ respectively. In addition, we have the soft breaking terms:

$$V_{\text{soft}} = \sum_i m_i^2 |\phi_i|^2 + m_h^2 |H|^2 + m_{h'}^2 |H'|^2 + \hat{m}^2 (HH' + \text{h.c.}) + \text{trilin. terms} \tag{3}$$

where ϕ_i denote all squark and slepton fields.

Non-trivial VEVs will develop particularly easily in directions in field space in which the contribution of V_D vanishes. These are given by $\boxed{a}$ and b denote $SU(2)$ -indices with $a \neq b$:

$$\begin{aligned}
 \text{a)} \quad & H_a = H'_b \\
 \text{b)} \quad & Q_L^a = H_b = U_R \\
 \text{c)} \quad & Q_L^a = H'_b = D_R \\
 \text{d)} \quad & L_L^a = H_b \\
 \text{e)} \quad & L_L^a = H'_b = E_R
 \end{aligned} \tag{4}$$

In the directions a) and d) no positive quartic term in the fields exist, and appropriate choices of the soft supersymmetry breaking masses together with radiative corrections can easily generate VEVs in the desired direction a)^{1),8)}. In the directions b), c) and e), terms quartic in the fields exist from (1), but, at least for squarks and sleptons of the first generation, they are very small. Hence terms trilinear in the fields, which depend on the part of V_{soft} not specified in (3), can cause undesired VEVs already at tree level⁹⁾, since they can always be made negative by an appropriate choice of the phase of the fields. These trilinear terms in (3), however, are very model-dependent, and in order to be as general as possible we will neglect their destabilizing contribution to the potential in the following.

Instead we will concentrate on radiative contributions to the potential and their destabilizing effects. If we denote the VEVs collectively by ϕ , the one loop contribution to the potential reads after minimal subtraction:

$$\Delta V = \frac{1}{64\pi^2} \sum_j (-1)^{2j} (2j+1) M_j^4(\phi) \ln(M_j^2(\phi)/\mu^2) \quad (5)$$

It is convenient to choose μ^2 such that ΔV vanishes. For ϕ much larger than all masses in the theory, this corresponds to

$$\mu^2 = c \phi^2 \quad (6)$$

where c depends on the coupling constants of the theory; to one loop order we can put $c = 1$ selfconsistently. After this choice of the renormalization point the one loop effective potential is identical with the tree level one, except that now all parameters are running with a renormalization point as given by (6). This simple picture only becomes inconsistent, if ϕ approaches the order of magnitude of the masses of the theory and if at the same time the anomalous dimensions of some parameter become as large as the parameter itself¹⁰⁾.

In the case of the scalar potential under consideration, the most scale-dependent parameters are the squark masses, which evolve mainly due to the effect of the strong interactions⁸⁾:

$$\mu \frac{d m_{sq}^2}{d \mu} \simeq - \frac{8}{3\pi} \alpha_s m_g^2 \quad (7)$$

The contribution of SU(2) and U(1) gauge interactions as well as Yukawa couplings to the r.h.s. of (7) is less than a few percent for the first generation. m_g denotes the gluino mass; for $m_g = 0$ the gluinos would cancel exactly the gluon contribution to the effective potential, and no effect would arise.

Due to the negativity of the r.h.s. of (7), the effective potential in the directions b) and c) is the most likely one to become unstable due to radiative effects. In the case of direction b), however, part of the radiative corrections (the scale-dependence of m_h^2) depend on the unknown mass of the top quark via the corresponding Yukawa coupling constant. Hence we will investigate the effective potential in the direction c) in the following.

After denoting

$$\langle Q_L^1 \rangle = \langle H \rangle = \langle D_R^1 \rangle = \phi \quad (8)$$

we can write the effective potential as follows:

$$V(\phi^2) = (m_{Q_L}^2(\phi^2) + m_{Q_R}^2(\phi^2) + m^2(\phi^2))\phi^2 + 2\ell_{11}^2(\phi^2)\phi^4 \quad (9)$$

where we have introduced

$$m^2 = m_h^2 + m'^2 \quad (10)$$

We estimate ℓ_{11}^2 as follows: we assume that the diagonal entry of the down quark mass matrix for the first generation before diagonalization is not more than 20 MeV. Hence we have

$$\ell_{11}^2 \lesssim \frac{20 \text{ MeV}}{\langle H' \rangle} \quad (11)$$

From the known value of M_W we obtain

$$\langle H' \rangle = (175 \text{ GeV}) \sin \theta \quad (12)$$

where θ is defined by⁸⁾

$$\tan \theta = \langle H \rangle / \langle H' \rangle, \quad \sin 2\theta = 2m^2 / (m_h^2 + m_h'^2 + 2m'^2) \quad (13)$$

Hence we find

$$\ell_{11}^2 \lesssim 1.3 \cdot 10^{-8} \sin^{-2} \theta \quad (14)$$

Due to its smallness the scale dependence of ℓ_{11}^2 turns out to be negligible. The most important part of the scale dependence of m_{QL}^2 and m_{DR}^2 can be obtained from (7) together with the renormalization group equations for α_s and m_g ⁸⁾:

$$\mu \frac{d\alpha_s}{d\mu} = -\frac{3}{2\pi} \alpha_s^2 \quad (15)$$

$$\mu \frac{dm_g}{d\mu} = -\frac{3}{2\pi} \alpha_s m_g$$

In the class of models discussed here, the scale dependence of m^2 [see (10)] turns out to be negative, if the following conditions are met [see (8)]: the trilinear coupling constants from (3) as well as the b quark Yukawa coupling constant times the squark masses have to be smaller than the $SU(2)$ gaugino masses. Unless these conditions are violated by some orders of magnitude, the scale dependence of m^2 will be negligibly small, hence we will consider m^2 (as well as ℓ_{11}^2) to be approximately constant in the following.

Now it is straightforward to calculate $V(\phi^2)$ as given by (9); starting with $\alpha_s(\mu^2 \cong M_V^2) \cong .12$ we find the following condition for the absence of a

non-trivial minimum of $V(\phi^2)$, which depends slightly on θ :

$$m_{Q_L}^2 + m_{D_R}^2 + m^2 > m_g^2 \cdot \begin{matrix} .85 & \text{for } \sin^{-2}\theta = 1 \\ .77 & \text{for } \sin^{-2}\theta = 10 \\ .67 & \text{for } \sin^{-2}\theta = 100 \end{matrix} \quad (16)$$

The masses in (16) are defined at $\mu^2 \cong m_g^2$. If the bound (16) is violated by more than a few percent, ϕ develops a VEV of the order of $11_2^{\ell-1} \times m_{sq}$ and the order of magnitude of $V(\phi^2)$ at its minimum is $\sim 11_2^{\ell-2} \times m_{sq}^4$. Hence this minimum will be much deeper than the one in which only H and H' develop VEVs in order to break $SU(2) \times U(1)$, which will involve mass scales of $O(m)$ only.

However, although such a non-trivial minimum of $V(\phi^2)$ might exist, the tunnelling rate into it could be small compared to the age of the universe.^{*)} [Temperature effects in the early universe disfavour VEVs in the direction of field space considered here compared with $SU(2) \times U(1)$ breaking ones.] In order to calculate the corresponding tunnelling rate¹²⁾, we have neglected the difference between the $SU(2) \times U(1)$ breaking minimum of V and the origin in field space; furthermore we have used the thin wall approximation¹²⁾. Instead of (16) we obtain after requiring the tunnelling rate to be negligible compared with the age of the universe:

$$m_{Q_L}^2 + m_{D_R}^2 + m^2 > m_g^2 \cdot \begin{matrix} .78 & \text{for } \sin^{-2}\theta = 1 \\ .70 & \text{for } \sin^{-2}\theta = 10 \\ .62 & \text{for } \sin^{-2}\theta = 100 \end{matrix} \quad (17)$$

Although the masses on the l.h.s. of (16) and (17) are defined at low scales, they are not yet the physical ones due to contributions of the D-terms after $SU(2) \times U(1)$ breaking through VEVs of H and H' . However, with the help of the results of Ref. (8), the following relation can be established

$$(2Q_L \equiv D_L): \quad m_{D_R}^2 + m_{Q_L}^2 + m^2 = m_{D_R}^2 + m_{D_L}^2 + m_2^2 \cos^2\theta \Big|_{\text{Phys}} \quad (18)$$

Here m_2 is the physical mass of the second of the three neutral Higgs scalars in this scenario, i.e. the one whose mass is in between the masses of the other two. Hence the bounds (16) and (17) are valid for physical masses after replacing m^2 by m_2^2 and $m_{Q_L}^2$ by m_{DL}^2 .

*) In the case of a tree-level instability of $V^9)$, this has been discussed by Claudson, Hall and Hinchliffell).

Clearly, non-trivial minima of $V(\phi^2)$ as discussed here are due to the fact that the running squark masses squared can become negative at high scales because of (7). In the context of low energy supergravity⁷⁾, however, one expects positive scalar masses squared even at the Planck scale. Hence, in these scenarios it is possible to arrive at more stringent bounds. Let us, e.g., assume a minimal particle content up to 10^{16} GeV, which is the GUT scale under this assumption. Then it is straightforward to complete the r.h.s. of (7) by the contributions of all three gauge interactions (again, for the first generation the contribution of Yukawa coupling constants is negligible), to solve the renormalization group equations for all three gauge coupling constants and gaugino masses (which are related at 10^{16} GeV) and hence to obtain the running squark masses up to 10^{16} GeV. Requiring positive squark masses squared at 10^{16} GeV and starting with $\alpha_s = .12$, $\alpha_{\text{QED}} = 1/128$ and $\sin^2 \theta_W = .23$ at M_W we arrive at the following condition:

$$\frac{1}{3} (m_{u_R}^2 + 2 m_{d_R}^2) > .8 m_g^2 \quad (19)$$

Again the squark and gluino masses are defined at M_W , and we choose a particular combination of squark masses, which is not affected by contributions from D-terms after $SU(2) \times U(1)$ breaking. Note that in some supergravity scenarios we could have $m_{\text{sq}}^2 \ll m_g^2$ at the Planck scale¹³⁾.

Let us finally discuss the model dependence of our results. The essential ingredients for obtaining (16) and (17) have been the tree level potential (1) - (3) in the particular direction c) and the renormalization group equations (7) and (15). Hence modifications of the superpotential which do not affect the tree level potential for $\langle Q_L \rangle = \langle D_K \rangle = \langle H' \rangle$ and vanishing VEVs for all other fields hardly influence our results. In particular this concerns models with singlet fields coupled to H and H' ; in fact we have not been able to find any model, where for vanishing VEVs of all fields except for the three mentioned above, additional contributions to the potential arise.

The effect of additional matter transforming under $SU(3)_c$ would be to make the coefficients of the r.h.s. of (15) less negative; consequently α_s and m_g would decrease more slowly, and, from (7), m_{sq}^2 would decrease more rapidly with increasing mass scales. Hence our bounds would even be enforced. In particular this concerns our result (19) in the context of low energy supergravity.

The only reason for an invalidation of our bounds could be the appearance of large Yukawa coupling constants or large trilinear couplings in terms of the

tree potential involving Q_L^1 , D_R^1 or H' , since these couplings would lead to a non-negligible positive contribution to the evolution equations of the corresponding masses.

On the other hand, trilinear terms in the tree level potential have a destabilizing effect, and a consideration of these terms would again enforce our bounds. Hence we conclude that a violation of the bound (17) would require drastic modifications of standard low energy supersymmetric theories, whereas a violation of the bound (19) would correspond to negative squark masses squared at the GUT scale and hence probably rule out $N = 1$ low energy supergravity.

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