Ref.TH.3759-CERN

FINITE TEMPERATURE GREEN FUNCTIONS BY STOCHASTIC QUANTIZATION

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A B S T R A C T

The stochastic quantization method of Parisi and Wu is extended to include thermal effects. The proof of equivalence between stochastic and usual quantization at $T \neq 0$ and some examples of one-loop calculations for gauge theories are discussed.

1. INTRODUCTION

The stochastic quantization method of Parisi and Wu¹⁾ has mainly been introduced for gauge theories to avoid gauge fixing, and therefore the Gribov ambiguity²⁾. Though there are no Faddeev-Popov ghosts in this formulation, it has been shown in second order that their contributions show up in the calculation of Green functions coming from longitudinal terms of the stochastic version of the gauge field propagator.³⁾ There exist proofs of equivalence with the usual method for gauge theories^{4), 5)}, but they rely on a modification of the original scheme. For scalar theories the situation is much simpler and there are different methods to establish equivalence⁵⁾⁻⁷⁾.

The aim of the present paper is to introduce thermal effects in stochastic quantization. In order to obtain finite temperature T Green functions, we modify the Langevin equation by considering Gaussian random variables which are periodic (or antiperiodic for fermions) in the Euclidean time, with period length $\beta = 1/T$. We prove that the free field Green functions obtained in this way are just the finite temperature Green functions in the so-called imaginary time formalism⁸⁾. For interacting fields, we show that the proofs of equivalence between stochastic and canonical quantization are valid also at $T \neq 0$. We discuss some explicit examples of one-loop calculations for the electric mass of gauge bosons based on the Langevin equation with periodic random variables.

The plan of the paper is as follows: in section 2 the basic formalism and the free field case are discussed. In section 3 we consider a self-interacting scalar field theory and section 4 contains a discussion of interacting gauge field theories.

2. STOCHASTIC QUANTIZATION AT $T \neq 0$: FREE FIELD THEORIES

To begin with we recall the basic features of stochastic quantization. Let $S[\phi]$ be the action for a Euclidean field theory in D dimensions^{*)} (we assume ϕ to be a scalar field for simplicity). The stochastic quantization of Parisi and Wu consists in introducing a fictitious "time" t ; the evolution of the fields in the extra time is assumed to be governed by the Langevin equation

$$\frac{\partial \phi(x, t)}{\partial t} = - \frac{\delta S[\phi]}{\delta \phi(x, t)} + \eta(x, t) \quad (2.1)$$

*) In the examples of section 4 we will put $D = 4$.

where $\eta(x, t)$ is a Gaussian random variable and $x = (x_0, x_1, \dots, x_{D-1})$. Let $\Phi(x, t)$ be a solution of the Langevin equation; then one can construct correlation functions of the fields

$$\langle \phi(x_1, t_1) \dots \phi(x_L, t_L) \rangle_\eta = \frac{\int [d\eta] \phi(x_1, t_1) \dots \phi(x_L, t_L) \exp\left(-\frac{1}{4} \int d^D x dt \eta^2(x, t)\right)}{\int [d\eta] \exp\left(-\frac{1}{4} \int d^D x dt \eta^2(x, t)\right)} \quad (2.2)$$

and the equivalence between stochastic and canonical quantization is proved if (2.2) converges to the usual Green function in Euclidean space in the limit $t_1 = \dots = t_L \rightarrow \infty$.

Let us now suppose that the Gaussian noise is periodic^{*)} in the x_0 -variable with period $\beta > 0$:

$$\eta(x_0, \vec{x}, t) = \eta(x_0 + \beta, \vec{x}, t) \quad (2.3)$$

where $\vec{x} = (x_1, \dots, x_{D-1})$. Then it is clear that also the solution of the Langevin equation is periodic in x_0 with period β , provided that the initial condition of (2.1) is periodic too: $\Phi(x_0, \vec{x}, 0) = \Phi(x_0 + \beta, \vec{x}, 0)$ ^{**)} . We define new correlation functions by modifying the η -averages as follows:

$$\langle A \rangle_\eta = \frac{\int [d\eta]_\beta A[\eta] \exp\left(-\frac{1}{4} \int_0^\beta dx_0 \int d^{D-1} \vec{x} dt \eta^2(x, t)\right)}{\int [d\eta]_\beta \exp\left(-\frac{1}{4} \int_0^\beta dx_0 \int d^{D-1} \vec{x} dt \eta^2(x, t)\right)} \quad (2.4)$$

where the subscript β means that we integrate over all the η 's which are periodic in x_0 with period β . Then the properties

$$\begin{aligned} \langle \eta(x, t) \eta(x', t') \rangle_\eta &= 2 \delta(x - x') \delta(t - t') \\ \langle \eta(x_1, t_1) \dots \eta(x_{2n+1}, t_{2n+1}) \rangle_\eta &= 0 \\ \langle \eta(x_1, t_1) \dots \eta(x_{2n}, t_{2n}) \rangle_\eta &= \sum_{\substack{\text{possible} \\ \text{pair comb.}}} \prod \langle \eta(x_i, t_i) \eta(x_j, t_j) \rangle_\eta \end{aligned} \quad (2.5)$$

which hold for the $\beta = \infty$ case are obviously still valid.

To see the effect of our modifications we consider first a free scalar field theory described by

$$S[\phi] = \int d^D x \left\{ \frac{1}{2} (\partial \phi)^2 + \frac{m^2}{2} \phi^2 \right\} \quad (2.6)$$

*) For Bose fields; for Fermi fields antiperiodicity must be imposed.

**) However, the large time limits are independent of the choice of initial value.

The Langevin equation is given by

$$\frac{\partial \phi(x, t)}{\partial t} = (\partial^2 - m^2) \phi(x, t) + \eta(x, t) \quad (2.7)$$

where $\eta(x, t)$ obeys (2.3).

We introduce the retarded Green function

$$G(x, t) = \theta(t) \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^{D-1}k}{(2\pi)^{D-1}} e^{ik_n \cdot x} e^{-t(k_n^2 + m^2)} \quad (2.8)$$

where

$$k_n = (\omega_n^+, \vec{k}) \quad (2.9a)$$

$$\omega_n^+ = \frac{2\pi n}{\beta} \quad (2.9b)$$

The solution of (2.7) together with the initial condition $\phi(x, 0) = 0$ is given by

$$\phi(x, t) = \int_0^{\infty} d\tau \int_0^{\beta} dy_0 \int d^{D-1}y G(x-y, t-\tau) \eta(y, \tau) \quad (2.10)$$

as can be immediately checked.

One can now evaluate the correlation function

$$D(x-x'; t, t') = \langle \phi(x, t) \phi(x', t') \rangle_{\eta} \quad (2.11)$$

and one obtains after some algebra

$$D(y; t, t') = \frac{1}{\beta} \sum_n \int \frac{d^{D-1}k}{(2\pi)^{D-1}} D(\omega_n^+, \vec{k}; t, t') e^{ik_n \cdot y} \quad (2.12)$$

where

$$D(\omega_n^+, \vec{k}; t, t') = \frac{1}{k_n^2 + m^2} \left\{ \exp(-(k_n^2 + m^2)|t-t'|) - \exp(-(k_n^2 + m^2)|t+t'|) \right\} \quad (2.13)$$

Now it is clear that for $t = t' \rightarrow \infty$

$$\lim_{t \rightarrow \infty} D(y; t, t) = \frac{1}{\beta} \sum_n \int \frac{d^{D-1}k}{(2\pi)^{D-1}} e^{ik_n \cdot y} \frac{1}{k_n^2 + m^2} \quad (2.14)$$

Eq. (2.14) shows that the equal time correlation function (2.11) tends to the finite temperature Green function for the free scalar field in the imaginary time formalism⁸⁾.

Then we turn to a free gauge field theory described by the action

$$S = \frac{1}{4} \int d^D x F_{\mu\nu}^a F_{\mu\nu}^a \quad (2.15)$$

where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a$ ($a = 1, \dots, N$). The fields A_μ^a satisfy the Langevin equation

$$\frac{\partial A_\mu^a(x, t)}{\partial t} = (\partial^2 \delta_{\mu\nu} - \partial_\mu \partial_\nu) A_\nu^a(x, t) + \eta_\mu^a(x, t) \quad (2.16)$$

where $\eta_\mu^a(x, t)$ is a Gaussian noise

$$\begin{aligned} \langle \eta_\mu^a \rangle_\eta &= 0 \\ \langle \eta_\mu^a(x, t) \eta_\nu^b(x', t') \rangle_\eta &= 2 \delta_{\mu\nu} \delta^{ab} \delta(x-x') \delta(t-t') \end{aligned} \quad (2.17)$$

and periodic in the x_0 -variable with period β . Following the same reasoning as before, one obtains

$$A_\mu^a(x, t) = \int_0^\infty d\tau \int_0^\beta dy_0 \int d^{D-1} y G_{\mu\nu}^{ab}(x-y, t-\tau) \eta_\nu^b(y, \tau) \quad (2.18)$$

where

$$G_{\mu\nu}^{ab}(y, \tau) = \frac{1}{\beta} \sum_n \int \frac{d^{D-1} k}{(2\pi)^{D-1}} e^{ik_n \cdot y} G_{\mu\nu}^{ab}(k_n, t) \quad (2.19)$$

and

$$G_{\mu\nu}^{ab}(k, t) = \theta(t) \delta^{ab} \left\{ (\delta_{\mu\nu} - k_\mu k_\nu / k^2) e^{-k^2 t} + k_\mu k_\nu / k^2 \right\}. \quad (2.20)$$

Constructing the correlation function of two gauge fields

$$D_{\mu\nu}^{ab}(x-x'; t, t') = \langle A_\mu^a(x, t) A_\nu^b(x', t') \rangle_\eta \quad (2.21)$$

one obtains

$$D_{\mu\nu}^{ab}(y, t, t') = \frac{1}{\beta} \sum_n \int \frac{d^{D-1} k}{(2\pi)^{D-1}} e^{ik_n \cdot y} D_{\mu\nu}^{ab}(k_n, t, t') \quad (2.22)$$

$$D_{\mu\nu}^{ab}(k, t, t') = \delta^{ab} \left\{ (\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \frac{1}{k^2} (e^{-k^2 t - t'} - e^{-k^2(t+t')}) + 2 \frac{k_\mu k_\nu}{k^2} \min(t, t') \right\}. \quad (2.23)$$

In the limit $t = t' \rightarrow \infty$, $D_{\mu\nu}^{ab}(k; t, t')$ tends to the finite temperature vector boson Green function in the imaginary time formalism in the Landau gauge, plus a longitudinal term proportional to t

$$\delta^{ab} \left\{ \frac{1}{k^2} (\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) + 2t \frac{k_\mu k_\nu}{k^2} \right\} \quad (2.24)$$

Formulae (2.22) ÷ (2.24) are the $T \neq 0$ version of the analogue results obtained by Parisi and Wu at zero temperature.^{*})

Finally we investigate the free fermion case. The Euclidean action is

$$S = - \int d^D x \bar{\psi} (i\partial - m) \psi \quad (2.25)$$

where the γ_μ matrices satisfy

$$\{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu} \quad (2.26)$$

The straightforward approach to stochastic quantization fails for fermion fields because the operator $(i\partial - m)$ has both positive and negative eigenvalues. This leads one to consider the following modified Langevin equation⁹⁾:

$$\begin{aligned} \frac{\partial \psi(x,t)}{\partial t} &= - (i\partial + m) \frac{\delta S}{\delta \bar{\psi}(x,t)} + \eta_\psi(x,t) = \\ &= (\partial^2 - m^2) \psi(x,t) + \eta_\psi(x,t) \end{aligned} \quad (2.27)$$

We can consider the equation for $\bar{\psi}$ too:

$$\frac{\partial \bar{\psi}(x,t)}{\partial t} = (\partial^2 - m^2) \bar{\psi}(x,t) + \bar{\eta}_\psi(x,t) \quad (2.28)$$

and the anticommuting random variables $\eta, \bar{\eta}$ are assumed to satisfy

$$\begin{aligned} \langle \eta_\alpha(x,t) \bar{\eta}_\beta(x',t') \rangle_\eta &= 2 [-i\partial_x + m]_{\alpha\beta} \delta(x-x') \delta(t-t') \\ \langle \eta \eta \rangle_\eta &= \langle \bar{\eta} \bar{\eta} \rangle_\eta = 0 \\ \langle \eta \rangle_\eta &= \langle \bar{\eta} \rangle_\eta = 0. \end{aligned} \quad (2.29)$$

Moreover, we assume that $\eta_\psi, \bar{\eta}_\psi$ are antiperiodic in x_0 , in order to introduce finite temperature effects:

$$\eta_\psi(x_0, \vec{x}, t) = - \eta_\psi(x_0 + \beta, \vec{x}, t) \quad (2.30)$$

and a similar equation for $\bar{\eta}_\psi$.

One can easily check that the solution of (2.27) is

$$\psi(x,t) = \int_0^\infty d\tau \int_0^\beta d\gamma_0 \int d^{D-1} \gamma G(x-\gamma, t-\tau) \eta_\psi(\gamma, \tau) \quad (2.31)$$

^{*}) Of course Eq. (2.24) is formally divergent with $t \rightarrow \infty$; this is due to the fact that (2.21) is not a gauge invariant quantity.

where $G(x, t)$ is the retarded Green function defined in Eq. (2.8) with

$$\begin{aligned} k_n &= (\omega_n^-, \vec{k}) \\ \omega_n^- &= (2n+1)\pi/\beta. \end{aligned} \quad (2.32)$$

If we now construct the correlation function of the two fermion fields $\psi, \bar{\psi}$:

$$S_{\alpha\beta}(x-x'; t, t') = \langle \psi_\alpha(x, t) \bar{\psi}_\beta(x', t') \rangle_\eta \quad (2.33)$$

we obtain

$$S_{\alpha\beta}(y; t, t') = \frac{1}{\beta} \sum_n \int \frac{d^{D-1}k}{(2\pi)^{D-1}} S_{\alpha\beta}(k_n'; t, t') e^{ik_n' \cdot y} \quad (2.34)$$

with $k_n' = (\omega_n^-, \vec{k})$ and

$$S_{\alpha\beta}(k; t, t') = \frac{(\not{k} + m)_{\alpha\beta}}{k^2 + m^2} \left\{ \exp(-(k^2 + m^2)|t - t'|) - \exp(-(k^2 + m^2)(t + t')) \right\}. \quad (2.35)$$

It is now clear that for $t = t' \rightarrow \infty$, we get

$$\lim_{t \rightarrow \infty} S_{\alpha\beta}(y; t, t) = \frac{1}{\beta} \sum_n \int \frac{d^{D-1}k}{(2\pi)^{D-1}} e^{ik_n' \cdot y} \frac{(\not{k}_n' + m)_{\alpha\beta}}{k_n'^2 + m^2} \quad (2.36)$$

which is the finite temperature free fermion propagator in the imaginary time formalism⁸⁾.

3. SELF-INTERACTING SCALAR FIELD THEORY

As a first example of interacting fields, we now consider the action of a self-coupled scalar field

$$S[\phi] = \int d^D x \left\{ \frac{1}{2} (\partial\phi)^2 + \frac{m^2}{2} \phi^2 + \frac{1}{3!} g \phi^3 + \frac{1}{4!} \lambda \phi^4 \right\} \quad (3.1)$$

and obtain the Langevin equation

$$\frac{\partial \phi(x, t)}{\partial t} = (\partial^2 - m^2) \phi(x, t) - \frac{1}{2} g \phi^2(x, t) - \frac{1}{6} \lambda \phi^3(x, t) + \eta(x, t) \quad (3.2)$$

where η is periodic in x_0 with period length β . Defining the Fourier transform of a function f by

$$f(\omega_n, \vec{k}) = \int_0^\beta dx_0 e^{-i\omega_n x_0} \int d^{D-1}x e^{-i\vec{k} \cdot \vec{x}} f(x) \quad (3.3)$$

we can write down Eq. (3.2) in momentum space

$$\begin{aligned}
 \frac{\partial \phi(\omega_n^+, \vec{k}, t)}{\partial t} = & -(k_n^2 + m^2) \phi(\omega_n^+, \vec{k}, t) - \\
 & - \frac{1}{2} g \frac{1}{\beta} \sum_r \int \frac{d^{D-1} p}{(2\pi)^{D-1}} \phi(\omega_r^+, \vec{p}, t) \phi(\omega_{n-r}^+, \vec{k} - \vec{p}, t) - \\
 & - \frac{1}{6} \lambda \frac{1}{\beta^2} \sum_{r,s} \int \frac{d^{D-1} p}{(2\pi)^{D-1}} \int \frac{d^{D-1} q}{(2\pi)^{D-1}} \phi(\omega_r^+, \vec{p}, t) \phi(\omega_s^+, \vec{q}, t) \phi(\omega_{n-r-s}^+, \vec{k} - \vec{p} - \vec{q}, t) + \\
 & + \eta(\omega_n^+, \vec{k}, t).
 \end{aligned} \tag{3.4}$$

The correlation function between the η 's is now given by

$$\langle \eta(\omega_r, \vec{p}, t) \eta(\omega_s, \vec{q}, t') \rangle_\eta = 2\beta (2\pi)^{D-1} \delta_{r+s,0} \delta(\vec{p} + \vec{q}) \delta(t-t') \tag{3.5}$$

and r, s are integer numbers. Now we see that we get the finite temperature case from the $T = 0$ case by the substitutions

$$\int \frac{dp_0}{2\pi} \rightarrow \frac{1}{\beta} \sum_n, \quad p \rightarrow p_n = \begin{pmatrix} \omega_n^+ \\ \vec{p} \end{pmatrix} \tag{3.6}$$

for all integrations and momenta in the Langevin equation, and

$$(2\pi) \delta(p_0 + q_0) \rightarrow \beta \delta_{r+s,0} \tag{3.7}$$

in the correlation function of the η 's. Therefore, the momentum and t structure of the theory are quite the same as in the case $T = 0$, and we can overtake the diagrammatic proof of equivalence between stochastic and canonical quantization from Ref. 6). In conclusion, we obtain that for scalar theories, the imaginary time formalism is obtained by stochastic quantization with random sources periodic in Euclidean time with period length $\beta = 1/T$.

4. GAUGE THEORIES

We shall now discuss thermal effects in the stochastic quantization of interacting gauge fields. Let us consider the Langevin equation for the gauge field A_μ^a

$$\frac{\partial A_\mu^a(x, t)}{\partial t} = - \frac{\delta S[A, \dots]}{\delta A_\mu^a(x, t)} + \eta_\mu^a(x, t) \tag{4.1}$$

where S is the Euclidean action containing both free and interaction parts (dots represent other fields). Now equivalence between stochastic and usual quantization is only expected if gauge invariant quantities are considered. This conjecture has been proved for self-interacting non-Abelian gauge fields obeying Eq. (4.1) at the one-loop level in Ref. 3). A general proof has been given in Ref. 4); it relies on the following modification of Eq. (4.1).

$$\frac{\partial A_\mu^a(x,t)}{\partial t} = - \frac{\delta S[A, \dots]}{\delta A_\mu^a(x,t)} - D_\mu^{ab} V^b + \eta_\mu^a(x,t) \quad (4.2a)$$

where

$$D_\mu^{ab} = \delta^{ab} \partial_\mu - g f^{abc} A_\mu^c(x,t) \quad (4.2b)$$

and $V^b = V^b[A, x]$ is an arbitrary (gauge non-invariant) functional of the fields A_μ^a . The insertion of the new piece $D_\mu^{ab} V^b$ in the Langevin equation does not change random averages of gauge invariant functionals $F[A]$, but it does allow us to calculate the limit $t \rightarrow \infty$ of $\langle F[A] \rangle_\eta$ where F is not gauge invariant, provided V^a breaks gauge invariance. For special V^b 's one obtains the usual description with gauge fixing and Fadeev-Popov ghosts⁴⁾ for $t \rightarrow \infty$.

Let us now choose a random variable $\eta_\mu^a(x, t)$ which is periodic in Euclidean time x_0 , a periodic functional V^b

$$V^b[A; x_0, \vec{x}] = V^b[A; x_0 + \beta, \vec{x}] \quad (4.3)$$

and periodic initial conditions for Eq. (4.2); then it is clear that its solution will be periodic too. From this, it follows that the proof of equivalence between stochastic and usual quantization of Refs. 4) and 5) can be overtaken with these modifications and A_μ^a naturally describes stochastic fields in a heat bath at temperature $T = 1/\beta$ ⁸⁾.

First it might be instructive, however, to see what happens if the original equation (4.1) is considered without modification. Even though the formal proof of equivalence seems to be difficult in this case, Eq. (4.1) could be useful for diagrammatic calculation because the corresponding Feynman rules look simpler. As an illustration, we shall consider a one-loop calculation in spinor QED. The action is given by

$$S = \int d^D x \left\{ \frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \bar{\Psi} (i \not{\partial} - m) \Psi - e \bar{\Psi} \gamma_\mu \Psi A_\mu \right\}. \quad (4.4)$$

The Langevin equations in momentum space are⁹⁾

$$\begin{aligned} \frac{\partial A_\mu(k,t)}{\partial t} &= (k_\mu k_\nu - \delta_{\mu\nu} k^2) A_\nu(k,t) + Y_\mu(k,t) \\ \frac{\partial \psi(k,t)}{\partial t} &= -(k^2 + m^2) \psi(k,t) + \Lambda(k,t) \\ \frac{\partial \bar{\psi}(k,t)}{\partial t} &= -(k^2 + m^2) \bar{\psi}(k,t) + \bar{\Lambda}(k,t) \end{aligned} \quad (4.5)$$

with

$$\begin{aligned} Y_\mu(k,t) &= \eta_\mu(k,t) + e \int \hat{d}q \bar{\psi}(q,t) \gamma_\mu \psi(k-q,t) \\ \Lambda(k,t) &= \eta_\psi(k,t) + e \int \hat{d}q (\not{k} + m) \gamma_\mu \psi(q,t) A_\mu(k-q,t) \end{aligned} \quad (4.6)$$

where $k = (\omega_n^\pm, \vec{k})$ (ω_n^+ for bosons, ω_n^- for fermions) and the correlation functions of the η 's are modified according to (3.5). The integral over q contains a discrete sum

$$\int \hat{d}q \equiv \frac{1}{\beta} \sum_n \int \frac{d^{D-1}q}{(2\pi)^{D-1}}. \quad (4.7)$$

We can now apply the set of Eqs. (4.5) to the calculation of the electric mass of the photon

$$m_{el}^2 = \Pi_{00}(k_0=0, \vec{k} \rightarrow \vec{0}) \quad (4.8)$$

which is different from zero at $T \neq 0$. Such a calculation is feasible by stochastic methods, because the vacuum polarization tensor $\pi_{\mu\nu}$ is a gauge invariant quantity in QED. We can solve Eqs. (4.5) by iteration as applied in Refs. 1), 3), 6) and 9). In the one-loop approximation, we have the set of diagrams a), b) of Fig. 1 which describe the correlation function

$$\langle A_\mu(x,t) A_\nu(x',t') \rangle_\eta \quad (4.9)$$

In the stochastic diagrams of Fig. 1, a crossed wavy (solid) line means a factor $D_{\mu\nu}(p; \tau, \tau')$ ($S_{\alpha\beta}(p; \tau, \tau')$), and a wavy (solid) line without crosses represents a factor $G_{\mu\nu}(p; \tau, \tau')$ ($G(p; \tau, \tau')$) (see Sect. 2 for their definitions). Loop integrations are performed according to (4.7) and at each vertex we have a "time" t integration and conservation of momentum and discrete energy. Finally, Feynman rules for the vertices can easily be read off from the Langevin equations. Taking the limit $t = t' \rightarrow \infty$ in (4.9) we can extract the gauge invariant vacuum polarization tensor and obtain the result⁹⁾

$$\begin{aligned} \Pi_{\mu\nu}(k) &= e^2 \int \hat{d}q V_{\mu\nu}(q, q') \frac{1}{q^2 + q'^2 + k^2 + 2m^2} \cdot \\ &\cdot \left\{ \frac{k^2}{q'^2 + m^2} \frac{1}{q^2 + m^2} + \frac{1}{q^2 + m^2} + \frac{1}{q'^2 + m^2} \right\} = \\ &= e^2 \int \hat{d}q V_{\mu\nu}(q, q') \frac{1}{(q^2 + m^2)(q'^2 + m^2)} \end{aligned} \quad (4.10)$$

with

$$V_{\mu\nu}(q, q') = \text{Tr} \left\{ \gamma_\mu (\not{q}' + m) \gamma_\nu (\not{q} + m) \right\} \quad (4.11)$$

which is just the analytic expression for the usual Feynman diagram. The sum over the discrete energies in (4.10) can be performed by using the formula¹⁰⁾

$$\frac{1}{\beta} \sum_{n=-\infty}^{\infty} f(\omega_n^\pm) = \int_{-\infty}^{\infty} \frac{dz}{2\pi} f(z) \pm \int_{-\infty+i\epsilon}^{\infty+i\epsilon} \frac{dz}{2\pi} (f(z) + f(-z)) \frac{1}{e^{-i\beta z} \mp 1} \quad (4.12)$$

where upper signs refer to bosons, lower signs to fermions; this formula explicitly separates $T = 0$ and $T \neq 0$ contributions. Thus we obtain the electric mass of the photon

$$m_d^2 = \frac{2e^2 T^2}{\pi^2} \int_0^\infty dy \frac{y^2 e^{\sqrt{y^2 + \lambda^2}}}{(e^{\sqrt{y^2 + \lambda^2}} + 1)^2} \quad (4.13)$$

with $\lambda = m/T$. In the limit $\lambda \rightarrow \infty$ *) one gets

$$m_d^2 \simeq \frac{4\sqrt{2} \alpha}{\sqrt{\pi}} m^{3/2} T^{1/2} e^{-m/T}, \quad (4.14)$$

($\alpha = e^2/4\pi$) which agrees with the result of Ref. 11), and for $\lambda \rightarrow 0$

$$m_d^2 = \frac{e^2 T^2}{3}. \quad (4.15)$$

Stochastic quantization at finite temperature can be applied to the scalar QED case in the same way. Now also the diagrams (e) of Fig. 1 contribute and after a straightforward calculation, one again obtains for a massless particle¹²⁾ the result (4.15).

On the other hand, in non-Abelian gauge theories, the electric mass of the gauge boson m_{el} cannot be calculated in this way. The reason is that for non-Abelian gauge theories neither the gauge boson propagator $D_{\mu\nu}^{ab}(k)$ nor the self-energy $\pi_{\mu\nu}^{ab}(k)$ are gauge invariant quantities and the limit $t = t' \rightarrow \infty$ of $\pi_{\mu\nu}^{ab}$ does not exist. One could try to calculate the one-loop contribution to

*) T could be assumed to be of the order of $3 \text{ } ^\circ\text{K}$.

m_{el}^2 as

$$m_{el}^2 = \lim_{t \rightarrow \infty} \lim_{\vec{k} \rightarrow \vec{0}} |\vec{k}|^4 (-D_{00}'(k_0=0, \vec{k}; t, t)) \quad (4.16)$$

where $D_{\mu\nu}'$ is the one-loop propagator, because $|\vec{k}|^4$ amputates the external legs and $\pi_{00}(k_0=0, \vec{k} \rightarrow \vec{0})$ is gauge invariant^{13),14)}. Even though (4.16) looks reasonable, it can easily be shown that it leads to the wrong result, $m_{el} = 0$. As a matter of fact, taking the limit $k \rightarrow 0$ for fixed t implies that the "damping force" $(-k^2 \delta_{\mu\nu} + k_\mu k_\nu) A_\nu^a(k, t)$ in the Fourier transform of Eq. (2.16) is absent even for non-interacting fields. On the other hand, the validity of the stochastic quantization approach is based upon the presence of a damping force which ensures the existence of a limiting value of the fields irrespective of the initial values^{*}.

The above difficulties can be avoided by using the modification (4.2a) of the Langevin equation, because in this case the limit $t \rightarrow \infty$ does always exist, even for non-gauge invariant quantities. Introducing

$$V^b(x) = -\frac{1}{\alpha} \partial_\mu A_\mu^b(x) \quad (4.17)$$

in (4.2a) ($\alpha = 1$ would correspond to the Feynman gauge), one obtains for the gauge boson self-energy $\pi_{\mu\nu}^{ab}$ in a pure gauge theory, the usual result plus the following term¹⁵⁾:

$$\tilde{\pi}_{\mu\nu}^{ab}(k) = g^2 f^{acd} f^{bcd} (k_\mu k_\nu - 2k^2 \delta_{\mu\nu}) \int d\hat{q} \frac{1}{q^2 (k-q)^2}. \quad (4.18)$$

It is clear, however, that

$$\lim_{\vec{k} \rightarrow \vec{0}} \tilde{\pi}_{00}^{ab}(k_0=0, \vec{k}) = 0 \quad (4.19)$$

so that the usual result^{13),14)} in $SU(N)$

$$m_{el}^2 = \frac{g^2}{3} T^2 N \quad (4.20)$$

for the one-loop gauge boson electric mass at $T \neq 0$ is obtained.

Let us finish this discussion by stating that even though the original Parisi-Wu approach cannot be used to calculate thermal masses in non-Abelian gauge theories through two-point correlation functions, it can be used to calculate gauge invariant Green functions and whenever the method is applicable at

^{*}) For discussions see Refs. 1), 3), and 5).

$T = 0$, the extension to $T \neq 0$ is allowed and straightforward, as explained in the previous sections. This can be seen explicitly in the example

$$\lim_{t \rightarrow \infty} \langle F_{\mu\nu}^a(x,t) F_{\mu\nu}^b(x,t) \rangle_\eta \quad (4.21)$$

which has been considered in detail in Ref. 3) at $T = 0$.

In conclusion we want to say that finite temperature effects can be incorporated quite naturally in the stochastic quantization scheme by choosing random variables which are (anti) periodic in Euclidean time x_0 with period $\beta = 1/T$ and the theory obtained in this way is equivalent to the imaginary time formalism of a finite temperature field theory.

We thank D. Amati for a critical reading of the manuscript and for discussions; one of us (G.N.) thanks the Theoretical Physics Division at CERN, where this work was done, for its hospitality.

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Figure Caption

Fig. 1 Stochastic diagrams contributing to the photon propagator up to $O(e^2)$. Diagrams (c) appear only in scalar QED.

Fig. 1