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SYMMETRY ANTI-RESTORATION IN SUPERSYMMETRIC THEORIES

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ABSTRACT

We show that, as a general phenomenon in supersymmetric theories, global symmetries which are present at zero temperature are broken at some finite temperature. We illustrate this mechanism, called symmetry anti-restoration, in a simple $N = 1$ globally supersymmetric model with a global $U(1)$ symmetry. Some applications of symmetry anti-restoration are pointed out.

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Finite temperature effects break supersymmetry and they generally remove the degeneracy of vacua which typically occurs in global supersymmetry theories at $T = 0$ ¹⁾. In this letter we point out that this shift of the degenerate vacua presents the peculiar and interesting feature that, in a quite general way, some of the global symmetries, which occur at $T = 0$, are broken at $T \neq 0$, being restored at low temperature. We call this phenomenon symmetry anti-restoration, since it constitutes the opposite of the appearance of some new symmetry when T increases (symmetry restoration).

The importance of this phenomenon in supersymmetric theories is enhanced by its complete generality: as it will be shown explicitly below, for any range of the parameters of the potential, there exists correspondingly an interval of temperature during which some global symmetry of the theory is broken. These symmetries can then be restored when temperature effects become less important.

The simplest model we can choose to illustrate such a phenomenon is an $N = 1$ globally supersymmetric²⁾ theory with a $U(1)$ global symmetry. The supermultiplets ϕ_+ and ϕ_- transform with opposite charge under this $U(1)$, while the supermultiplet χ is neutral. The superpotential P is defined as:

$$P = g \phi_+ \chi \phi_- - \mu \chi + \frac{f}{6} \chi^3 \quad (1)$$

The mass term $m \phi_+ \phi_-$ can always be eliminated by a convenient shift of χ in the linear term. We omit a quadratic term in χ for simplicity.

From (1), we get the $T = 0$ potential:

$$V_0 = g^2 |\chi \rho_-|^2 + g^2 |\chi \rho_+|^2 + |g \rho_+ \rho_- - \mu + \frac{f}{2} \chi^2|^2 \quad (2)$$

where ρ_+ , ρ_- and χ denote the scalar components of the supermultiplets ϕ_+ , ϕ_- and χ , respectively. At finite temperature T , the additional contribution¹⁾:

$$V_T = \frac{T^2}{8} \left[f^2 |\chi|^2 + 2g^2 (|\rho_+|^2 + |\rho_-|^2 + |\chi|^2) \right] + \frac{\pi^2}{8} T^4 \quad (3)$$

must be added.

V_0 vanishes for:

$$\langle \rho_+ \rangle = \langle \rho_- \rangle = 0, \quad \langle \chi \rangle^2 = \frac{2\mu}{f} \quad (4)$$

and

$$\langle \rho_+ \rangle \cdot \langle \rho_- \rangle = \mu/g, \quad \langle \chi \rangle = 0. \quad (5)$$

The minimum in (4) corresponds to the U(1) unbroken phase, whereas the minimum (5) breaks this U(1).

We want to show that there always exists a range of T for which the system is in the U(1) broken phase. To prove that, it is not necessary to go through the whole minimization of $V \equiv V_0 + V_T$, but it suffices to show that for a range of T there exists a minimum of V corresponding to the U(1) broken phase lower than any local minimum of the U(1) unbroken phase.

We start by examining the local minimum of V with U(1) unbroken. So we fix $\langle \rho_+ \rangle = \langle \rho_- \rangle = 0$ and we let χ be the only complex variable. Differentiating V with respect to χ and $\bar{\chi}$, one gets:

$$\frac{\partial V}{\partial \chi} (\rho_+ = \rho_- = 0) = \left(-\mu + \frac{f}{2} \bar{\chi}^2 \right) f \chi + \frac{T^2}{8} (f^2 + 2g^2) \bar{\chi} = 0 \quad (6)$$

and

$$\frac{\partial V}{\partial \bar{\chi}} (\rho_+ = \rho_- = 0) = \left(-\mu + \frac{f}{2} \chi^2 \right) f \bar{\chi} + \frac{T^2}{8} (f^2 + 2g^2) \chi = 0 \quad (7)$$

From the Eqs. (6) and (7), one gets:

$$-\mu f (\chi^2 - \bar{\chi}^2) = 0, \quad (8)$$

i.e., χ is real or purely imaginary. If χ is purely imaginary, $\chi = ib$, with b real, from Eq. (6) one derives:

$$\mu f + \frac{f^2}{2} b^2 + \frac{T^2}{8} (f^2 + 2g^2) = 0. \quad (9)$$

If $f\mu > 0$, Eq. (9) has no solution. This is the only assumption we make on the parameters of P [Eq. (1)]: we take μ , f and g real, with μ and f of the same sign. We come now to the solution with χ real. Equation (6) [or Eq. (7)] gives:

$$\chi \left[-\mu f + \frac{T^2}{8} (f^2 + 2g^2) + \frac{f^2}{2} \chi^2 \right] = 0. \quad (10)$$

Equation (10) presents two solutions according to the range of values of T :

$$\chi = 0 \quad \text{if} \quad T^2 > 8\mu f / (f^2 + 2g^2), \quad (11)$$

$$\chi = \pm \left[\frac{2}{f^2} \left(\mu f - \frac{T^2}{8} (f^2 + 2g^2) \right) \right] \equiv A \text{ and } \chi = 0 \text{ if } T^2 < 8\mu f / (f^2 + 2g^2) \quad (12)$$

To establish whether $\chi = A$ or $\chi = 0$ is the point of minimum for $T^2 < 8\mu f / (f^2 + 2g^2)$, we need to calculate $V(A)$ and $V(0)$:

$$V(\rho_+ = \rho_- = 0, \chi = A) = \mu^2 - \frac{1}{f^2} \left[\mu f - \frac{T^2}{8} (f^2 + 2g^2) \right]^2 + \frac{\pi^2}{8} T^4 \quad (13)$$

$$V(\rho_+ = \rho_- = \chi = 0) = \mu^2 + \frac{\pi^2}{8} T^4. \quad (14)$$

Being $V(A) < V(0)$, we conclude that in the $U(1)$ unbroken phase the following situation arises:

$$V_{\min}(\rho_+ = \rho_- = 0, \chi = A) = \mu^2 - \frac{1}{f^2} \left[\mu f - \frac{T^2}{8} (f^2 + 2g^2) \right]^2 + \frac{\pi^2}{8} T^4 \text{ for } T^2 < 8\mu f / (f^2 + 2g^2)$$

and

$$V_{\min}(\rho_+ = \rho_- = \chi = 0) = \mu^2 + \frac{\pi^2}{8} T^4 \text{ for } T^2 > 8\mu f / (f^2 + 2g^2).$$

We show now that there exists at least one direction in the $U(1)$ broken phase, $\rho_+ = \rho_- = v \in \mathbb{R}$ and $\chi = 0$, along which we get values for V lower than $V(\rho_+ = \rho_- = 0, \chi = A)$ and $V(\rho_+ = \rho_- = \chi = 0)$ in the intervals $T^2 < 8\mu f / (f^2 + 2g^2)$ and $T^2 > 8\mu f / (f^2 + 2g^2)$, respectively. The value $\rho_+ = \rho_- = v$ is given in terms of the P parameter by the minimization condition:

$$\frac{\partial V}{\partial v^2}(\rho_+ = \rho_- = v, \chi = 0) = 2(gv^2 - \mu)g + \frac{T^2}{2}g^2 = 0 \quad (15)$$

One gets:

$$v^2 = \left(\mu - \frac{g}{4} T^2 \right) / g \text{ for } T^2 \leq 4\mu/g. \quad (16)$$

Correspondingly, the potential gets the value:

$$\begin{aligned} V_{\min}(\rho_+ = \rho_- = v = \left(\left(\mu - \frac{g}{4} T^2 \right) / g \right)^{1/2}, \chi = 0) &= \\ &= \mu^2 - \left(\mu - \frac{g}{4} T^2 \right)^2 + \frac{\pi^2}{8} T^4. \quad (T^2 \leq 4\mu/g) \end{aligned} \quad (17)$$

Remembering the value of $V_{\min}(\rho_+ = \rho_- = \chi = 0)$, Eq. (14), we conclude that:

$$V_{\min}(\beta_+ = \beta_- = v, \chi = 0) < V_{\min}(\beta_+ = \beta_- = \chi = 0),$$

and so, in the range $8\mu f/(f^2+2g^2) < T^2 \leq 4\mu/g$, the system is in the U(1) broken phase.

Let us consider now the interval $T^2 < 8\mu f/(f^2+2g^2)$ with $T^2 \leq 4\mu/g$. In this range of T , the $V_{\min}$ of the U(1) unbroken phase is $V(\rho_+ = \rho_- = 0, \chi = A)$, Eq. (13). If we compare $V_{\min}(\rho_+ = \rho_- = v, \chi = 0)$ and $V(\rho_+ = \rho_- = 0, \chi = A)$, recalling the reality of μ , f and g , we get easily:

$$V_{\min}(\beta_+ = \beta_- = v, \chi = 0) < V_{\min}(\beta_+ = \beta_- = 0, \chi = A)$$

for

$$T^2 < 16\mu f / (f^2 + 2g^2 + 2gf). \quad (18)$$

But, since $(16\mu f) / (f^2+2g^2+2gf)$ is greater than $(8\mu f) / (f^2+2g^2)$ for any value of f , g and μ , we conclude that:

$$V_{\min}(\beta_+ = \beta_- = v, \chi = 0) < V_{\min}(\beta_+ = \beta_- = 0, \chi = A)$$

for $T^2 < 8\mu f/(f^2+2g^2)$.

Therefore we have proved that $V_{\min}(\rho_+ = \rho_- = v, \chi = 0)$ is lower than any $V_{\min}$ of the U(1) unbroken phase for all temperature $T < 2\sqrt{\mu/g}$.

At low T , some supersymmetry breaking terms can always favour the U(1) symmetric minimum, $\rho_+ = \rho_- = 0, \chi \neq 0$, thus restoring the U(1) symmetry. We stress that this result is achieved without any tuning of the parameters of the superpotential. We made only the mild requirement of the reality and of the same sign on μ and f .

Symmetry anti-restoration appears at first sight to be a phenomenon rich in important applications. The violation at some temperature of presently good quantum numbers can yield dramatic consequences for the primordial Universe.

A particularly interesting example is provided by the baryon number. Its non-conservation at some finite temperature can be the clue for the generation of a cosmological matter-antimatter asymmetry³⁾ even at temperatures much below 10^{13} GeV. Contrarily to what happens in the ordinary grand unified models, this late production of a cosmological baryon asymmetry does not cause any trouble to the present matter stability since B is again a good quantum number at $T \rightarrow 0$.

Needless to say, this mechanism would be crucial in inflationary models where there is not enough reheating to produce the superheavy bosons which are commonly considered responsible for the B asymmetry.

We think that what has been said above can apply also to the case of local symmetries. The only difference is given by the appearance of D terms, but, at least for ranges of the parameters of the potential, the anti-symmetry restoration should occur. The problem is presently under study. If this is the case, the mechanism could yield some other interesting consequences: for instance, a violation of electric charge at some T could provide a solution to the monopole problem⁴⁾.

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