Ref. TH.3756-CERN

HOPF INSTANTON AND PROTON DECAY

Jihn E. Kim ^{*)}
CERN - Geneva

and

Hae Won Lee
Department of Physics
Seoul National University
Seoul 151, Korea

A B S T R A C T

We point out that the Hopf instanton can generate new types of $\Delta B = 1, 2$, etc. processes, such as proton decay. However, the rate is completely negligible in the SU(5) model.

*) On leave from Department of Physics,
Seoul National University, Seoul 151,
Korea.

1. Introduction

Baryon number (B) violation in non-Abelian gauge theories is the focus of current theoretical¹⁾⁻³⁾ and experimental⁴⁾ interest. In this paper, we argue that there exists another mechanism for B violation in grand unified theories (GUTs).^{*)}

The best understood and unambiguous mechanism for B violation is through exchange of superheavy gauge bosons X and Y with estimated mass of $10^{14} \sim 10^{15}$ GeV, or through coloured Higgs with mass $> 10^{11}$ GeV. These mechanisms via tree Feynman diagrams are the most clean experimental window towards the GUT scale, and may even have played an important role in generating B asymmetry of our universe⁶⁾. Another mechanism is the catalysis of $\Delta B \neq 0$ processes by GUT monopoles. The rate of B violation through this process depends on the monopole flux of our universe, which is believed to be tiny⁷⁾. Proton decay rate by these two processes in many GUTs can be large enough for experimental tests⁴⁾. The third, an extremely feeble and hence experimentally uninteresting process for $\Delta B = 2, 4, \dots$ is through 't Hooft's determinant interaction²⁾ generated by an instanton solution in the electroweak theory $SU(2)_L \times U(1)$ ⁸⁾.

Let us briefly review 't Hooft's mechanism, which our new B violation associated with GUT instantons will resemble. The $SU(2) \times U(1)$ gauge theory has an instanton solution, since $\pi_3(SU(2) \times U(1)/U(1)_{em}) = \pi_3(S^3) = \mathbb{Z}$. Note that S^3 is a coset space of G/H , where $G = SU(2) \times U(1)$ and $H = U(1)_{em}$. Breaking of G into the unbroken subgroup H is realized by the vacuum expectation value (v.e.v.) $v \approx 250$ GeV of a Higgs doublet. We argue that the instanton size of the electroweak theory is g_2/v , based on an educated guess from the monopole size when a monopole appears as $G \rightarrow H$. A non-perturbative effect of this action is manifested in the factor

$$f = e^{-8\pi^2/g_2^2} = e^{-2\pi \sin^2 \theta_w / \alpha_{em}} \quad (1)$$

where $\sin^2 \theta_w$ and α_{em} are evaluated at $M_w/\alpha_2 \approx 2322$ GeV. Taking $\alpha_{em}(M_w/\alpha_2) \approx 1/125.75$ and $\sin^2 \theta_w \approx 0.23$ as an illustration, we obtain $f \approx 10^{-79}$. If this is the only mechanism for B violation, τ_p is $\approx 10^{140}$ years and no proton would have ever decayed in our universe.

*) Langacker, Segre, and Weldon⁵⁾ have constructed models with a stable proton. The global symmetries have been introduced. In such models the GUT scale can be low as far as $\sin^2 \theta_w$ is renormalized to a correct value. See Kim and Song⁵⁾. If the imposed global symmetry is not anomalous as in the Langacker-Segre-Weldon model, our instanton does not generate the proton decay, but $\Delta B \neq 0$ processes can occur through the 't Hooft method²⁾.

However, GUTs liberated us from the realm of small gauge coupling constants. For example, α_{em} will be much larger than $1/137$ at the GUT scale. If there are many charged fermions between M_W and the GUT scale $\tilde{M}$, $\alpha_{em}(\tilde{M})$ can be of order 1. Therefore, one may anticipate a large proton decay rate due to an instanton. For this purpose, let us try to embed an instanton in GUTs so that we can use a large value of coupling constant in Eq. (1). We will have an additional suppression factor in the $\Delta B \neq 0$ process resulting from the instanton size integration.

As a prototype example, we will refer to the SU(5) model of Georgi and Glashow¹⁾. In this model, $\{a, b, c\} = \{1, 2, 3\}$ is the colour index set and $\{i, j\} = \{4, 5\}$ is the weak index set. The SU(2) space of the 't Hooft mechanism is embedded in $\{4, 5\}$ space. If we embed the instanton in $\{1, 2, 3\}$ space, we obtain the Belavin, Polyakov, Schwartz, and Tyupkin (BPST) solution⁹⁾. This BPST solution is important at scales $\lesssim 1$ GeV where the colour coupling α_c is large. Note that the BPST instanton does not have a unique scale, because the mapping $\pi_3(SU(2)) = \mathbb{Z}$ does not introduce any v.e.v. The GUT instanton which we are looking after must be embedded in the $\{a, i\}$ subspace. In extended GUTs¹⁰⁾, the choice of this subspace can be easily generalized.

The GUT instanton ($\equiv$ Hopf instanton) which we propose is based on the Hopf mapping¹¹⁾,

$$\pi_3(S^2) = \mathbb{Z} \quad (2)$$

There is no simple group whose topology is equivalent to S^2 . S^2 must be generated as a coset space G/H , thus introducing a v.e.v. which we would like to identify as the GUT scale. When the Georgi-Glashow SU(2) model¹²⁾ is broken by an adjoint Higgs field, we have $\pi_3(SU(2)/U(1)) = \mathbb{Z}$ and obtain the Hopf instanton. This feature cannot be made phenomenologically viable in electroweak theories, since we already know that the electroweak Higgs field is not an SU(2) adjoint Higgs field but an SU(2) doublet. However, the Hopf instanton can be embedded in GUTs through the adjoint Higgs field [e.g., 24_H of SU(5)] of the grand unification, as the 't Hooft-Polyakov monopole¹³⁾ is realized in GUTs¹⁴⁾. As in the monopole case, the Hopf instanton is embedded in the $\{a, i\}$ subspace. In our case, $\{a, i\}$ must be taken as $\{a, 4\}$ since 5_H develops a v.e.v in the 5th direction.

II. The Pontryagin Index

Let us first show that one topological number, the Pontryagin index, is sufficient to describe the BPST instanton and the Hopf instanton, though their core topology may be very different. The bosonic part of the Lagrangian in Euclidean space-time is

$$\mathcal{L}_E = \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}_\mu \Phi) + V(\Phi) \quad (3)$$

where Φ is written as a column vector.

An instanton solution satisfies the following equations of motion,

$$(\mathcal{D}_\mu F_{\mu\nu})^a = -g J_\nu^a \quad (4)$$

$$\mathcal{D}_\mu \mathcal{D}_\mu \Phi = \frac{\partial V}{\partial \Phi^\dagger} \quad (5)$$

where $J_\nu^a = \Phi^\dagger T^a i \overleftrightarrow{D}_\nu \Phi$, and has a finite action $S_E = \int d^4x \mathcal{L}_E < \infty$. In view of the finite action, we impose the following asymptotic behaviours as $(|x_\mu| \equiv r) \rightarrow \infty$,

$$F_{\mu\nu}^a \longrightarrow O(1/r^2) \quad (6)$$

$$\mathcal{D}_\mu \Phi \longrightarrow O(1/r^2) \quad (7)$$

$$\Phi^\dagger \Phi \longrightarrow v^2 \quad (8)$$

Let the v.e.v. v break the gauge group G to H . Then we can classify the finite action configurations of the gauge and Higgs fields by the two topological numbers;

- (i) the Pontryagin index which is associated with $\pi_3(G) = \mathbb{Z}$,

$$g[A] = \frac{1}{32\pi^2} \int d^4x F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a \quad (9)$$

- (ii) another topological number associated with the mapping $\Phi :$

$$S^3 \rightarrow G/H, \text{ i.e., } \pi_3(G/H).$$

Let us show that these two topological numbers are not independent. The Higgs topology is determined by the Pontryagin index ^{9), 15)}. From the exact homotopy sequence,

$$\pi_3(G) \longrightarrow \pi_3(G/H) \longrightarrow \pi_2(H) \quad (10)$$

and

$$\pi_2(H) = 0 \quad \text{for most gauge groups.} \quad (11)$$

we find that

$$\pi_3(G) \xrightarrow{p} \pi_3(G/H) \quad (12)$$

is surjective. The mapping p is found as follows. From Eq. (6), we can write $A_\mu \rightarrow 0 (1/r^2) + g^{-1} (\partial_\mu g)$ for some gauge function g . The condition $D_\mu \Phi \rightarrow 0$ gives the relation

$$\partial_\mu (g \Phi) = 0 \quad \Rightarrow \quad g \Phi = v \quad (13)$$

Therefore, we can write

$$\Phi \underset{r \rightarrow \infty}{=} g^{-1} v \quad (14)$$

for some fixed v , which defines the mapping p in Eq. (12).

We have just seen that finite action configurations are classified only by the Pontryagin index. As in the pure Yang-Mills theory, our GUT instanton gives the lower bound of the action

$$S_E \geq \frac{8\pi^2}{g^2} |q| \quad (15)$$

where q is the Pontryagin index. But, in our case, this bound does not have to be saturated. The existence of the finite action configuration with A and Φ guarantees the existence of the instanton solution.

III. Instanton Induced Proton Decay

In this section, we estimate the effective action of B decay due to the GUT instanton. Let us introduce Higgs fields $\Phi (= 24_H)$ and $H (= 5_H)$. The Lagrangian for the gauge and Higgs fields is

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \text{Tr}(W_{\mu\nu} W^{\mu\nu}) + \text{Tr} [D_\mu \Phi]^2 \\ & + (D_\mu H)^\dagger (D^\mu H) - V(\Phi, H) \end{aligned} \quad (16)$$

where

$$\begin{aligned}
 W_\mu &= W_\mu^a T^a \\
 W_{\mu\nu} &= \partial_\mu W_\nu - \partial_\nu W_\mu - ig [W_\mu, W_\nu] \\
 D_\mu &= \partial_\mu - ig W_\mu \\
 \Phi &= \Phi^a T^a
 \end{aligned} \tag{17}$$

The T^a 's are the Hermitian matrices generating the fundamental representation 5 of $SU(5)$, and are normalized according to

$$\text{Tr } T^a T^b = \frac{1}{2} \delta^{ab} \tag{18}$$

As usual $V(\Phi, H)$ is chosen such that it breaks $SU(5)$ to $SU(3)_c \times U(1)_{em}$. In the unitary gauge, the vacuum values of Φ and H are

$$\langle \Phi \rangle = \tilde{v} \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -\frac{3}{2} + \xi & \\ & & & & -\frac{3}{2} - \xi \end{bmatrix}, \tag{19}$$

$$\langle H \rangle = v \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The fermions are

$$\begin{aligned}
 \bar{5} = \psi_\alpha &= \begin{bmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ e \\ \nu \end{bmatrix}_L, \quad 10 = \psi^{\alpha\beta} = \begin{bmatrix} 0 & \bar{u}_3 & -\bar{u}_2 & -u^1 & -d^1 \\ -\bar{u}_3 & 0 & \bar{u}_1 & -u^2 & -d^2 \\ \bar{u}_2 & -\bar{u}_1 & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -e^+ \\ d^1 & d^2 & d^3 & e^+ & 0 \end{bmatrix}_L
 \end{aligned} \tag{20}$$

As discussed before, an instanton solution exists for any topological number. Instead of finding the exact solutions of the combined system (i.e. the gauge field and Higgs fields), we follow the philosophy adopted by 't Hooft. We embed

the BPST instanton in the gauge field first and then seek an appropriate solution of the Higgs fields. To study the B violating process, let us embed the SU(2) into SU(5) via

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \left[\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ \hline & & \boxed{SU(2)} & & \\ \hline & & & & \end{array} \right] \quad (21)$$

including proper gauge rotations in $SU(3)_c \times U(1)_{em}$.

Under this embedding, the fermions are decomposed according to

$$\bar{5} \longrightarrow 2 + 1 + 1 + 1 \quad (22)$$

$$10 \longrightarrow 2 + 2 + 2 + 1 + 1 + 1 + 1 \quad (23)$$

where 2 in Eq. (22) is

$$2 = \begin{pmatrix} e^- \\ -\bar{d}_3 \end{pmatrix} \quad (24)$$

and 2's in Eq. (23) are

$$\begin{pmatrix} \bar{u}_2 \\ u^1 \end{pmatrix}_L, \quad \begin{pmatrix} -\bar{u}_1 \\ u^2 \end{pmatrix}_L, \quad \begin{pmatrix} -d^3 \\ -e^+ \end{pmatrix}_L \quad (25)$$

The kinetic energy for (24) and (25) is described by two 4-component SU(2) doublet spinors,

$$\mathcal{L}_f = \sum_{r=1}^2 \bar{\Psi}_r i \gamma_\mu \bar{D}^\mu \Psi_r \quad (26)$$

where $\bar{D}_\mu = \partial_\mu - ig A_\mu^a \tau^a$ and $\tau^a = \sigma^a/2$. The Ψ_r 's are two SU(2) doublets of 4 component Dirac fields,

$$\Psi_1 = \begin{pmatrix} d^3 \\ e^+ \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} \bar{u}_2 \\ u^1 \end{pmatrix} \quad (27)$$

Similarly, we can also include the second and the third generations,

$$\psi_3 = \begin{pmatrix} s^3 \\ \mu^+ \end{pmatrix}, \psi_4 = \begin{pmatrix} \bar{c}_2 \\ c^1 \end{pmatrix}, \psi_5 = \begin{pmatrix} b^3 \\ z^+ \end{pmatrix}, \psi_6 = \begin{pmatrix} \bar{t}_2 \\ t^1 \end{pmatrix} \quad (28)$$

Since the masses of the second and the third generations are heavy, we will treat them separately.

Under the embedding (21), the Higgs fields decompose as follows:

$$24_H \longrightarrow \begin{matrix} (3) \\ t=1 \end{matrix} + 3 \begin{matrix} (2) \\ t=\frac{1}{2} \end{matrix} + 9 \begin{matrix} (1) \\ t=0 \end{matrix} + 3 \begin{matrix} (2) \\ t=\frac{1}{2} \end{matrix} \quad (29)$$

$$5_H \longrightarrow \begin{matrix} (2) \\ t=\frac{1}{2} \end{matrix} + 3 \begin{matrix} (1) \\ t=0 \end{matrix}$$

The v.e.v. of 24_H decomposes under the $SU(2)$ as $\langle \Phi \rangle \rightarrow (3) + \text{singlets}$

$$\langle \Phi \rangle = \frac{5-2\varepsilon}{4} \text{diag.}(0, 0, 1, -1, 0) + \dots \quad (30)$$

Let us denote this isotriplet as Φ^a ($a = 1, 2, 3$). Then we have the asymptotic condition

$$|\Phi| \xrightarrow{r \rightarrow \infty} \frac{5-2\varepsilon}{2} \tilde{v} \quad (31)$$

The Higgs field Lagrangian for this triplet is

$$\mathcal{L}_H = \frac{1}{2} (\tilde{\mathcal{D}}_\mu \Phi)^T (\tilde{\mathcal{D}}^\mu \Phi) - V(\Phi) \quad (32)$$

where $(\tilde{\mathcal{D}}_\mu)_{ab} = \partial_\mu \delta_{ab} - ig_{abc} A_{c\mu}$ and $V(\Phi)$ is minimized at $|\Phi| = \tilde{v} (5-2\varepsilon)/2$.

In the Euclidean space, the action of the approximate solution of the Higgs field is given by¹⁴⁾ ($\varepsilon = 0$)

$$\begin{aligned} S_H &= \int d^4x \left\{ -\frac{1}{2} (\tilde{\mathcal{D}}_\mu \Phi)^T (\tilde{\mathcal{D}}_\mu \Phi) - V(\Phi) \right\} \\ &= -\frac{1}{2} \cdot 4\pi^2 \left(\frac{5}{2}\right)^2 \tilde{v}^2 \rho^2 + O(\lambda \tilde{v}^4 \rho^4) \end{aligned} \quad (33)$$

where ρ is an instanton size and a factor 1/2 is included to take into account the reality of Φ .

Before finding an effective action, we note the well-known result for a single instanton in SU(N) - QCD with N_f flavours with small masses^{*}

$$\begin{aligned} J \equiv \langle n+1 | n \rangle &= \frac{4}{\pi^2 (N-1)! (N-2)!} \int d^4 z d\rho \frac{1}{\rho^5} (\rho m_F)^{N_f} \\ &\cdot \left(\frac{4\pi^2}{g^2} \right)^{2N} \exp \left[-\frac{8\pi^2}{g^2(\mu)} + \frac{1}{3} (11N - 2N_f) \ln(\mu\rho) \right. \\ &\quad + \frac{1}{3} (N_f - N) \left\{ \gamma - \frac{17}{12} + \ln \pi - \frac{6}{\pi^2} \zeta'(2) \right\} \\ &\quad \left. + \frac{5}{6} - \ln 2 + O(m_F) \right] \end{aligned} \quad (34)$$

where μ is the subtraction point for the coupling constant definition. In our case, we can take $N = 5$ and $N_f = 4$ for two heavy generations of fermions with common mass. The representation (28) is real under the embedding subgroup SU(2), even though $2(10 + \bar{5})$ is not real under SU(5). We have separated out the contribution of the first generation fermions (27), because these light fermions can be taken as massless ones. The contribution to the effective action due to light fermions is the well-known 't Hooft's determinant interaction. The effective interaction due to n_f massless isodoublets together with (34) is

$$\begin{aligned} \int \mathcal{L}_{\text{eff}} d^4 x &= J \cdot \frac{1}{\mu} \cdot \exp(2n_f W^{\frac{1}{2}}) \rho^4 \cdot \\ &\cdot \langle (\text{'t Hooft } 2n_f \text{ point vertex}) \rangle \end{aligned} \quad (35)$$

where

$$W^{\frac{1}{2}} = \frac{1}{6} \ln(\mu\rho) + \frac{1}{6} \left\{ \gamma - \frac{17}{12} + \ln \pi - \frac{6}{\pi^2} \zeta'(2) \right\} \quad (36)$$

with the Euler constant $\gamma = 0.5772156649$ and the derivative of ζ function

^{*}) See, for example, Ref. 16).

$\zeta'(2) = -0.9375482543$. The position and size integrations in the symbol J in Eq. (35) act also on the rest of Eq. (35). The symbol $\langle \rangle$ in Eq. (35) takes the average over the appropriate subgroup of $SU(5)$. In our case, the unbroken subgroup $SU(3)_c \times U(1)_{em}$ should be contained in the averaging. Since the $SU(2)$ instanton can be embedded in various ways in $SU(5)$, we also average over the $SU(2)$ where the instanton is embedded. The following Higgs contribution is multiplied to Eq. (35) with the same convention as before,

$$\exp [-W^t] \quad (37)$$

for each massless (or small mass) complex scalar with the isospin t .

For $t = 1$, we have

$$W^1 = \frac{2}{3} \ln(\mu \rho) + \frac{2}{3} \left\{ \gamma - \frac{17}{12} + \ln \pi - \frac{6}{\pi^2} \zeta'(2) \right\} - \frac{5}{6} + \ln 2 \quad (38)$$

The Higgs field in Eq. (29) is complicated. Both heavy and light Higgs fields appear in the same isomultiplets. Therefore, we present, for simplicity, the result for the $t = 1$ Higgs field only.

Combining all these contributions, we obtain

$$\begin{aligned} \int \mathcal{L}_{eff} d^4x = & \frac{1}{36\pi^2} \left(\frac{4\pi^2}{g^2(\mu)} \right)^{10} \int d^4z d\rho \frac{1}{\rho^5} (\rho m)^4 \rho^6 \\ & \cdot \exp \left[-\frac{8\pi^2}{g^2(\mu)} + 14 \ln(\mu \rho) - \frac{1}{3} \left(\gamma - \frac{17}{12} \right. \right. \\ & \left. \left. + \ln \pi - \frac{6}{\pi^2} \zeta'(2) \right) + \frac{5}{3} - 2 \ln 2 \right. \\ & \left. - 4\pi^2 \left(\frac{25}{8} \tilde{v}^2 \rho^2 \right) \right] \times \langle (t \text{ Hooft} \\ & 4\text{-point vertex}) \rangle \end{aligned} \quad (39)$$

After integrating over ρ , we obtain

$$\begin{aligned} \int \mathcal{L}_{eff} d^4x = & \frac{1}{36\pi^2} \left(\frac{4\pi^2}{g^2} \right)^{10} \frac{1}{2} \Gamma(10) \int d^4z \left(\frac{m}{\Lambda} \right)^4 \left(\frac{1}{\Lambda} \right)^2 \\ & \cdot \exp \left[-\frac{8\pi^2}{g^2(\mu)} + 14 \ln(\mu/\Lambda) + \frac{5}{4} - \frac{3}{2} \ln 2 \right] \langle \rangle \end{aligned} \quad (40)$$

where

$$\Lambda = \frac{5}{2} \sqrt{2} \pi \tilde{v} = 2 M_X / g(M_X) \quad (41)$$

This is one of the results of our investigation.

If we treat all the three generations as massless fermions, we will obtain $1/\Lambda^{14}$ order instead of (40). Therefore, the dominant contribution is Eq. (40).

IV. Proton Decay

Simplifying Eq. (40), we obtain a nonlocal effective interaction

$$\begin{aligned} \mathcal{L}_{\text{eff}} = C \frac{1}{M_X^2} \left(\frac{m}{M_X} \right)^4 e^{-8\pi^2/g^2(M_X)} \\ \cdot \langle \varepsilon^{st} \{ 2 \bar{\Psi}_1^\alpha (1+\gamma_5) \psi_s^\alpha \bar{\Psi}_2^\beta (1+\gamma_5) \psi_t^\beta \\ - \bar{\Psi}_1^\alpha (1+\gamma_5) \psi_s^\beta \bar{\Psi}_2^\beta (1+\gamma_5) \psi_t^\alpha \} \rangle_{\text{color}} \end{aligned} \quad (42)$$

where "colour" means the average over $SU(3)_c$. The $SU(2)$ average has already been taken into account. The numerical constant C is

$$C = \frac{\pi^{18}}{72 \cdot 24} \Gamma(10) \exp\left(\frac{5}{4} - \frac{3}{2} \ln 2\right) = 2.3 \times 10^{11} \quad (43)$$

The $SU(5)$ unification gives the coefficient $\sim 10^{-153} \text{ GeV}^{-2}$, which is much smaller than $10^{-30} \text{ GeV}^{-2}$ allowed by the stability of the proton.

If we take $\alpha_{\text{GUT}} \approx 0.15$ which may be realized in extended GUTs, we have a condition on M_X from the requirement that the coefficient be less than $10^{-30} \text{ GeV}^{-2}$. For $m \approx 10 \text{ GeV}$, we obtain $M > 3.4 \times 10^4 \text{ GeV}$. Even in this case, the bound is not very useful.

If we take $\alpha_{\text{GUT}} \approx 1$, for $m \approx 10 \text{ GeV}$ we obtain $M > 1.3 \times 10^7 \text{ GeV}$.

Finally, let us look at the 't Hooft's determinant term in Eq. (42) more "intuitively". This is a chirality flipping interaction. Let us rewrite

Eq. (27) in terms of two left-handed and two right-handed multiplets

$$\begin{pmatrix} \bar{u}_2 \\ u^1 \end{pmatrix}_L \xrightarrow{\quad} \begin{pmatrix} \bar{u}_2 \\ u^1 \end{pmatrix}_R ; \quad \begin{pmatrix} d^3 \\ e^+ \end{pmatrix}_L \xrightarrow{\quad} \begin{pmatrix} d^3 \\ e^+ \end{pmatrix}_R \quad (44)$$

The 't Hooft interaction in Eq. (42) by a Hopf instanton changes left-handed fields to right-handed ones, as shown by dashed arrows in Eq. (44). The anti-instanton changes the right-handed fields to left-handed ones. The four-fermion operator (42) is schematically shown in Fig. 1. This gives the process

$$p \longrightarrow e^+ + \text{hadrons} \quad (45)$$

Let us compare this process with the one discussed by 't Hooft²⁾. The 't Hooft process gives $\Delta B = 2, 4$, etc. while the Hopf instanton gives $\Delta B = 1$, etc.

Acknowledgements

We thank Dimitri Nanopoulos for his interest and a careful reading of the manuscript. We also thank Prof. Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at ICTP, Trieste, where the work was initiated.

References

1. H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438;
J.C. Pati and A. Salam, Phys. Rev. Lett. 31 (1973) 661; Phys. Rev. D8
(1973) 1240; Phys. Rev. D10 (1974) 275;
P. Langacker, Phys. Rep. 72C (1981) 185.
2. G. 't Hooft, Phys. Rev. Lett. 37 (1976) 8.
3. V.A. Rubakov, Nucl. Phys. B203 (1982) 311; Pis'ma Zh. Eksp. Teor. Fiz. 33
(1981) 658 [JETP Lett. 33 (1981) 644];
C.G. Callan, Phys. Rev. D26 (1982) 2058; D25 (1982) 2141.
4. R.M. Bionta et al., Phys. Rev. Lett. 51 (1983) 27;
S. Errede et al., Phys. Rev. Lett. 51 (1983) 245.
5. P. Langacker, G. Segre and A. Weldon, Phys. Lett. 73B (1978) 87;
J.E. Kim and H.S. Song, Phys. Rev. D22 (1980) 1753.
6. A.D. Sakharov, Zh. Eksp. Teor. Fiz. Pis'ma Red. 5 (1967) 32 [JETP Lett. 5
(1967) 24];
M. Yoshimura, Phys. Rev. Lett. 41 (1978) 281; 42 (1979) 746(E);
S. Barr, G. Segre and A. Weldon, Phys. Rev. D20 (1979) 2494;
D.V. Nanopoulos and S. Weinberg, Phys. Rev. D20 (1979) 2484.
7. E.N. Parker, Astrophys. J. 160 (1970) 383;
M.S. Turner, E.N. Parker, and T.J. Bogdan, Phys. Rev. D26 (1982) 1296.
8. S.L. Glashow, Nucl. Phys. 22 (1961) 579;
S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264;
A. Salam, in Nobel Symposium No. 8, ed. by N. Svartholm (Almqvist and
Wiksell, Stockholm, 1969) p. 367.
9. A.A. Belavin, A.M. Polyakov, A.S. Schwartz, and Y.S. Tyupkin, Phys. Lett.
59B (1975) 85.
10. J.E. Kim, Phys. Rev. Lett. 45 (1980) 1916; Phys. Rev. D23 (1981) 2706.
11. G. Woo, J. Math. Phys. 18 (1977) 1756 and references therein.
12. H. Georgi and S.L. Glashow, Phys. Rev. Lett. 28 (1972) 1494.
13. G. 't Hooft, Nucl. Phys. B79 (1974) 276;
A.M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. 20 (1974) 430 [JETP Lett. 20
(1974) 194].
14. C.P. Dokos and T.N. Tomaras, Phys. Rev. D21 (1980) 2940;
M. Daniel, G. Lazarides and Q. Shafi, Nucl. Phys. B170 (1980) 156;
K. Choi and J.E. Kim, Seoul National Univ. Preprint SNUHE 83/02 (1983).
15. G. 't Hooft, Phys. Rev. D14 (1976) 3432.
16. C. Bernard, Phys. Rev. D19 (1979) 3013.

Figure 1

The 't Hooft non-local interaction
for $p \rightarrow e^+ \dots$ decay