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HOW TO SAVE MINIMAL SU(5)

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ABSTRACT

The minimal SU(5) model with the canonical particle content is considered. It is argued that non-renormalizable terms with a coupling constant governed by a mass M larger than the unification mass M_X must be introduced. Making a symmetry assumption, we show that these non-renormalizable terms naturally produce strong mixing between fermions belonging to different generations. As a result, the following conclusions are derived:

- (i) The proton lifetime is increased by at least three orders of magnitude with the dominant decay modes being $p \rightarrow K^+ \bar{\nu}$ or $p \rightarrow K^0 e^+$.
- (ii) The difficulties with fermion masses in canonical SU(5) can be resolved.
- (iii) All Yukawa coupling constants can be made of the same order of magnitude, in contrast to canonical SU(5).

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1. - Minimal $SU(5)$ ¹⁾ is an attractive model. It unifies electroweak and strong interactions within the group of the smallest possible rank and with the minimal number of structures (fields). It predicts unambiguously $\sin^2\theta_W$ and this prediction is in excellent agreement with the experimental value. It would be strange if it happened just by accident.

However, minimal $SU(5)$ is in serious disagreement with proton decay experiments. The predicted upper limit $\tau(p \rightarrow e^+\pi^0) < 0.8 \cdot 10^{31}$ yr [Ref. 2)] [or according to Ref. 3) even an order of magnitude less] is considerably smaller than the experimental lower limit $\tau(p \rightarrow e^+\pi^0) > 2 \cdot 10^{32}$ yr⁴⁾.

All the previously suggested ways to increase the proton lifetime⁵⁾ within the $SU(5)$ model involve new structures: a Higgs 45-plet, new fermion multiplet(s), etc. In other words, in these attempts, the minimal $SU(5)$ model is replaced by an extended version. Here we explore how the proton lifetime may be increased within the minimal $SU(5)$.

We begin with a definition of minimal $SU(5)$. This is the $SU(5)$ model in which the particle content at $M < M_X$ (M_X is the unification mass) is described by 24 gauge bosons, three generations of fermions, each containing a 5-plet and a 10-plet, and a 5-plet and a 24-plet of Higgs fields. Undoubtedly, this model must be considered as a low-energy limit of some model with extended symmetry and with new fields of masses $M > M_X$. Our aim is to show that the relics of this extended model in the low-energy domain, taken in the form of non-renormalizable terms^{*)} with definite symmetry, resolve the difficulties of minimal $SU(5)$ with proton decay and fermion masses.

*) Non-renormalizable terms in minimal $SU(5)$ were suggested by Ellis and Gaillard⁶⁾. We shall not discuss here the nature of non-renormalizable terms; it can be assumed that they are produced by renormalizable interactions at $s > M_X^2$.

Our model is the following one. At $M < M_X$, the particle content is that of minimal SU(5). The general structure of the non-renormalizable terms we consider is

$$\frac{\hat{f}_{\alpha\beta}}{M_n} \chi_\alpha^T C \chi_\beta \Phi \varphi, \quad \frac{f_{\alpha\beta}}{M_n} \bar{\chi}_\alpha \psi_\beta \Phi \varphi \quad (1)$$

where α and β are generation indices [SU(5) indices are omitted], $\hat{f}_{\alpha\beta}$ and $f_{\alpha\beta}$ are dimensionless coupling constants, χ and ψ are the 10-plet and the 5-plet of fermions respectively, Φ and φ are the Higgs 24-plet and 5-plet respectively and $M_n > M_X$ is the mass scale of the non-renormalizable interactions considered here.

The crucial point of the model is the assumption that non-renormalizable terms (1) respect some additional symmetry, while all usual SU(5) interactions (and Yukawa interactions in particular) do not know about it. We describe this symmetry by conservation in the interactions (1) of "generation charge" Q_μ ($\mu = 1, 2, 3$ is the generation index), for which the prescription is the following:

$$Q_1 = +1, \quad Q_2 = 0, \quad Q_3 = -1, \quad Q_\varphi = 0, \quad Q_\Phi = 0 \quad (2)$$

where Q_φ, Q_Φ are Higgs charges. The SU(5) Yukawa terms produce a small breaking of generation charge conservation.

2. - Let us turn to calculations of fermion masses and fermion mixings. Consider SU(5) Yukawa terms together with non-renormalizable terms (1) in which the conservation of Q_μ is taken into account.

$$\begin{aligned} \mathcal{L}_{up}^{non} = & \frac{1}{M_n} (\chi_\alpha^T)_{ij} C(\chi_\beta)_{kl} \Psi_m \Phi_r^n (\hat{f}_{\alpha\beta} \delta_n^i \in_{rjklm} + \\ & \hat{f}'_{\alpha\beta} \delta_n^m \in_{rijkl}) + \frac{1}{M_n} (\chi_2^T)_{ij} C(\chi_2)_{kl} \Psi_m \Phi_r^n (\hat{f}_{22} \delta_n^i \in_{rjklm} + \\ & f'_{22} \delta_n^m \in_{rijkl}) \end{aligned} \quad (3)$$

$$\mathcal{L}_{up}^{ren} = \hat{h}_{\mu\nu} (\chi_\alpha^T)_{ij} C(\chi_\beta)_{kl} \Psi_m \in_{ijklm} \quad (4)$$

$$\begin{aligned} \mathcal{L}_{down}^{non} = & \frac{1}{M_n} (\bar{\chi}_\mu)^{ij} (\Psi_\nu)_k \Psi_m \Phi_r^n (f_\mu \delta_i^k \delta_j^r \delta_n^m + \\ & f'_\mu \delta_i^r \delta_j^m \delta_n^k) \end{aligned} \quad (5)$$

$$\mathcal{L}_{down}^{ren} = h_{\mu\nu} (\bar{\chi}_\mu)^{ij} (\Psi_\nu)_j \Psi_i \quad (6)$$

where $\mathcal{L}_{up}$ and $\mathcal{L}_{down}$ are the terms relevant to the masses of the up and down quarks respectively, the superscripts "non" and "ren" indicate non-renormalizable and renormalizable interactions, the generation indices α and β run through 1 and 3 only, while μ and ν run through 1,2,3; C is a charge conjugation matrix, $\hat{f}_{\mu\nu}$ and f_μ are dimensionless coupling constants of non-renormalizable interactions in the up and down sectors respectively, $\hat{h}_{\mu\nu}$ and $h_{\mu\nu}$ are Yukawa couplings for the up and down sectors respectively, and i,j,k,l,m,n,r are SU(5) indices.

The interactions (3)-(6) produce the fermion mass matrices through the VEV of Higgs fields $\langle \varphi \rangle_0 = v_5/\sqrt{2}$ and $\langle \Phi \rangle_0 = 1/\sqrt{30} \text{diag}(2V_{24}, 2V_{24}, 2V_{24}, -3V_{24}, -3V_{24})$.

The elements of these mass matrices for up-quarks ($m_{\mu\nu}$), down-quarks ($m_{\mu\nu}$) and charged leptons ($m_{\mu\nu}$) are:

$$\hat{m}_{13} = \frac{V_5 V_{24}}{\sqrt{60} M_n} [2\hat{f}_{13} - 8\hat{f}_{31} + 12(\hat{f}'_{13} + \hat{f}'_{31})] + \frac{4}{\sqrt{2}} (\hat{h}_{13} + \hat{h}_{31}) V_5$$

$$\hat{m}_{31} = \frac{V_5 V_{24}}{\sqrt{60} M_n} [-8\hat{f}_{13} + 2\hat{f}_{31} + 12(\hat{f}'_{13} + \hat{f}'_{31})] + \frac{4}{\sqrt{2}} (\hat{h}_{13} + \hat{h}_{31}) V_5 \quad (7)$$

$$\hat{m}_{22} = \frac{V_5 V_{24}}{\sqrt{60} M_n} [-6\hat{f}_{22} + 12\hat{f}'_{22}] + \frac{4}{\sqrt{2}} \hat{h}_{22} V_5$$

$$m_{\nu\nu} = \frac{V_5 V_{24}}{\sqrt{60} M_n} (-3f_{\nu\nu} + 2f'_{\nu\nu}) + \frac{h_{\nu\nu}}{\sqrt{2}} V_5 \quad (8)$$

$$\tilde{m}_{\nu\nu} = -\frac{3V_5 V_{24}}{\sqrt{60} M_n} (f_{\nu\nu} + f'_{\nu\nu}) + \frac{h_{\nu\nu}}{\sqrt{2}} V_5$$

All other mass terms (with $\mu \neq \nu$) are produced through Yukawa couplings and V_5 :

$$\hat{m}_{\mu\nu} = \frac{4}{\sqrt{2}} (\hat{h}_{\mu\nu} + \hat{h}_{\nu\mu}) V_5 \quad (9)$$

$$\tilde{m}_{\mu\nu} = m_{\mu\nu} = \frac{1}{\sqrt{2}} h_{\mu\nu} V_5$$

From the assumption of weakly-broken conservation of generation charge, one has $h \sim fV_{24}/M$, and hence the mass terms (9) are much smaller than (8).

Now let us turn to the diagonalization of the mass matrices and the calculation of rotation matrices. The main effect consists of the interchange of right components of u and t quarks produced by non-renormalizable terms and described by the matrix

$$\Pi = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad (10)$$

By the fine tuning of the constants $\hat{f}_\alpha$ and $\hat{f}'_\alpha$, one can obtain $m_u \approx \hat{m}_{13} \approx 0$ and $m_t \approx \hat{m}_{31}$. Similarly, by adjusting the constants f_ν and f'_ν in Eq. (7) for $\tilde{m}_{\nu\nu}$, the electron and the τ lepton can also be interchanged. We shall denote this possibility as case 1 and the possibility without $e \leftrightarrow \tau$ interchange as case 2.

The Yukawa terms produce relatively small rotations (of the order of the Kobayashi-Maskawa angles) described by the matrices

$$O = \begin{pmatrix} \cos d_2 & \sin d_2 & 0 \\ -\sin d_2 \cos d_1 & \cos d_1 \cos d_2 & \sin d_1 \\ \sin d_1 \sin d_2 & -\sin d_1 \cos d_2 & \cos d_1 \end{pmatrix} \quad (11)$$

for left up-quarks (O_L^U), right up-quarks (O_R^U), left down-quarks (O_L^D), etc. Three rotation angles α_{1L} , α_{2L} (for left components of down-quarks) and α_{2L} (for left components of up-quarks) can be calculated from the K-M angles. They yield mass terms m_{21} , m_{32} and $\hat{m}_{21}$ of the same order of magnitude (~ 100 MeV). This result allows us to fix our model by choosing all other mass terms (9) to be of the same order of magnitude. Thus we discover that all Yukawa coupling constants may be of comparable magnitude, $h_{\mu\nu} \sim \hat{h}_{\mu\nu} \sim 10^{-4}$.

3. - The physically important parameter of the model is the mass scale M_n of non-renormalizable interactions (3) and (5). It can be estimated from the mass of the t quark

$$m_t \approx \hat{m}_{31} \approx \frac{20}{\sqrt{60}} \frac{V_5 V_{24}}{M} f,$$

where $f = f_3 \approx -f_1$. For $f \sim 1$ and $m_t \approx 30$ GeV, one obtains $M \approx 20 M_X$.

4. - Consider now proton decay. In terms of the fermion mass eigenstates in generation space U (up-quarks), D (down-quarks) and L (charged leptons), the effective four-fermion Lagrangian for proton decay can be written in the following form:

$$\begin{aligned} \mathcal{L} = \frac{4G_x}{\sqrt{2}} \epsilon_{ijk} \left\{ (\bar{U}_{Li}^c \gamma^\mu R_{UU} U_{Lj}) [(\bar{E}_L^c \gamma^\mu R_{ED} D_{Lk}) - \right. \\ \left. (\bar{D}_{Lk}^c \gamma^\mu R_{DE} E_L)] + \right. \\ \left. (\bar{U}_{Li}^c \gamma^\mu R_{UD} D_{Lj}) [(\bar{D}_{Lk}^c \gamma^\mu N_L) - (\bar{E}_L^c \gamma^\mu R_{EU} U_{Lk})] \right\} \end{aligned} \quad (12)$$

where $R_{UU} = O_R^{U+} \Pi O_L^U$, $R_{UD} = O_R^{U+} \Pi O_L^D$, $R_{ED} = O_R^{E+} \Pi O_L^D$, $R_{DE} = O_R^{D+} O_L^E$, $R_{EU} = O_R^{E+} \Pi O_L^U$ and i, j, k are the colour indices. A very general analysis of proton decay suppression by rotation matrices for the case of horizontal symmetry was performed in Ref. 7). In our case, the diquark matrix elements relevant to proton decay are :

$$\begin{aligned} (R_{UU})_{11} &\approx \sin \hat{\alpha}_{2R} \sin \hat{\alpha}_{2L} \approx \frac{\hat{m}_{23}}{m_c} \frac{\sin \theta_1^{KM} \sin \theta_3^{KM}}{\sin \theta_2^{KM} - \sin \theta_3^{KM}} \lesssim 0.01 \\ (R_{UD})_{11} &\approx \sin \alpha_{1L} \sin \alpha_{2L} \approx \sin \theta_1^{KM} \sin \theta_2^{KM} \lesssim 0.01 \\ |(R_{UD})_{12}| &\approx |\sin \alpha_{1L} + \sin \hat{\alpha}_{2R}| \approx \left| \frac{\hat{m}_{23}}{m_c} + \sin \theta_2^{KM} \right| \end{aligned} \quad (13)$$

Here the θ_i^{KM} are K-M angles. Numerical values in Eq. (13) correspond to limits for K-M angles derived in our model. Unless there is an accidental compensation between $\sin \alpha_{1L}$ and $\sin \hat{\alpha}_{2R}$, $|(R_{UD})_{12}| \gtrsim 0.03$.

From Eq. (12), it now follows that in case 1 ($e \leftrightarrow \tau$ interchange), the proton decay is dominated by the $p \rightarrow K^+ \bar{\nu}$ channel. In comparison with $p \rightarrow e^+ \pi^0$ in the canonical version of minimal SU(5), its width is suppressed by $(R_{UD})_{12}^2$ and by a factor of order 0.1 which comes from matrix element calculations. Therefore, the proton lifetime increases by a factor $\sim 10^3 - 10^4$.

If the charged leptons are not interchanged (case 2), the rate for $p \rightarrow K^0 e^+$ is comparable to $p \rightarrow K^+ \bar{\nu}$.

5. - The non-renormalizable terms (3) and (5) produce, through the VEV of Φ , new Yukawa interactions with large coupling constants ($\sim fV_{24}/M$). In contrast to the usual Yukawa interactions, these violate SU(5). These interactions induce proton decay through the exchange of coloured Higgs bosons. The effective four-fermion Lagrangian, keeping only the most important terms, is

$$\mathcal{L} = \frac{2}{9} \frac{g^2}{m_h^2} \left(\frac{m_t}{m_W} \right)^2 (U_L^T \bar{R}_{UD} D_L) \left[(U_R^T \bar{R}_{UE} E_R) + \frac{1}{2} \frac{m_\tau}{m_t} (N_L^T R_{ND} D_L) \right] \quad (14)$$

where $\bar{R}_{UD} \approx \Pi' M_{KM}$ and $(\bar{R}_{ND})_{31} \approx 1$, $(\bar{R}_{UE})_{11} \approx -\sin \tilde{\alpha}_{1R} \sin \tilde{\alpha}_{2R}$, $(R_{UE})_{12} \approx \sin \tilde{\alpha}_{1R}$ for case 1 ($e \leftrightarrow \tau$ interchange), while $(R_{ND})_{11} \approx 0$ and $(\bar{R}_{UE})_{11} \approx 1$ for case 2, and

$$\Pi' = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

The elements of the matrix $\bar{R}_{UD}$ relevant to proton decay are

$$(\bar{R}_{UD})_{11} \approx \sin \theta_1^{KM} \sin \theta_2^{KM} \quad (15)$$

$$(R_{UD})_{12} \approx \sin \theta_2^{KM} - \sin \theta_3^{KM}$$

From (14) and (15), one concludes that if the charged leptons are interchanged, the main contribution to proton decay comes from the channels $p \rightarrow K^+ \bar{\nu}$, $p \rightarrow K^0 e^+$, and $p \rightarrow K^0 \mu^+$, otherwise the case 2 $p \rightarrow K^0 e^+$ dominates. The ratio of the width of Higgs mediated decay to that of gauge boson mediated decay in the case 1 is

$$\frac{\Gamma_H}{\Gamma_X} \approx \frac{1}{81} \left(\frac{m_t m_\tau}{m_W^2} \right)^2 \left(\frac{m_X}{m_H} \right)^4$$

and therefore the Higgs mediated decay dominates if $M_H/M_X \lesssim 0.02$.

6. - Conclusions

We have considered minimal SU(5) defined as a model with SU(5) symmetry and the canonical content of fields. It is argued that non-renormalizable interactions between these fields exist as the low energy limit of new renormalizable interactions at $s > M_X^2$. We assume that these interactions conserve a "generation charge" which reflects some new symmetry of renormalizable interactions at $s > M_W^2$. The symmetry is softly broken by SU(5) Yukawa interactions. The non-renormalizable interactions under these assumptions produce the strong mixings of fermions, described by the interchange of fermion states in different generations, while the usual SU(5) Yukawa interactions result in relatively small mixings (rotations) of the states. As a result, the proton lifetime is increased by three to four orders of magnitude. For the principle variant of our model, the dominant decay mode is $p \rightarrow K^+ \bar{\nu}$; in other variants, $p \rightarrow K^+ \bar{\nu}$ and/or $p \rightarrow K^0 e^+$, $p \rightarrow K^0 \mu^+$ can dominate.

All Yukawa coupling constants in the model are of the same order ($h \sim 10^{-4}$), and all fermion masses can be explained. The mass scale of non-renormalizable interactions must be $M_n \approx 20 M_X$.

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