HAMILTONIAN MONTE CARLO STUDY OF 1+1 DIMENSIONAL MODELS
WITH RESTRICTED SUPERSYMMETRY ON THE LATTICE

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ABSTRACT

Lattice versions with restricted supersymmetry of simple 1+1 dimensional supersymmetric models are numerically studied using a local Hamiltonian Monte Carlo method. The pattern of supersymmetry breaking closely follows the expectations of Bartels and Bronzan obtained in an alternative lattice formulation.

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There are now many attempts to construct grand unified theories on the basis of supersymmetry¹⁾. In order to lift in such theories the mass degeneracy between fermions and bosons at low energy, supersymmetry should be broken. Mechanisms for the breaking of supersymmetry are therefore of interest. We address ourselves here to 1+1 dimensional supersymmetric models. Several examples for such breaking mechanisms which are relevant in low space-time dimensions have been discussed recently²⁾⁻⁵⁾.

The lattice regularization is a method for the non-perturbative study of field theoretic models. However, serious problems arise when supersymmetric models are to be formulated on the lattice⁶⁾. The continuum supersymmetric algebra does not exist on the lattice, since it contains the momenta as continuous translation operators. Nevertheless, several attempts on the usual hypercubical lattice have been made to avoid these difficulties⁷⁾⁻¹⁰⁾. However, none of these seems to be completely satisfactory. In Refs. 7) and 8), it was proposed to work in the Hamiltonian lattice formulation with continuous time and discretized space and to use an algebra involving only the Hamiltonian. This algebra is restricted compared to the full one, defined in the continuum. Here we will use this method for a lattice Monte Carlo study of supersymmetric models in 1+1 dimensions.

The behaviour of the energy expectation value can be used to study whether supersymmetry is present or broken. While it might be difficult to verify by Monte Carlo calculations beyond a certain accuracy the presence of unbroken supersymmetry ($\langle E \rangle = 0$), one should be able to verify the breaking of supersymmetry ($\langle E \rangle > 0$) if $\langle E \rangle$ becomes big enough.

We use in our study the local Hamiltonian Monte Carlo method¹¹⁾⁻¹⁴⁾ which is efficient to study field theoretic models in 1+1 dimensions on the lattice involving fermions, gauge bosons and scalars. Since this method uses the Hamiltonian formulation, it is suitable to study lattice models with the restricted supersymmetry algebra.

We start with the $N = 1$ supersymmetric two-dimensional version of the Wess-Zumino model defined by the Lagrangian^{15),16)}

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) + \frac{i}{2} \bar{\psi} \not{\partial} \psi - \frac{1}{2} (V(\phi))^2 - \frac{1}{2} V'(\phi) \bar{\psi} \psi \quad (1)$$

where $V(\phi)$ is an arbitrary function of the real scalar field ϕ and ψ is a Majorana spinor. From the conserved supersymmetric current Q_α^μ (μ -Lorentz index, α -spinor index)

$$Q_\alpha^\mu = [(\not{\partial} \phi + i V(\phi)) \gamma^\mu \psi]_\alpha \quad (2)$$

the fermionic supercharge Q_α is obtained as

$$Q_\alpha = \int d^3x Q_\alpha^0 \quad (3)$$

The standard $N = 1$ supersymmetric algebra takes the form

$$\{Q_\alpha, Q_\beta\} = 2 (\not{P} C)_{\alpha\beta} \quad (4)$$

where C is the charge conjugation matrix and P_μ the two-momentum. Throughout this paper, we use the notations

$$g_{00} = -g_{11} = 1, \quad P^\mu = (H, \mathbf{P}), \quad P_\mu = (H, -\mathbf{P}) \quad (5)$$

and a Majorana basis

$$\gamma^0 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad C = \gamma^0, \quad \gamma^3 = \gamma^0 \gamma^1 \quad (6)$$

so that the spinor components are real. From Eq. (4) we obtain the algebra

$$Q_1^2 = H \quad (7a)$$

$$Q_2^2 = H, \quad \{Q_1, Q_2\} = 2P. \quad (7b)$$

If a supersymmetrically invariant state exists for which $Q_\alpha |vac\rangle = 0$ it is the true ground state and supersymmetry is not spontaneously broken. For models without topological configurations this means that the ground state energy should be zero

$$\langle E \rangle = \langle vac | H | vac \rangle = 0. \quad (8)$$

Therefore, a positive ground state energy is a signal for the spontaneous breaking of supersymmetry.

Following Ref. 8), we shall keep on the lattice only relation (7a), and expect (7b) to be recovered in the continuum.

Our lattice Hamiltonian is defined by the restricted supersymmetry algebra relation (7a) where the supercharge component operator Q_1 has to be discretized. Introducing a spatial lattice with lattice spacing a and using the basis (6) we get

$$Q_1 = \frac{1}{\sqrt{a}} \sum_n \left\{ \pi_n \psi_n^1 - a \Delta \phi_n \psi_n^2 - a V(\phi_n) \psi_n^2 \right\}. \quad (9)$$

π_n is the momentum operator conjugate to the Higgs field ϕ_n at site n , $\psi_n^{1,2}$ are the real Majorana fermion components, the sum runs over all lattice sites. The only non-vanishing discretized commutation and anticommutation relations are

$$[\pi_n, \phi_m] = -i \delta_{n,m}, \quad \{\psi_n^\alpha, \psi_m^\beta\} = \delta^{\alpha\beta} \delta_{n,m} \quad (10)$$

Choosing the difference operator $\Delta\phi_n$ over next-to-nearest or nearest neighbour sites, one obtains doubling of bosonic and fermionic degrees of freedom⁷⁾ in the continuum or not. Since both choices do not destroy the restricted supersymmetry properties we take

$$\Delta\phi_n = \frac{1}{2a} (\phi_{n+1} - \phi_{n-1}) . \quad (11)$$

Therefore the naive continuum limit of our model will not be (1) but our lattice model still fulfils the restricted lattice supersymmetric algebra (7a). The form (11) has the advantage that the local Hamiltonian method can be used straightforwardly.

Using (7a), (9)-(11) we obtain the lattice Hamiltonian

$$H = Q_1^2 = \sum_n \left\{ \frac{1}{2a} \pi_n^2 + \frac{a}{2} \left(\frac{\phi_{n+1} - \phi_{n-1}}{2a} \right)^2 + \frac{a}{2} (V(\phi_n))^2 + \right. \\ \left. i V'(\phi_n) \psi_n^1 \psi_n^2 - i \psi_n^1 \frac{\psi_{n+1}^2 - \psi_{n-1}^2}{2a} + a V(\phi_n) \frac{\phi_{n+1} - \phi_{n-1}}{2a} \right\} \quad (12)$$

The last term in (12) would not be obtained discretizing the Hamiltonian directly instead of the supercharge. To get this term in the continuum formulation, central charges have to be considered. For a function $V(\phi)$, allowing soliton formation, one has to include into the algebra (4) topological quantum numbers as central charges T ¹⁷⁾

$$\{Q_\alpha, Q_\beta\} = 2(PC)_{\alpha\beta} + 2i(\gamma^3 C')_{\alpha\beta} T . \quad (13)$$

The operator T is defined as a surface contribution

$$T = \int dx \frac{\partial \phi}{\partial x} V(\phi) \quad (14)$$

which vanishes in the absence of topological configurations. The last term in (12) is just the lattice version of T .

Now we introduce fermion creation and annihilation operators χ_n^+ and χ_n at the lattice sites. They have to be of such a form that the reality of the components ϕ_n^1 and ϕ_n^2 is respected and H is suitable for the local Hamiltonian method. With

$$\psi_n^1 = \frac{1-i(-1)^n}{2i^n} (\chi_n^+ + i\chi_n), \quad \psi_n^2 = \frac{1+i(-1)^n}{2i^n} (\chi_n^+ - i\chi_n) \quad (15)$$

we obtain from (9) the anticommutators

$$\{\chi_n, \chi_m^+\} = \delta_{nm}, \quad \{\chi_n, \chi_m\} = \{\chi_n^+, \chi_m^+\} = 0 \quad (16)$$

and derive the lattice Hamiltonian

$$H = \sum_n \left\{ \frac{1}{2a} \pi_n^2 + \frac{a}{2} \left(\frac{\phi_{n+1} - \phi_{n-1}}{2a} \right)^2 + \frac{a}{2} (V(\phi_n))^2 + (-1)^n V'(\phi_n) (\chi_n^+ \chi_n - \frac{1}{2}) + \right. \\ \left. + \frac{1}{2a} (\chi_n^+ \chi_{n+1} + \chi_{n+1}^+ \chi_n) + a V(\phi_n) \frac{\phi_{n+1} - \phi_{n-1}}{2a} \right\}. \quad (17)$$

A detailed discussion of the Monte Carlo algorithm can be found in Refs. 11)-14). The energy expectation value has the form

$$\langle E \rangle = \text{Tr} (H e^{-\beta H}) / \text{Tr} (e^{-\beta H}) \quad (18)$$

where the inverse temperature β should be sufficiently large. The traces are calculated by dividing the imaginary time interval into $2N$ subintervals ($N\Delta\tau = \beta$) and splitting the Hamiltonian into two pieces $H = H_1 + H_2$

$$\text{Tr} (e^{-\beta H}) = \sum_{t_1, \dots, t_{2N}} \langle t_1 | e^{-\Delta\tau H_1} | t_{2N} \rangle \langle t_{2N} | e^{-\Delta\tau H_2} | t_{2N-1} \rangle \dots \\ \dots \langle t_3 | e^{-\Delta\tau H_1} | t_2 \rangle \langle t_2 | e^{-\Delta\tau H_2} | t_1 \rangle + O(\Delta\tau^3). \quad (19)$$

The sum runs over complete sets of states at each time slice. The matrix elements in (19) become local by a suitable choice of the decomposition of H into H_1 and H_2 and of the intermediate states. As pointed out in Ref. 13), a different break-up of the Hamiltonian for fermions and Higgs bosons has to be used.

The fermions are described by occupation numbers, the Higgs fields are given by their actual values on the sites. One Monte Carlo iteration is performed by updating the fermions in one sweep through the lattice holding the Higgs fields fixed followed by a sweep updating the Higgs fields keeping the fermions fixed. As starting lattice, we use a half-filled fermion band¹²⁾ with occupied odd sites and all Higgs fields equal to zero.

For most of our calculations, we perform 5000 to 20 000 iterations and measure after each fifth iteration. The lattices used for measurements consist of 12 spatial lattice sites and 100 or 50 sites in time direction keeping the inverse temperature fixed to $\beta = 3.5$ ($\Delta\tau/a = 0.07$ or 0.14). Varying β , $\Delta\tau$ and the lattice size, we checked that the energy expectation values are essentially unchanged for the parameter range of function $V(\phi)$ discussed below.

We study models with odd and even functions $V(\phi)$. They determine the effective potential $V_{\text{eff}}(\phi) = \frac{1}{2}[V(\phi)]^2$. Let us first discuss the model with the odd function of ϕ

$$V(\phi) = \lambda \phi^3. \quad (20)$$

The results for the total energy expectation value $\langle E \rangle$ and the most important contributions to the energy are plotted in Fig. 1 as function of the parameter λ in (20). The Higgs kinetic energy refers to the first two terms of Eq. (17), the Higgs potential energy to the third, the Higgs-fermion coupling energy to the fourth and the fermion kinetic energy to the fifth term of the same equation. The last term in (17) related to the surface term (14) and introduced to preserve the restricted supersymmetric algebra fluctuates around zero and can be numerically neglected within our accuracy. This is in agreement with the form of the effective potential corresponding to (20) which does not permit the formation of solitons.

The vanishing of the total lattice energy $\langle E \rangle$ in Fig. 1 indicates within our statistical errors that we obtain unbroken supersymmetry with the restricted lattice algebra (7a). This agrees with strong coupling calculations¹⁸⁾ for such models. There are no mechanisms known which would break supersymmetry in the continuum model. Considering the size of the different non-vanishing contributions to the energy expectation value in Fig. 1, it is quite remarkable how these cancellations work.

Next, we discuss the model with the even function

$$V(\phi) = \lambda \phi^2 + \alpha. \quad (21)$$

One expects at the tree level breaking of supersymmetry for $\alpha > 0$ and no breaking for $\alpha \leq 0$. In Fig. 2a, the total energy expectation values $\langle E \rangle$ obtained in the Monte Carlo calculation are given as function of the parameter α in (21). We find non-vanishing energies $\langle E \rangle$ also for $\alpha = 0$ and not too large negative values of α . For another even function $V(\phi) = 2\phi^4$ the energy is greater than zero as well.

Also for a function similar to (21)

$$V(\phi) = \lambda \phi^2 + m\phi \quad (22)$$

supersymmetry is unbroken for all λ/m in the tree approximation. Bartels and Bronzan¹⁰⁾ find breaking of supersymmetry on the one-loop level of the effective potential and by strong coupling calculations for large λ/m in the range

$$\lambda/m \geq (\lambda/m)_{crit}. \quad (23)$$

In their study, they used a lattice formulation of the Wess-Zumino model which preserves the full superalgebra by introducing derivatives with long range correlations and non-local interaction terms. A similar spontaneous breakdown of supersymmetry appears in the $d+1$ dimensional continuous $O(N)$ Wess-Zumino model^{4),5)}. Our Monte Carlo calculations for the model (22) given in Fig. 2b seems to confirm these predictions and clearly indicate supersymmetry breaking.

The functions (21) and (22) differ only by a shift in ϕ , therefore, the calculations in Ref. 10) make also the breaking of supersymmetry for the model (21) plausible.

We would like to remark that the model (21) for $\alpha < 0$ and the model (22) for $m \neq 0$ correspond to double well effective potentials and allow therefore the formation of solitons. Indeed, we are able to observe these solitons in our lattice calculations¹⁴⁾. For topologically non-trivial states on the lattice, the surface term referring to the last part in (17), becomes clearly non-zero indicating the presence of solitons as well.

The pattern of supersymmetry breaking which we find in 1+1 dimensions is similar to the behaviour of supersymmetric quantum mechanics ($d = 0+1$)³⁾, where breaking is expected only for functions $V(\phi)$ with an even number of zeros.

We conclude, the Monte Carlo calculation of our lattice models seems to confirm qualitatively all features expected by a completely different lattice formulation respecting the naive continuum limit and this in spite of our doubling of fields. We stress also, that it is not formally proven that the lattice model with restricted supersymmetry will converge to the full supersymmetric model in the continuum.

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REFERENCES

- 1) J. Ellis, Talk at the International Symposium on Lepton and Photon Interactions, Cornell, CERN preprint TH.3718 (1983).
- 2) E. Witten, Nucl. Phys. B185 (1981) 513;
I. Affleck, J. Harvey and E. Witten, Nucl. Phys. B206 (1982) 413.
- 3) P. Salomonson and J.W. van Holten, Nucl. Phys. B196 (1982) 50.
- 4) A.C. Davis, P. Salomonson and J.W. van Holten, Nucl. Phys. B208 (1982) 48.
- 5) K. Higashima and T. Uematsu, Phys. Lett. 123B (1983) 209.
- 6) P.H. Dondi and H. Nicolai, Nuovo Cimento 41A (1977) 1;
T. Banks and P. Windey, Nucl. Phys. B198 (1982) 226.
- 7) V. Rittenberg and S. Yankielowicz, CERN preprint TH.3263 (1982).
- 8) S. Elitzur, E. Rabinovici and A. Schwimmer, Phys. Lett. 119B (1982) 165.
- 9) S. Cecotti and L. Girardello, Nucl. Phys. B226 (1983) 1;
S. Elitzur and A. Schwimmer, Nucl. Phys. B226 (1983) 109;
N. Sakai and M. Sakamoto, Harvard preprint HUTP-83/A013 (1983);
H. Aratyn and A.H. Zimerman, DESY preprint DESY 83-075 (1983).
- 10) J. Bartels and G. Kramer, Z. Phys. C20 (1983) 159;
J. Bartels and J.B. Bronzan, Phys. Rev. D28 (1983) 818;
- 11) I.E. Hirsch, D. Scalapino, R. Sugar and R. Blankenbecler, Phys. Rev. B26 (1982) 5033.
- 12) J. Ranft and A. Schiller, Phys. Lett. 122B (1983) 403;
A. Schiller and J. Ranft, Nucl. Phys. B225 [FS9] (1983) 204;
J. Ranft, CERN preprint TH.3721 (1983), to be published in Phys. Lett. B.
- 13) J. Ranft and A. Schiller, CERN preprint TH.3752 (1983).
- 14) J. Ranft and A. Schiller, to be published.
- 15) J. Wess and B. Zumino, Nucl. Phys. B78 (1974) 1.
- 16) S. Ferrara, Lett. Nuov. Cimento 13 (1975) 629.
P. Di Vecchia and S. Ferrara, Nucl. Phys. B130 (1977) 93;
J. Hruby, Nucl. Phys. B131 (1977) 275; *ibid.* B162 (1980) 449.
- 17) E. Witten and D. Olive, Phys. Lett. 78B (1978) 97.
- 18) C.M. Bender, F. Cooper and A. Das, Phys. Rev. Lett. 50 (1983) 397.

FIGURE CAPTIONS

- Fig. 1 : The total energy expectation value as well as important contributions to the energy as function of the parameter λ for the odd function of ϕ $V(\phi) = \lambda\phi^3$.
- Fig. 2a : The energy expectation value for a model with $V(\phi) = 2\phi^2 + \alpha$ as function of the parameter α .
- Fig. 2b : The energy expectation value for a model with $V(\phi) = 2\phi^2 + m\phi$ as function of the parameter m .

Fig. 1

Fig. 2a

Fig. 2b