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We investigate the structure of phenomenological supergravity models which permit the hierarchy problem to be “solved” in the sense that $m_{3/2}$ and m_W are determined dynamically to be $\exp[-O(1)/\alpha] \times m_P$. Such models must have a flat hidden sector potential, which is only possible if the theory has an underlying $SU(1,1)$ invariance. Flat $SU(1,1)$ theories necessarily have a zero cosmological constant and the hidden sector is an Einstein space with $R_{zz} = \frac{2}{3} G_{zz}$. The $SU(1,1)$ invariance is necessarily broken down to $U(1)$ by the gravitino mass. If $m_{3/2}$ is the only source of $SU(1,1)$ breaking then the tree-level gaugino masses are small and $A = \frac{3}{2}$, while values of A up to 3 and non-zero gaugino masses are possible if other sources of $SU(1,1)$ breaking are tolerated. Yukawa couplings may scale as some power of $m_{3/2}/m_P$ in these models where $m_{3/2}$ is generated dynamically, which may explain the hierarchy of Higgs-fermion Yukawa couplings: $m_t/m_W = O(m_W/m_P)^{1>0}$? These models also permit the spontaneous violation of CP in the Yukawa coupling matrix. Numerical studies yield $20 \text{ GeV} < m_t < 100 \text{ GeV}$ in these phenomenological $SU(1,1)$ supergravity models. Speculations are presented about their relation to a fundamental theory based on extended supergravity.

1. INTRODUCTION

We seek a “no-scale” unified theory in which all other mass scales in physics are numerically related to a primordial scale of order the Planck mass m_P . We envisage the existence of a fundamental theory at m_P which includes gravitation. This fundamental theory is supposed to yield a phenomenological low energy theory effective at energies $E \ll m_P$. If this effective theory is to have a dynamic range extending much below m_P , it must be renormalizable and hence take the form of a spontaneously broken gauge theory. We would like this effective theory to contain no intrinsic mass parameters—the “no-scale” ansatz—but would hope to derive m_W and any other small scales dynamically. A natural strategy is to exploit the logarithmically slow variation of couplings in a renormalizable gauge theory to obtain a hierarchical ratio

$$m_W/m_P = \exp\left(-\frac{O(1)}{\alpha}\right) : \alpha = O(\alpha_{em}) \quad (1.1)$$

This dimensional transmutation strategy has been pursued^{1,2)} in the absence of supersymmetry, but then one can object that scalar boson masses are intrinsically unstable, and that it is technically unnatural to postulate the existence of light scalars. Supersymmetry³⁾ has the capability to protect the masses of these light scalars by virtue of no-renormalization theorems⁴⁾ and the absence of quadratic divergences, and therefore seems to be a desirable ingredient in the construction of “no-scale” theories. Phenomenology requires supersymmetry to be softly broken in the effective low energy theory, and we presume that this falls out from the spontaneous breakdown of an initially supersymmetric theory. It has been shown^{5,6)} how to implement dimensional transmutation by exploiting the logarithmic variation of supersymmetry breaking mass parameters $\tilde{m}$, even though the Yukawa and multiple scalar couplings are not renormalized in a supersymmetric theory. In this way one can obtain a result like (1.1) for any arbitrary choice of supersymmetry breaking mass scale $\tilde{m} \lesssim m_W$. This however leaves us with the problem of calculating $\tilde{m}$, which was put by hand into previous supersymmetric dimensional transmutation models.

We would therefore like to determine $\tilde{m}$ dynamically via a formula analogous to (1.1), but we know this is impossible in a globally supersymmetric theory. This is because global supersymmetry cannot be broken by radiative corrections, but must be broken already at the tree level. Thus we turn to a spontaneously broken locally supersymmetric supergravity theory where $\tilde{m} = O(m_{3/2})$, for which no-renormalization theorems⁴⁾ do not forbid the dynamical determination of the supersymmetry breaking scale. Supergravity theories are in any case the only framework for

combining supersymmetry with gravitation, and the most plausible candidates for an eventual unification of all particle interactions are the extended supergravity theories, particularly the $N = 8$ theory^{7,8,9}). However, our effective low energy theory must have a chiral fermion spectrum and presumably only possesses simple $N = 1$ supersymmetry. Our primary goal in this paper is to elucidate the conditions necessary for “solving” the hierarchy problem by using¹⁰) dimensional transmutation to determine $m_{3/2}$ as well as m_w (1.1):

$$m_{3/2}/m_p = \exp\left(-\frac{O(1)}{\alpha}\right) : \alpha = O(\alpha_{em}) \quad (1.2)$$

A secondary goal is to cast light on the relationship between our effective $N = 1$ supergravity and the possibly fundamental $N > 1$ supergravity theory, by identifying features of the conjectured fundamental theory that may usefully be extracted as desiderata for the low energy effective theory.

It was recently shown¹¹) how to write down a tree-level $N = 1$ supergravity theory with a flat scalar potential and an undetermined gravitino mass. This theory has¹¹) a global non-compact $SU(1,1)$ symmetry, and its scalar sector closely resembles that of the $N = 4$ extended supergravity¹²). Subsequently it was demonstrated¹⁰) that one could determine $m_{3/2}$ and m_w dynamically [(1.1), (1.2)] by $SU(3)_C \times SU(2)_L \times U(1)_Y$ radiative corrections in the context of such a “flat” theory. In section 2 of this paper we argue that an approximate global $SU(1,1)$ symmetry may not only be *sufficient*, but also *necessary* for such a “solution” to the hierarchy problem. We show that such a “solution” requires the existence of a “hidden” sector as well as the observable $SU(3)_C \times SU(2)_L \times U(1)_Y$ fields, and start section 3 with a clarification of the connection between the “hidden” and “observable” sectors. We discuss in particular the phenomenological requirement that at least some chiral Yukawa couplings have a non-trivial limit when the dynamically determined $m_{3/2} \rightarrow 0$. We also raise the interesting possibility that some chiral Yukawa couplings actually vanish in this limit, which may help explain the observed hierarchy of light fermion masses: perhaps

$$m_f/m_w = O(m_w/m_p)^{p_f} : 1 > p_f > 0 \quad (1.3)$$

It is also possible to arrange for spontaneous CP violation in the chiral Yukawa coupling matrix. While non-compact $SU(1,1)$ must be broken, it is attractive to conjecture that it may be broken down to a non-compact $\tilde{U}(1)$ subgroup, in which case the observable sector couplings are more constrained. The conventional supersymmetry breaking parameter A ¹³) takes the value $3/2$, while the

CORRIGENDA TO

PHENOMENOLOGICAL SU(1,1) SUPERGRAVITY

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Please replace the printed text by

p. 4, 2 lines from bottom: $m_w \sim \mu_0$

p. 7, line 8: $n(n+1)/2$ is the dimensionality of a real manifold based on n component fields. In general supergravity models are based on complex manifolds with $n(n+4)/4$ dimensions. These coincide for the case $n = 2$ discussed here.

p. 10, 5 lines from bottom: $\Sigma = -\ln(g/\lambda)$

equation (2.30)

$$3(z+z^*) \rightarrow -3(z+z^*)$$

p. 12, equation (2.40)

$$A_\mu = \frac{3}{2} i \frac{\partial_\mu (z^* - z)}{(z + z^*)}$$

p. 15, line 7: $m_{3/2} = e^{G(z,z^*)/2}$

p. 16, equation (3.10)

$$Q_{zz^*} = \left(\frac{\Phi_z}{G_z} + \frac{\Phi_{z^*}}{G_{z^*}} - \frac{\Phi_{zz^*}}{G_{zz^*}} \right) G_z$$

p. 18, equation (3.18)

$$\frac{15}{2} \rightarrow \frac{15}{4}$$

p. 20, equation (3.31)

$$e^{-\frac{3}{2}(1-p)i\theta} \rightarrow e^{+\frac{3}{2}(1-p)i\theta}$$

equation (3.32)

$$e^{-i(\theta + \frac{\pi}{2})} \rightarrow e^{-i(\theta - \frac{\pi}{2})}$$

p. 21, line 3: $z \rightarrow \frac{1}{2}(m_{3/2})^{-2/3} + i\infty$

p. 22, equation (3.37)

$$A \approx \frac{3}{2} \left(1 + \arccos \Theta \cos\left(\frac{3r+2}{2}\Theta\right) \right) \exp\left(-i \arccos \Theta \sin\left(\frac{3r+2}{2}\Theta\right)\right)$$

equation (3.38)

$$\cos \frac{3r}{2} \Theta = -1$$

p. 24, line 3: $\hat{m}_4 = m_3^2/2m_{3/2}^2$

p. 27, equation (5.5a)

$$+ y_i^* y_i$$

equation (5.7)

$$\left(\frac{m_f}{m_w}\right) \propto g_{H\bar{f}f} \propto \left(\frac{m_w}{m_p}\right)^{\frac{q_i+q_j+q_k+3}{3}}$$

p. 31: Q should be replaced by μ on this page and in fig. 1.

Line 4: $\hat{m}_4 = m_3^2/2m_{3/2}^2$

gaugino mass acquires no contribution from the hidden sector and may be $\ll m_{3/2}$ at the tree level. Section 4 discusses applications of the philosophy outlined above, discussing in particular the phenomenology of a “no-scale” standard model with $A = 3/2$ and contrasting it with previous results¹⁰⁾ obtained with $A = 3$. Finally, in section 5 we speculate about the possible compatibility of our philosophy with the existence of a fundamental extended supergravity theory⁷⁾. All such theories with $N \geq 4$ contain a global non-compact $SU(1,1)$ symmetry, which is however broken if the supergravities are gauged¹⁴⁾. Thus it may not dismay us that even the largest non-compact group $E_{7(7)}$ of the $N = 8$ theory, whose maximal compact subgroup is $SU(8)$, does not contain an $SU(5) \times SU(1,1)$ subgroup. We propose that global $SU(1,1)$ be added to the list of desiderata

$$\begin{array}{ccc}
 (N = 8 \text{ susy}) & \times & SU(8)^{\text{local}} \times E_{7(7)}^{\text{global}} \\
 \downarrow [\text{refs } 9,15] & & \downarrow \quad \downarrow [\text{ref. 15}] \\
 (N = 1 \text{ susy}) & \times & SU(5)^{\text{local}} \times SU(1,1)^{\text{global}}
 \end{array} \quad (1.4)$$

which may ultimately be abstracted from a fundamental $N = 8$ theory.

2. THE HIDDEN SECTOR, FLATNESS AND $SU(1,1)$

Let us first recall some relevant elements of the general structure¹⁶⁾ of $N = 1$ supergravity theories. In addition to the graviton–gravitino and gauge supermultiplets, they contain a set of chiral superfields ϕ^i which are coordinates on a Kähler manifold. The associated Kähler potential $G(\phi^i, \phi_j^*)$ determines couplings through its derivatives

$$G_i \equiv \frac{\partial}{\partial \phi^i} G, \quad G^j \equiv \frac{\partial}{\partial \phi_j^*} G, \quad G_i{}^j \equiv \frac{\partial}{\partial \phi^i} \frac{\partial}{\partial \phi_j^*} G, \text{ etc.} \quad (2.1)$$

Of particular interest to us is the scalar potential

$$V = e^G \left[G_i G_i{}^j G_j - 3 \right] + \frac{1}{2} D^\alpha D_\alpha \quad (2.2)$$

where $G_i{}^j$ is the inverse of the matrix G_i , and

$$D^\alpha \equiv G_i (T^\alpha)^i_j \phi^j \quad (2.3)$$

determines the D term in equation (2.2). The negative term in equation (2.2) enables one^{16,17)} to have local SUSY breaking with a gravitino mass $m_{3/2} = e^{G/2} M$:

$$m_{3/2} = e^{G/2} M \quad : \quad M \equiv \frac{m_P}{\sqrt{8\pi}} \quad (2.4)$$

as well as global SUSY breaking, while maintaining $V = 0$ at the global minimum and hence having a zero cosmological constant.

Included among the chiral superfields ϕ^i are the “observable” fields y^i which make up the supersymmetric standard model: matter fields for quarks and leptons, and two light doublets of Higgs fields. We know phenomenologically that the characteristic scale of this observable sector is determined by Higgs vacuum expectation values to be $O(m_w) = O(10^2)$ GeV. The hierarchy problem is that of understanding why m_w is so much less than M . The gravitino mass should not be much smaller than m_w : if it were, the theory would look globally supersymmetric and be fated to have a non-zero cosmological constant.

The observable fields only make a contribution to the potential $\delta V = O(m_w^4)$, which is not by itself sufficient to cancel the negative term in the potential (2.2) if $e^G = m_{3/2}^2 = O(m_w^2)$. We must therefore invoke the existence of a “hidden” sector of fields z^i which must bear the brunt of the cancellation in the cosmological constant:

$$V(z^i, y^i = 0) = O(m_w^4) = O(m_{3/2}^4) \quad (2.5)$$

The hierarchical ratio of scales $m_w, m_{3/2} \ll m_P$ suggests that radiative corrections to the potential (2.2) cannot be neglected. It is possible to construct models in which m_w and $m_{3/2}$ are fixed by hand to be $O(100)$ GeV, but this is unaesthetic. The “no-scale” hypothesis which we explore here is that m_w and $m_{3/2}$ can in fact be determined by radiative corrections¹⁰⁾.

It is known how to use radiative corrections due to heavy fermion (t quark¹⁸⁾?) couplings to determine m_w by dimensional transmutation^{5,6)}: $m \sim \mu_0$, where μ_0 is the scale at which Higgs mass parameters have evolved sufficiently to permit gauge symmetry breaking:

$$\det m_H^2(\mu_0) = 0 \quad (2.6)$$

given by

$$\frac{m_W}{M} \sim \frac{\mu_0}{M} = \exp\left(-\frac{O(1)}{\alpha_t}\right) : \alpha_t \equiv \frac{g_{H\bar{t}t}^2}{4\pi} \quad (2.7)$$

Our objective in this paper is to analyze the conditions under which the gravitino mass $m_{3/2}$ can also be determined dynamically by radiative corrections, analogously to equation (2.7). To achieve this, we need a theory with local and global supersymmetry breaking possible at the tree level, but with the local SUSY breaking order parameter (2.4) undetermined at the tree level. To achieve this, we need a theory with a flat potential. Flatness here means that the potential must be no larger than the available radiative correction effects, which are $O(m_W^4) = O(m_{3/2}^4) = O(\mu_0^4)$ [see equation (2.5)], whereas the natural order of magnitude of the potential (2.2) is $O(m_{3/2}^2 M^2)$. We will find that we are led necessarily to the flat theories discovered in ref. 11.

Let us first look at the hidden sector z^1 which bears the brunt (2.5) of cancelling the cosmological constant. We simplify to the case of a single hidden field z :

$$V(z, 0) = e^{G(z, z^*)} \left[\frac{G_z G_{z^*}}{G_{zz^*}} - 3 \right] + O(m_{3/2}^2 / \mu_0^2) \quad (2.8)$$

The dynamical determination of $m_{3/2} = e^{G/2}$ is possible if and only if

$$\left[\frac{G_z G_{z^*}}{G_{zz^*}} - 3 \right] = O(\mu_0^2) \sim O(e^G) \quad (2.9)$$

which must be less than about 10^{-32} for sufficiently large field variation. This degree of flatness imposes stringent constraints on the Kähler curvature

$$R_{zz^*} \equiv \partial_z \partial_{z^*} \ln G_{zz^*} \quad (2.10)$$

We have the following mass sum rule^{16,11)} for the gravitino and its two spin-0 partners X and Y :

$$m_x^2 + m_y^2 - 4m_{3/2}^2 = -2e^{\eta} \left(\frac{\zeta_z \zeta_{z^*}}{\zeta_{zz^*}} \right) \left(\frac{R_{zz^*}}{\zeta_{zz^*}} \right) \quad (2.11)$$

Flatness (2.9) requires $m_x^2, m_y^2 = O(m_{3/2}^4/M^2)$, so that substituting equation (2.9) into (2.11) we deduce that

$$R_{zz^*} = \left(\frac{2}{3} + O\left(\frac{m_{3/2}^2}{M^2}\right) \right) \zeta_{zz^*} \quad (2.12)$$

Thus, in order for the hierarchy problem to be “solved” by radiative corrections determining $m_{3/2}$, it is *necessary* that the hidden sector be approximately an Einstein space with curvature $R \approx \frac{2}{3}$:

$$R \equiv \frac{R_{zz^*}}{\zeta_{zz^*}} = \frac{2}{3} + O(10^{-32}) \quad (2.13)$$

It is easy to find¹¹⁾ the hidden sector $G(z, z^*)$ which gives $R = \frac{2}{3}$ automatically, by noticing that

$$V(z) = 9e^{\frac{1}{3}\eta} \zeta_{zz^*}^{-1} \partial_z \partial_{z^*} e^{-\frac{1}{3}\eta} \quad (2.14)$$

which vanishes if

$$\eta = -3 \ln \left(f(z) + f^*(z^*) \right) \quad (2.15)$$

where $f(z)$ is an arbitrary analytic function. In this case

$$\zeta_z = -3 \frac{f_z}{(f+f^*)}, \quad \zeta_{zz^*} = 3 \frac{f_z f_{z^*}}{(f+f^*)^2} \quad (2.16)$$

and hence the Lagrangian for the z field is just the kinetic term

$$\mathcal{L}_{zz^*} \partial_\mu z \partial^\mu z^* = 3 \frac{\partial_\mu f \partial^\mu f^\dagger}{(f + f^\dagger)^2} \quad (2.17)$$

All such theories are equivalent up to trivial field redefinitions $z \leftrightarrow f(z)$.

We would expect *a priori* that such a flat theory would possess some elegant symmetry. Since the potential is supposed to be flat throughout the complex z plane, we would expect the symmetry group to be non-compact, containing at least one non-compact $\tilde{U}(1)$ corresponding to translations and/or dilatations. The locally supersymmetric structure tells us more about this group. Since the flatness condition imposes the geometric structure of an Einstein space with curvature $R = 2/3$, this symmetry group must be an isometry group with dimension $n(n+1)/2 = 3$, as the complex field z has $n = 2$ components. The relevant group is in fact the non-compact $SU(1,1)$ group of isometric transformations:

$$f \rightarrow \frac{\alpha f + i\beta}{i\gamma f + \delta} : \alpha\delta + \beta\gamma = 1 \quad (\alpha, \beta, \gamma, \delta \text{ real}) \quad (2.18)$$

It is useful for our later discussion to exhibit explicitly the following three $SU(1,1)$ transformations:

$$(a) \text{ imaginary translations: } f \rightarrow f + i\beta \quad (\gamma = 0, \alpha = \delta = 1) \quad (2.19a)$$

$$(b) \text{ dilatations: } f \rightarrow \alpha^2 f \quad (\beta = \gamma = 0, \delta = 1/\alpha) \quad (2.19b)$$

$$(c) \text{ conformal transformations: } \delta f = -it \left(\frac{f^2 - 1}{1 + itf} \right) \quad (2.19c)$$

$$(t \equiv \tan \Theta : \beta = \gamma = \sin \Theta, \alpha = \delta = \cos \Theta)$$

The Kähler potential G is invariant under the action of the first transformation (imaginary translation):

$$\delta_a \mathcal{L} = 0 \quad (2.20a)$$

but not under dilatations:

$$\delta_b \mathcal{L} = -6 \ln \alpha \quad (2.20b)$$

nor under conformal transformations

$$\delta_c \mathcal{L} = 3 i t (f - f^\dagger) + O(t^2) \quad (2.20c)$$

Thus the gravitino mass $m_{3/2} = e^{G/2}$ changes when one makes a dilatation or conformal transformation.

Indeed, the total hidden sector Lagrangian is invariant under $SU(1,1)$ except for the gravitino mass term

$$- e^{G/2} \bar{\psi}^\mu \sigma_{\mu\nu} \psi^\nu \quad (2.21)$$

which breaks this symmetry in a characteristic way. The generator corresponding to imaginary translations (a) does not transform the fermionic fields, so the mass term (2.21) is invariant under the corresponding non-compact $\tilde{U}_a(1)$. The fermionic fields are also invariant under the non-compact $\tilde{U}_b(1)$ of dilatations (b), so the gravitino mass term transforms with definite weight (-3) . The helicity $\pm 3/2$ components ψ_μ and the helicity $\pm 1/2$ components χ of the gravitino transform*) differently under the conformal transformations (c):

$$\left. \begin{aligned} \delta \psi_\mu &= \frac{3}{4} i t (f + f^\dagger) \gamma_5 \psi_\mu \\ \delta \chi &= \frac{1}{4} i t (f + f^\dagger) \gamma_5 \chi \end{aligned} \right\} \quad (2.22)$$

which means that the gravitino mass term (2.21) is comprehensively non-invariant under the transformations (c). Thus we see that we have soft breaking of $SU(1,1) \rightarrow \tilde{U}_a(1)$ by a single term with dilatation weight (-3) under $\tilde{U}_b(1)$. The form of this $SU(1,1)$ breaking will be relevant when we discuss later the coupling of the hidden and observable sectors.

We have already seen that “solving” the hierarchy problem requires an almost flat potential with a small cosmological constant (2.5), and that a zero potential (2.14) requires an $SU(1,1)$

*) Since the entire non-compact group manifold can be covered with a single coordinate patch, there is no subtlety with the global definition of fermion field transformations: our manifold is trivially Hodge¹⁹⁾.

invariant Einstein space with curvature $R = \frac{2}{3}$. One can ask the converse question: does $SU(1,1)$ imply a flat potential and, if so, does the cosmological constant necessarily vanish? In general, $SU(1,1)$ invariance implies an Einstein-Kähler manifold with curvature

$$R \equiv \frac{R_{zz^*}}{G_{zz^*}} = \frac{2}{n} \quad (2.23)$$

corresponding to a unique Kähler potential, up to field redefinitions $z \rightarrow f(z)$:

$$G = -n \ln(z + z^*) + h(z) + h^*(z^*) \quad (2.24)$$

If this yields a constant potential

$$V = e^G \left[\frac{G_z G_{z^*}}{G_{zz^*}} - 3 \right] = C \quad (2.25)$$

then it must *a fortiori* be constant in the real direction $\text{Im } z = 0$, $z = X$. One can then solve the differential equation

$$e^G \left[\frac{G_z G_{z^*}}{G_{zz^*}} - 3 \right]_{\text{Im } z = 0} = C \quad (2.25R)$$

to derive three independent classes of solutions, up to trivial field redefinitions $z \rightarrow cz$:

$$\left. \begin{aligned} \text{(i)} \quad & h(z) = 0 \quad \text{and} \quad n = 3 \\ \text{(ii)} \quad & h(z) = \ln z^{n/2} \\ \text{(iii)} \quad & h(z) = \ln z^{n/2} \left[z^{\sqrt{3}n} - z^{-\sqrt{3}n} \right] \end{aligned} \right\} \quad (2.26)$$

Solution (i) is that which gives $C = 0$ and $R = 2/3$. Solutions (ii) and (iii), while flat in the real direction $z = X$, are not constant when $\text{Im } z$ varies. Thus, extending the $SU(1,1)$ invariance of the kinetic terms (but not the gravitino mass term) in the Lagrangian to include the scalar potential requires it to be flat with zero cosmological constant. To get a constant non-zero potential (2.2) with $C \neq 0$, we must break $SU(1,1)$. It is simple to show that if $C \neq 0$ one has

$$R_{zz^*} = 2 \frac{G_{zz^*}^2}{G_z G_{z^*}^*} = \frac{2}{3} G_{zz^*} \left[\frac{1}{1 + \frac{C}{3} e^{-G}} \right] \quad (2.27)$$

which exhibits small deviations from (2.13) if $C = O(m_w^4)$.

Before proceeding to a discussion of its relationship to the observable sector, we would like to mention some interesting structural properties of the flat $SU(1,1)$ invariant theory. We return to the form¹⁶⁾ of the $N = 1$ supergravity Lagrangian before the elimination of the auxiliary fields of the vector and chiral supermultiplets:

$$\mathcal{L}_{\text{tot}} = \mathcal{L}^{\text{pot}} + \mathcal{L}^{\text{aux}} + \mathcal{L}^{\text{kin}} \quad (2.28)$$

where

$$\mathcal{L}^{\text{pot}} = \lambda e \left[\frac{1}{2} (u + u^*) + \frac{1}{2} \bar{\chi}_\mu \sigma^{\mu\nu} \chi_\nu \right] \quad (2.29)$$

is the contribution of the superpotential terms. In equation (2.29) e is the vierbein determinant, and u is the auxiliary field of the compensating scalar multiplet $\epsilon \equiv (a, \xi, 1/3 u)$. The other auxiliary fields are the A_μ and b_μ fields in the graviton vector multiplet $(e_{m\mu}, \psi_\mu, A_\mu, b_\mu)$: A_μ is a gauge field for a $U(1)$ subgroup of $SU(1,1)$ as we shall see shortly. In equation (2.29) we have set the $g = g^+ = \lambda$ by making a simplifying Kähler transformation $g \rightarrow g e^\Sigma$: $\Sigma = -\ln(\lambda)$. Thus we see that there are two meaningfully distinct theories with $\lambda = 0$ and $\lambda = 1$ respectively. The Poincaré supergravity is obtained by fixing the gauge of dilatation, s -supersymmetry and special conformal transformations: $a = 1$, $\xi = 0$ and $b_\mu = 0$. Finally, the auxiliary field h appears in the chiral multiplet (z, χ, h) . For our $SU(1,1)$ solution [2.27(i)] with

$$\phi \equiv -3 e^{-1/3 G} = 3(z + z^*) \quad (2.30)$$

which enables us to recast L^{aux} and L^{kin} in the following interesting form:

$$\begin{aligned} \mathcal{L}^{\text{aux}} = & \frac{1}{9} e \phi u u^\dagger - e(u h^\dagger + u^\dagger h) \\ & - \frac{1}{9} \phi A_\mu A^\mu - A^\mu \left[i(\partial_\mu z^* - \partial_\mu z) + i \bar{\psi}_\mu \gamma_5 \chi \right] \end{aligned} \quad (2.31)$$

and

$$\begin{aligned} \mathcal{L}^{\text{kin}} = & \frac{1}{12} e \phi R + \frac{1}{12} \phi \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma \\ & + \frac{3}{16} \epsilon^{\mu\nu\rho\sigma} (\bar{\psi}_\mu \gamma_\nu \psi_\rho) (\partial_\sigma z^* - \partial_\sigma z) \\ & + \text{torsion terms} + \chi - \psi_\mu \text{ terms} \end{aligned} \quad (2.32)$$

It is noteworthy that L^{kin} (2.32) contains *no kinetic terms* for the z field or for its spin $-1/2$ partner χ . The equations of motion yield the following values for auxiliary fields:

$$u = 0, \quad h = \frac{1}{2} \lambda, \quad A^\mu = -\frac{9}{2} \frac{1}{\phi} \left(i(\partial_\mu z^* - \partial_\mu z) + i \bar{\psi}_\mu \gamma_5 \chi \right) \quad (2.33)$$

We can distinguish two interesting cases. For $\lambda = 0$, the potential is flat but supersymmetry is unbroken. For $\lambda = 1$, supersymmetry is always broken since $h = 1/2$, but the potential remains flat and a gravitino mass term (2.22) is present and cannot vanish. Thus one can go to the superunitary gauge $\chi = 0$ for any value of z . The third auxiliary field equation (2.33) describes a composite vector field taking the following form in the superunitary gauge:

$$A_\mu = -\frac{9}{2} \frac{1}{\phi} i(\partial_\mu z^* - \partial_\mu z) = \frac{3i}{2} \frac{\partial_\mu z^* - \partial_\mu z}{(z + z^*)} \quad (2.34)$$

which transforms non-trivially under $SU(1,1)$ as we will see shortly. Using equations (2.33) L^{pot} (2.29) and L^{aux} (2.31) become:

$$\mathcal{L}^{\text{pot}} + \mathcal{L}^{\text{aux}} = \frac{e}{2} \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu + \frac{g}{4} \frac{e}{\phi} (\partial_\mu (z^* - z))^2 \quad (2.35)$$

and the rest of the Lagrangian can be written as

$$\mathcal{L}^R = \frac{\phi}{6} \left[e R + \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu \hat{D}_\rho \psi_\sigma \right] + \text{torsion terms} \quad (2.36)$$

where

$$\hat{D}_\rho \equiv D_\rho + \frac{i}{2} A_\rho \gamma_5 \quad (2.37)$$

At this stage no kinetic term for the real part of z has yet appeared. However, the Einstein term in equation (2.36) is not yet in its canonical form, so a rescaling of e and ψ_μ is needed:

$$e \rightarrow e^{\frac{2}{3}\eta} e, \quad \psi_\mu \rightarrow e^{\frac{1}{12}\eta} \psi_\mu : e^{\frac{1}{3}\eta} = \frac{-3}{\phi} \quad (2.38)$$

This rescaling provides a kinetic term for the real part of the z field and after some algebra the Lagrangian takes the canonical form

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \bar{\psi}_\mu \gamma_5 \gamma_\nu \hat{D}_\rho \psi_\sigma \epsilon^{\mu\nu\rho\sigma} - e \frac{3 \partial_\mu z \partial^\mu z^*}{(z + z^*)^2} \\ & - \frac{1}{2} e R + e e^{\eta/2} \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu \end{aligned} \quad (2.39)$$

where $\hat{D}_\rho$ is given by equation (2.37) with

$$A_\mu = -\frac{3}{2} i \frac{\partial_\mu (z^* - z)}{(z + z^*)} \quad (2.40)$$

together with the relevant spin-connection term. The vector field A_μ (2.40) remains invariant under the $SU(1,1)$ transformations (2.20a) and (2.20b). However, under the third transformation (2.20c) it changes by

$$\delta_c A_\mu = \frac{3}{2} t \partial_\mu (z + z^*) + O(t^2) \quad (2.41)$$

which is characteristic of a gauge transformation. The gravitino kinetic term in (2.39) remains invariant if we make the following chiral rotation on the gravitino field:

$$\delta \psi_\mu = i t \left(\frac{3}{4} \gamma_5 (z + z^*) \psi_\mu \right) + O(t^2) \quad (2.42)$$

The kinetic terms in the Lagrangian (2.39) have the structure of a non-linear $SU(1,1)/U(1)$ σ -model. As advertized previously, we see that the soft gravitino mass term in the Lagrangian violates $\tilde{U}_b(1)$ dilatation invariance (2.20b) and conformal invariance (2.20c), while preserving unbroken the $\tilde{U}_a(1)$ non-compact translations (2.20a). By making a change of variables $z \rightarrow \ln z$ we can identify $\tilde{U}_a(1)$ as a global R symmetry. We will see in section 3 that if this symmetry remains after the introduction of the observable sector then it will suppress any tree level mass for the gauginos. Note that in the case $\lambda = 0$, the theory is a special case of a non-linear σ -model invariant under $SU(1,1)$ transformations: supersymmetry is not broken and the gravitino mass term is absent.

3. COUPLING HIDDEN AND OBSERVABLE SECTORS

We have seen in section 2 that in order to “solve” the hierarchy problem¹⁰⁾ by generating dynamically m_w and $m_{3/2}$, the observable particle fields y^i must be supplemented by a hidden sector z^i which yields a flat potential. This can happen if and only if the hidden sector has a non-compact group invariance which is $SU(1,1)$ in the simplest case of a single field z . In this section we explore ways of coupling the hidden and observable sectors. The potential variation and dynamical determination of $m_{3/2} = e^{G/2}$ require a non-trivial coupling. They also suggest a new scenario for the dynamical determination of a hierarchy of fermion Yukawa couplings, and for introducing CP violation into the Yukawa coupling matrix.

To see the need for a non-trivial coupling between the hidden and observable sectors, let us first write down a popular ansatz²⁰⁾ for models where $m_{3/2}$ is put in by hand. Take the full Kähler potential G incorporating minimal kinetic terms for the y fields to be:

$$G(z, y^i) = G(z, z^*) + y^i y_i^* + (\mathcal{F}(y) + h.c.) \quad (3.1)$$

which gives the y potential (using $m_{3/2}^2 = e^G \approx e^G$)

$$V(y) \approx m_{3/2}^2 \left| \frac{\partial \mathcal{F}}{\partial y^i} + y_i^* \right|^2 + D\text{-terms} \quad (3.2)$$

To make the correspondence to the conventional global SUSY potential $V(y) = |\partial F / \partial y^i|^2$ with conventionally normalized Yukawa couplings $g_{ijk}^y: F = g_{ijk}^y y^i y^j y^k$, we must rescale the superpotential $\mathcal{F}(y)$:

$$\mathcal{F}(y) = F(y) / m_{3/2} \quad (3.3)$$

which enables us to rewrite (3.2):

$$V(y) = \left| \frac{\partial F}{\partial y^i} + m_{3/2} y_i^* \right|^2 + D\text{-terms} : \quad (3.4)$$

$$F = g_{ijk}^y y^i y^j y^k + \dots$$

Since $m_{3/2} = O(10^{-16})$ in natural units, while the Yukawa couplings g_{ijk}^y should be $O(10^{-5} \text{ to } 1)$, we must have chosen large trilinear couplings in the $\mathcal{F}(y)$ of the original ansatz (3.1). An analogous conclusion holds if we make another popular ansatz²¹⁾:

$$G(z, y^i) = G(z, z^*) + y^i y_i^* + (\ln(c + F(y)) + h.c.) \quad (3.1L)$$

which we can expand as

$$G(z, y^i) \approx G(z, z^*) + y^i y_i^* + \ln c c^* + \left(\frac{F(y)}{c} + h.c. \right) + \dots \quad (3.5)$$

The correct identification of the superpotential $F(y)$ requires $C = m_{3/2}$ in natural units. In previous scenarios where one was content to fix $m_{3/2}$ by hand, this *ad hoc* introduction of bizarre $m_{3/2}$ factors into the Kähler potential was unaesthetic but perfectly legitimate, since one did not ask the origin of $m_{3/2}$.

In our case¹⁰⁾, the dynamical determination of $m_{3/2} = e^{G/2}$ means that it is unreasonable to prescale the superpotential numerically as in equation (3.3). Instead, we introduce into the Kähler potential (3.1 or 3.1L) some functions of z and z^* which vary dynamically along with $m_{3/2} = e^{G(z,z^*)}$, and are chosen in such a way that the Yukawa couplings have the appropriate limits when $m_{3/2} \ll M$. Thus we can replace the ansatz (3.1) by the ansatz

$$G(z, y^i) = G(z, z^*) + y^i y_i^* + \left(\Phi(z, z^*) F(y) + h.c. \right) \quad (3.6)$$

or (3.1L) by

$$G(z, y^i) = G(z, z^*) + y^i y_i^* + \left(\ln(1 + \Phi F) + h.c. \right) \quad (3.6L)$$

where G is the $R = 2/3$ $SU(1,1)$ choice (2.16). The functions $\Phi(z, z^*)$ might actually be different for different trilinear terms in the superpotential. Taking (3.6) as an example, we note that the hidden and observable sectors in the potential (2.2) are not simply additive, since there is mixing between the two sectors in the Kähler metric:

$$G_{z^* y} = \Phi_{z^*} \frac{\partial F}{\partial y} \quad (3.7)$$

Nevertheless, to leading non-trivial order in the observable superpotential:

$$V = e^G \left[\left| y^* + Q_{z^*} \frac{\partial F}{\partial y} \right|^2 - Q_{z z^*} e^{-\frac{1}{3}G} F \right] + D\text{-terms} + \dots \quad (3.8)$$

where

$$Q(z, z^*) \equiv e^{-\frac{1}{3}G(z, z^*)} \Phi(z, z^*) \quad (3.9)$$

so that

$$Q_{z^*} = \left(\Phi - 3 \frac{\Phi_{z^*}}{G_{z^*}} \right), \quad Q_{zz^*} = \left(\frac{\Phi_z}{G_z} + \frac{\Phi_{z^*}}{G_{z^*}} - \frac{\Phi_{zz^*}}{G_{zz^*}} \right) \quad (3.10)$$

If we want a Yukawa coupling to have a non-trivial limit as $m_{3/2} = e^{G/2} \rightarrow 0$, then we need

$$Q_{z^*} \underset{G \rightarrow -\infty}{\sim} e^{-G/2} \quad (3.11)$$

for trilinear terms $F_3(y)$ in the superpotential $F(y)$. On the other hand, if

$$Q_{z^*} \underset{G \rightarrow -\infty}{\sim} e^{\frac{\lambda-1}{2}G} : \lambda > 0 \quad (3.12)$$

then the corresponding Yukawa coupling coming from $F_3(y)$ scales as:

$$g^Y \underset{G \rightarrow -\infty}{\sim} e^{\frac{\lambda}{2}G} \sim m_{3/2}^\lambda : \lambda > 0 \quad (3.13)$$

Clearly the known fermion-Higgs Yukawa couplings are not incompatible with scaling laws like (3.13), which could well explain the observed hierarchy of fermion masses if the functions Φ multiplying the trilinear coupling terms of different fermion generations scale differently:

$$\left. \begin{aligned}
 \lambda &= O(1/4) \quad \text{for the first generation} \\
 \lambda &= O(1/8) \quad \text{for the second generation} \\
 \lambda &= 0 \quad \text{for the third generation}
 \end{aligned} \right\} \quad (3.14)$$

Before discussing this idea further, it is as well to consider possible constraints on the form of the scaling functions $\Phi(z, z^*)$.

We recall that the hidden sector Kähler potential is invariant under the $\tilde{U}_a(1)$ transformation (2.20a) corresponding to $z \rightarrow z + i\beta$, and transforms with definite weight (-3) under the $\tilde{U}_b(1)$ of dilatations (2.20b). If we require that the full Kähler potential (3.6) or (3.6L) be $\tilde{U}_a(1)$ invariant, then we must demand that

$$\Phi(z, z^*) = \Phi(z + z^*) = \Phi(e^{-1/3\zeta}) \quad (3.15)$$

Thus all $SU(1,1)$ breaking arises through the gravitino mass $m_{3/2} = (z + z^*)^{-3/2}$. The most natural possibility is to give definite dilatation weights to the different terms in (3.6):

$$\Phi(z, z^*) = e^{\frac{\lambda-1}{2}\zeta} = |z + z^*|^{\frac{3}{2}(1-\lambda)} : \lambda \geq 0 \quad (3.16)$$

The existence of a finite, non-trivial limit as $m_{3/2} = e^{G/2} \rightarrow 0$ requires $\lambda = 0$ in equation (3.16) (perhaps only for the third generation? maybe only the t quark?), which means that the ansatz (3.6) becomes for the trilinear superpotential terms

$$G_T(z, y^i) = G(z, z^*) + y^i y_i^* + e^{-\zeta/2} (F_3(y) + h.c.) \quad (3.17)$$

The ansatz (3.6L) can now be written in a particularly compact and suggestive form:

$$G(z, y^i) = \left(\ln \left(e^{G/2} + F_3(y) \right) + \text{h.c.} \right) + y^i y_i^* \quad (3.17L)$$

Using equations (3.8) to (3.10) the observable potential becomes

$$V = e^G |y^*|^2 + \frac{5}{2} \frac{\partial F_3}{\partial y} e^{-G/2} - \frac{15}{2} e^{G/2} (F_3 + F_3^*) + \text{D-terms} \quad (3.18)$$

which corresponds to a theory with a finitely rescaled trilinear superpotential

$$\tilde{F}_3 \equiv \frac{5}{2} F_3 \quad (3.19)$$

and the conventional trilinear supersymmetry breaking parameter¹³⁾

$$A = \frac{3}{2} \quad (3.20)$$

Since $|A| < 3$ in this case, the effective potential (3.18) is positive semidefinite and the global minimum is at $\langle 0 | y^i | 0 \rangle = 0$ with $V = 0$ in the absence of radiative corrections. It is easy to check that the fermion-fermion-scalar couplings are also rescaled by the same factor (3.19) in the $m_{3/2} \rightarrow 0$ limit.

It is tempting to seek an additional group-theoretical basis for the ansatz (3.17) or (3.17L) by demanding that the superpotential terms in the Kähler potential have the same conformal weight as the hidden sector term: $\delta_b G = -6 \ln a$ (2.21b) implies

$$\delta_b e^{G/2} = \alpha^{-3} e^{G/2} \quad (3.21)$$

This can be arranged by assigning conformal weights (-1) to all the observable fields appearing in conventional trilinear superpotential terms:

$$\delta_b y = \alpha^{-1} y \Rightarrow \delta_b F_3(y) = \alpha^{-3} y^3 \quad (3.22)$$

in which case (3.17L) transforms as

$$\delta_\nu G = -6\ln\alpha + (\alpha^{-2}-1)yy^* \quad (3.23)$$

The simplicity of the dilatation transformation law (3.23) is unfortunately broken by the minimal kinetic terms heretofore assumed for the observable fields. The suggestion of a universal conformal weight for the superpotential terms in $G(z, y^i)$ would have interesting consequences for terms $F_{n \neq 3}(y)$ in the superpotential which were not trilinear:

$$G(z, y^i) = \left[\ln \left(e^{\eta/2} + \sum_n e^{\frac{\eta}{6}(3-n)} F_n(y) \right) \right] + y^i y_i^* + h.c. \quad (3.24)$$

The corresponding n -linear low energy coupling parameters would scale as $m_{3/2}^{1-n/3}$. This would not save bilinear mass terms $F_2(y)$, which one might naïvely have guessed to be $O(M)y^2$, from being too large [$O(m_{3/2}^{1/3} M^{2/3})y^2$] to be of relevance in the observable sector. On the other hand, the effects of quadrilinear terms $F_4(y)$, normally thought to be suppressed like $O(1/M)y^4$, would be enhanced to $O(m_{3/2}^{-1/3} M^{-2/3})y^4$.

The ansatz (3.15) of $\tilde{U}_a(1)$ invariance can be extended to other aspects of the theory. For example, $N = 1$ supergravity theories¹⁶⁾ can in general accommodate a non-minimal kinetic term for the gauge fields, which takes the form of a chiral function ρ multiplying the gauge fields:

$$\rho(\phi_i) VV \quad (3.25)$$

A priori, ρ might be a function of both the hidden and observable sector fields:

$$\rho = \rho(z^i, y^i) \quad (3.26)$$

However, in our class of $SU(1,1)$ models the gauge fields V must be inert, while the observable chiral fields y^i might scale (3.22) under dilatations, but are otherwise $SU(1,1)$ invariant. Residual $SU(1,1) \rightarrow \tilde{U}_a(1)$ invariance would forbid any z dependence in the chiral function ρ :

$$\rho = \rho(y^i) \quad (3.27)$$

This reflects our previous identification of $\tilde{U}_a(1)$ with an R-invariance of the hidden sector. To the extent that the vacuum expectation values of all the observable fields y^I relax to values $\ll M$, equation (3.27) tells us that the tree-level gaugino masses $m_{\tilde{g}}$ will be strongly suppressed. Non-zero gaugino masses $m_{\tilde{g}}$ will in general arise from loop diagrams, but this symmetry argument would lead us to expect that $m_{\tilde{g}}/m_{3/2} \ll 1$.

While the hypothesis (3.15) of $SU(1,1)$ breaking down to $\tilde{U}_a(1)$ via the gravitino mass alone is very attractive, at our present level of theoretical sophistication we are not able to exclude more general scenarios for $SU(1,1)$ breaking. A natural possibility in the view of the structure of equations [(3.6), (3.6L)] would be that $\Phi(z, z^*)$ is a chiral function:

$$\Phi = \Phi(z) \quad (3.28)$$

To get a scaling law such as (3.11) or (3.12) we must postulate that

$$\Phi \underset{\zeta \rightarrow -\infty}{\sim} e^{\frac{(\lambda-1)\zeta(z, \bar{z})}{2}} \underset{z \rightarrow \infty}{\sim} (2z)^{\frac{3}{2}(1-\lambda)} : \lambda \geq 0 \quad (3.29)$$

A set of choices of $\Phi(z, z^*)$ which interpolate between the possibilities (3.16) and (3.29) is (for $\lambda = 0$):

$$\Phi = |z + z^*|^{\frac{3}{2}p} (2z)^{\frac{3}{2}(1-p)} \quad (p \text{ real}) \quad (3.30)$$

Using equation (3.10) we find that the trilinear superpotential couplings g^y are rescaled by a factor

$$g^y \propto e^{\frac{-3}{2}(1-p)i\theta} (1 + \frac{3}{2}p)(\cos\theta)^{-\frac{3}{2}(1-p)} \quad (3.31)$$

where $\theta = (1/2i) \ln z/z^*$, while

$$A = 3 \left[\frac{p}{2} + \sin\theta (1-p) e^{-i(\theta + \frac{\pi}{2})} \right] \quad (3.32)$$

It is important to note that both the Yukawa coupling g^y and the A parameter have moduli which depend on θ . In particular, if $p < 1$ then $g^y \rightarrow \infty$ as $\cos \theta \rightarrow 0$ for any fixed value of the gravitino mass: $z \rightarrow (1/2 m_{3/2})^{-2/3} + i\infty$. This is unacceptable since the dimensional transmutation scale μ_0 increases as $|g^y|$ increases, and the radiatively corrected vacuum energy becomes more negative. Thus the dynamically determined value of the gravitino mass would increase without limit, until $|g^y|$ becomes so large that perturbation theory is no longer applicable. All θ dependence in [(3.31), (3.32)] of course vanishes when $p = 1$, corresponding to the $\tilde{U}_s(1)$ invariant choice $\Phi = e^{-G/2}$ (3.16). When $p > 1$, the largest value of $|g^y|$ (and hence of μ_0) is attained when $\theta = 0$, and therefore this phase is dynamically chosen by the requirement of minimizing the vacuum energy. The A parameter should not be much larger than 3 if our present vacuum is to be a global minimum of the potential, and hence we deduce from (3.32) that

$$1 \leq p \leq 2 \quad : \quad \frac{3}{2} \leq A = \frac{3}{2}p \leq 3 \quad (3.33)$$

in the class of models (3.31). In this range of values $|A| \leq 3$ and hence the potential is positive semidefinite with $m_{3/2}$ undetermined at the tree level.

Choices of more general functions $\Phi(z, z^*)$ open up the intriguing possibility of relaxing dynamically to non-zero values of θ , and hence introducing a new form of spontaneous CP violation if the different fermion Yukawa couplings relax to different θ values. We see from equation (3.10) that if $F_3(y)$ is rescaled by an angle factor $q(\theta)$:

$$Q_{z^*} = e^{-G/2} q(\theta) \quad (3.34)$$

then

$$A = \frac{3}{2} + i \cos \theta e^{-i\theta} \frac{q'(\theta)}{q(\theta)} \quad (3.35)$$

To illustrate the idea, let us take

$$q(\theta) = 1 + a e^{-\frac{3}{2}i\theta} \quad (3.36)$$

in which case

$$|Ag^Y| = \frac{9}{4} \left[1 + 2a \left[\cos \frac{3r}{2}\theta + 2r \cos \theta \cos \frac{sr}{2}\theta \right] + O(a^2) \right] \quad (3.37)$$

If a is negative and small, then the theory relaxes dynamically to one of the discrete solutions of

$$\cos \frac{3r}{2}\theta + 2r \cos \theta \cos \frac{sr}{2}\theta = 0 \quad (3.38)$$

One solution to (3.38) is

$$\theta = \frac{2\pi}{r} \quad \text{for } r \geq 4 \quad (3.39)$$

In this case we can take $A = \frac{3}{2}$ to be real, and then g^Y acquires a phase:

$$\delta = -\frac{1}{2} r |a| \sin \frac{4\pi}{r} \quad (3.40)$$

Clearly one can imagine playing more sophisticated games with the dynamical generation of CP violation and of a hierarchy of light fermion masses. However, before embarking on such an enterprise we prefer to hold our fire until our theoretical understanding of what lies beyond the phenomenological $N = 1$ supergravity theory discussed here advances sufficiently to constrain input assumptions in an interesting way.

4. PHYSICS OF THE OBSERVABLE SECTOR

In previous sections we have seen how our strategy¹⁰⁾ for “solving” the hierarchy problem requires a hidden sector which has approximate $SU(1,1)$ invariance, and we have seen how to couple this to the observable sector in such a way as to retain non-trivial (or possibly small) Yukawa couplings in the limit $m_{3/2}/M \rightarrow 0$. Our motivation has been to use radiative corrections to fix $m_{3/2}$ as well as m_w by dimensional transmutation, and now it is time to discuss this in more detail. In this paper we follow the approach of ref. 10, which we first summarize for the convenience of the reader.

We start with the observable sector parameters renormalized at the Planck (or GUT) scale: $m_q^2 = m_\ell^2 = m_H^2 = m_{3/2}^2$ with our choice (3.1) of minimal kinetic terms, $m_{\tilde{q}}/m_{3/2} \lesssim O(1)$, and the trilinear supersymmetry breaking parameter $A = O(1)$. The as yet undetermined value of $m_{3/2}$ sets an unknown overall scale to all the mass parameters. We then use the renormalization group²²⁾ to compute the multiplicative logarithmic scale variation in the mass parameters until μ_0 where the condition

$$\det m_H^2(\mu_0) = 0 \quad (4.1)$$

is satisfied (see fig. 1), assuming for the moment that $\mu_0 > m_{3/2}$. The value of μ_0 is determined by the t-quark Yukawa coupling:

$$\frac{\mu_0}{M} = \exp\left(-\frac{O(1)}{\alpha_t}\right) \quad (4.2)$$

If the theory is renormalized at some scale μ below μ_0 , the tree level potential gives $SU(2)_L \times U(1)_Y$ gauge symmetry breaking, with negative vacuum potential energy (see fig. 1):

$$V = (-)_{\mu} \times m_{3/2}^4 + (\text{higher orders}) \quad (4.3)$$

Starting from small $m_{3/2}$, this form clearly wants to drive $m_{3/2}$ as large as possible, but if it becomes much larger than μ_0 , then $\det m_H^2$ is never renormalized to zero or negative values, gauge symmetry breaking becomes impossible, and the vacuum potential energy is zero. This reflects the fact that the negative coefficient $(-)_{\mu}$ in equation (4.3) vanishes for some $\mu = O(\mu_0)$. As seen in fig. 1, the potential clearly has an absolute minimum for some value of $m_{3/2}$ between 0 and $O(\mu_0)$. We computed this in ref. 10 by choosing $\mu = O(m_{3/2})$ so as to cancel one-loop corrections to the tree-level potential (4.3), and then locating the value of $m_{3/2}$ where

$$\frac{d}{dm_{3/2}} V = 0 \quad : \quad V = (-)_{\mu=O(m_{3/2})} \times m_{3/2}^4 \quad (4.4)$$

We found that, as expected, this dynamically determined value of $m_{3/2}$ was $O(\mu_0)$ as shown in fig. 1, and hence fixed by dimensional transmutation as in equation (4.2).

This scenario was investigated numerically in ref. 10 for the specific case $A = 3$, which is seen from equation (3.33) to be a limiting case. Some of the previous results are reproduced in fig. 2. Three regions of $\xi \equiv m_{\tilde{\nu}}/m_{3/2}$, $\hat{m}_4 = m_3^2/2m_{3/2}$ parameter space are compatible with experiment, including in particular the particle physics constraint that all charged particles weigh more than 20 GeV (denoted by P), and the cosmological constraint that the present density of neutral sparticles be less than 2×10^{-29} g/cc (denoted by C). The central region of fig. 2 is favoured by the argument of section 3 that small $m_{\tilde{\nu}}$ may be favoured because of a residual $\tilde{U}_a(1)$ invariance. In the right-hand region our present vacuum is unstable, hence we disfavour it even though the lifetime of our vacuum is probably longer than the age of the Universe¹⁰. The left-hand region does not suffer from this vacuum instability problem, but is disfavoured by $\tilde{U}_a(1)$. In the preferred central region

$$33 \text{ GeV} < m_t < 103 \text{ GeV} \quad (4.5)$$

We present in fig. 3 the results of new calculations for the other extreme end of the allowed range (3.33), namely $A = 3/2$. As discussed in section 3, this would actually be the value of A preferred (3.20) if one imposes $\tilde{U}_a(1)$ invariance (3.15) on the coupling [3.6), (3.6L)] between the hidden sector and the observable superpotential. Comparing with fig. 2, we see that the values of m_t required for given values of ξ and $\hat{m}_4$ are qualitatively unchanged. The most noticeable difference is the shrinkage of the previously preferred central region to a thin sliver: $\xi \simeq -0.1$, $\hat{m}_4 \gtrsim 0.57$. In this range we have

$$\left. \begin{aligned} 43 \text{ GeV} &< m_t < 69 \text{ GeV} \\ 43 \text{ GeV} &< m_{H^0} < 54 \text{ GeV} \\ 8 \text{ GeV} &< m_{\chi^0} < 11 \text{ GeV} \\ m_{\chi^\pm} &\sim (20 \text{ to } 22) \text{ GeV} \end{aligned} \right\} \quad (4.6)$$

In this case $A = 3/2$ there is no danger of cosmological vacuum instability in the right-hand region of fig. 3, which extends down to $\xi = 0.1$ [compatible with approximate $\tilde{U}_a(1)$ invariance?] and allows

$$20 \text{ GeV} < m_t < 90 \text{ GeV} \quad (4.7)$$

The left-hand region of fig. 3 is still disfavoured by the large values of $|\xi|$ that it requires.

The results displayed in figs. 2 and 3 demonstrate that models compatible with all particle physics and cosmological constraints exist for all values of A in the preferred (3.33) range of $3/2$ to 3. Moderately small values of $|\xi| = O(0.1)$ are possible, and the top quark mass lies in the range 20 to 100 GeV.

5. RELATION TO EXTENDED SUPERGRAVITIES

In the previous sections we have only been concerned with $SU(1,1)$ $N = 1$ supergravity as an effective low energy theory. In this section we discuss the possible relation to an extended supergravity theory⁷⁾ interpreted as a fundamental theory^{8,9)}. It has already been observed¹¹⁾ that the $SU(1,1)$ invariant hidden sector with $R_{zz} = \frac{2}{3} G_{zz}$ considered here is isomorphic with the scalar sector of the $N = 4$ extended supergravity¹²⁾. The difference is that the $N = 4$ theory contains 4 gravitinos, 6 spin-1 bosons and 4 spin- $1/2$ fermions, whereas the corresponding numbers in our model are 1, 0 and 1 respectively. Now we can trace the absence of a scalar potential in the $N = 4$ theory to its possession of the non-compact $SU(1,1)$ invariance¹²⁾. The absence of a scalar potential and the vanishing of the cosmological constant in the higher $N > 4$ ungauged supergravities are linked to their larger non-compact global invariance groups⁷⁾ which contain $SU(1,1)$ as a subgroup. These non-compact invariances are of course broken in the gauged versions¹⁴⁾ of these theories, and the corresponding scalar potentials are non-trivial and difficult to interpret, since they are not bounded below. We believe that the previous sections have demonstrated the desirability of retaining at least an approximate residual $SU(1,1)$ invariance. This might perhaps be taken as an indication that one should discard the gauged extended supergravities, but that might be a premature reaction.

Since it has previously been proposed^{7,8)} that one extract the phenomenological low energy gauge group from the $SU(8)^{\text{local}}$ invariance of the $N = 8$ supergravity theory, it is reasonable to ask whether this can be reconciled with the extraction of a phenomenological $SU(1,1)^{\text{global}}$ from the $E_{7(7)}^{\text{global}}$ of the $N = 8$ theory. Our phenomenological hidden sector actually has an $SU(1,1)^{\text{global}}/U(1)^{\text{local}}$ structure, with a composite gauge connection

$$A_\mu = \frac{3}{2} i \frac{\partial_\mu z^* - \partial_\mu z}{(z + z^*)} \quad (5.1)$$

We are not committed to any definite viewpoint as to the detailed structure of the hidden sector, and it may well be that this $U(1)^{\text{local}}$ acquires a physical gauge boson as a result of quantum corrections, though we are aware of divergent viewpoints²³⁾ on this possibility. This $U(1)^{\text{local}}$ subgroup of $SU(8)^{\text{local}}$ would then behave analogously to the $SU(5)^{\text{local}}$ subgroup that we wish to identify with the conventional GUT gauge interactions. One might have hoped that $E_{7(7)}$ would contain an $SU(5) \times SU(1,1)$ subgroup, but this is not the case. The non-compact generators of $E_{7(7)}$ belong to a 4-fold antisymmetric tensor $\underline{70}$ representation of $SU(8)^{\text{local}}$ which breaks down into the following $SU(5)$ representations:

$$\underline{70} = 3(\underline{10} + \overline{\underline{10}}) + (\underline{5} + \overline{\underline{5}}) \quad (5.2)$$

Thus no non-compact generator is a singlet of $SU(5)$: $E_{7(7)}$ does not even contain an $SU(5) \times \tilde{U}(1)$ subgroup. However, $E_{7(7)}$ does contain an $SU(4) \times SU(1,1)$ subgroup as can be deduced from the knowledge that the $N = 8$ theory contains the $N = 4$ theory and its symmetries. The best that can be found would be an $SU(1,1)$ whose generators were all electromagnetically neutral, with the non-compact generators $Q_5, \overline{Q}^5$ coming from the $(\underline{5} + \overline{\underline{5}})$ of $SU(5)$ in the decomposition (5.2) and carrying neutrino-like quantum numbers. The compact generator obtained from their commutator is contained in $SU(8)$ and is partly inside, partly outside the $SU(5)$ subgroup:

$$[Q_5, \overline{Q}^5] \propto \frac{8\sqrt{10}}{15} T_{24} + \frac{4}{\sqrt{15}} T_{35} + \frac{4}{\sqrt{21}} T_{48} + \frac{2}{\sqrt{7}} T_{63} \quad (5.3)$$

where T_{n^2-1} is the diagonal generator

$$T_{n^2-1} = \text{diag} \sqrt{\frac{2}{n(n-1)}} \left(\underbrace{1, 1, 1, \dots, 1}_{n-1}, -(n-1) \right) \quad (5.4)$$

of $SU(n)$. It is not clear how to construct an acceptable hidden sector based on this $SU(1,1)$. One could take the point of view that since gauged supergravities are already known to have their non-compact invariances broken, maybe the same happens to our $SU(1,1)$. In the case of a conflict between a gauged symmetry and a global symmetry, one would expect the gauge symmetry [in our case $SU(5)$] to win and the global symmetry [in our case $SU(1,1)$] to lose. One would just have to

hope that the broken $SU(1,1)$ has enough strength to protect the flatness of the potential that was our focus in the previous sections. In the meantime, one could exploit the fact that $E_{7(7)}$ contains an $SU(4) \times SU(1,1)$ subgroup to construct a toy “flat” GUT based on the gauge group $SU(4)$. As hypothesized in section 3.2, one could postulate that a non-compact $\tilde{U}_b(1)$ subgroup of $SU(1,1)$ dilatations was also an approximate symmetry of the Kähler potential: $G \rightarrow G + \text{constant}$. One would then assign different $\tilde{U}_b(1)$ charges q_i to the different $SU(4)$ representations y_i in the toy GUT, and incorporate the necessary n^{th} order homogeneous function $\Phi(z, z^*)$ to preserve identical dilatation properties for all terms in the Kähler potential:

$$G = \ln \left((z + z^*)^{-3} + \sum_i a_{ijk} y_i y_j y_k \Phi_{n_{ijk}}(z, z^*) \right) + \text{h.c.} \quad (5.5a)$$

where

$$n_{ijk} = -6 - q_i - q_j - q_k \quad (5.5b)$$

Then terms for which

$$q_i + q_j + q_k = -3 \quad (5.6a)$$

will survive in the $m_{3/2} \rightarrow 0$ (cf. the t quark?) while terms for which

$$q_i + q_j + q_k > -3 \quad (5.6b)$$

will have Yukawa couplings, and hence masses, scaling as

$$\left(\frac{m_f}{m_W} \right) \propto g_{H\bar{f}f} \propto \left(\frac{m_W}{m_P} \right)^{q_i + q_j + q_k + 3} \quad (5.7)$$

(cf. the first two generations of quarks and leptons?).

It remains to be seen whether such an ambitious scenario²⁴⁾ could be embedded within any realistic fundamental extended supergravity theory. However, it is encouraging that all the $N \geq 4$

extended supergravity theories contain a non-compact global $SU(1,1)$ symmetry of the type whose phenomenological desirability we have argued in the previous sections: it may be the only way to “solve” the hierarchy problem.

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Figure captions

- Fig. 1 : Sketch of the variation of SUSY-breaking mass parameters m_i^2 with the renormalization scale Q . $\text{Det } m_H^2 = 0$ at a scale Q_0 , which determines the dynamically preferred value of $m_{3/2}$, as seen in the bottom half of the figure.
- Fig. 2 : Values of m_t (in GeV) for general values of $\hat{m}_4 \equiv m_{\tilde{g}}^2/2m_{3/2}$, $\xi = m_{\tilde{\nu}}/m_{3/2}$ and $A = 3$. Our present vacuum is unstable in the allowed region on the right of the figure. The shaded domain indicates values of $\hat{m}_4$ and ξ disallowed because of the absence of a charged sparticle with mass less than 20 GeV (P), and/or because of an excessive cosmological density of the lightest neutral sparticle (C).
- Fig. 3 : As for fig. 2, but now with $A = 3/2$. Note that the central allowed region has shrunk to a very thin domain, while the values of m_t obtained are qualitatively similar to those in fig. 2. The allowed domain on the right of the figure now has a stable vacuum.

Fig. 1

Fig. 2

Fig. 3