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THE GLUEBALL SPECTRUM IN THE BAG MODEL
AND IN LATTICE GAUGE THEORIES

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Abstract

Existing Bag model calculations for the masses of lowest lying glueball states are extended and reviewed and compared to the present lattice Monte Carlo calculations. The predicted spectra agree remarkably well.

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1. Introduction

Since the advent of Quantum Chromodynamics there has been interest in hadronic states containing gluons only, so called glueballs or gluonic mesons [1]. There are as yet no positively identified glueball states, although there exist some promising candidates [2]. Theoretical predictions for the spectrum are available from the MIT Bag Model [3,4] and Lattice Monte Carlo calculations [5,6,7]. In the Bag Model the glueball states are constructed from a few valence gluons (cavity modes) whereas in the lattice approach one extracts masses from correlation functions measured at large imaginary times. The purpose of this note is to elaborate somewhat on the Bag description and to compare the results of the two different approaches.

2. Bag Model Predictions

The MIT Bag Model is successful in explaining masses and other dimensional quantities for the lowest lying mesons and baryons [8,3,4]. As free parameters one typically has the Bag constant B , the strong coupling constant α_s (or alternatively Λ_{QCD} entering a running α_s), the quark self energy parameter e_q , and/or the geometrical or Casimir energy parameter Z_0 . The lowest lying glueball states are constructed from 2 or 3 valence gluons in the lowest transverse electric (TE) and transverse magnetic (TM) cavity modes. The only additional parameters entering are the corresponding self energies e_{TE} and e_{TM} . We now discuss the results of two different bag model fits to the glueball mass spectrum.

In ref. [4] e_{TE} and e_{TM} were fitted to $i(1440)$, which has the (TE)(TM) quantum numbers 0^{-+} , under the three different ad hoc assumptions

$e_{TE} = e_{TM}$, $e_{TE} = 2e_{TM}$ and $e_{TE} = \frac{1}{2}e_{TM}$. In our discussion we restrict ourselves to the latter since the two others correspond to the situation of having the TE gluon self energy larger than the TE kinetic energy. No correction for center of mass motion is made. The resulting spectrum is shown in Fig. 1 and labelled CS.

In ref. [3] a different approach, based on a specific model for the QCD vacuum, is employed. The basic assumption of this model is that the QCD vacuum is locally well described as a color and spin singlet $(TE)^2$ gluon state. Over long distance scales, these condensed "glueballs" are described by an effective lagrangian,

$$= \frac{1}{2} (\partial_\mu \phi)^2 - \frac{m^2}{2} \phi^2 - \frac{\lambda}{4} \phi^4. \quad (1)$$

By minimizing with respect to ϕ and assuming that the vacuum is densely filled by glueballs of radius R , one obtains

$$B = \frac{3(-m^2(R))^{1/2}}{8\pi R^3} \quad (2)$$

where $m^2(R) < 0$ because of the instability of the vacuum. So once $B^{1/4}$ and Λ_{QCD} are determined from fits to the meson and baryon spectra eq. (2) gives e_{TE} through m_2 . There will also be an observable $J^{PC} = 0^{++}$ state which is an excitation of ϕ . The mass of this object can be estimated by expanding around ϕ_0 , and finding a term $(\sqrt{2}\mu)^2 \frac{1}{2}(\phi - \phi_0)^2$ where $\mu^2 = -m^2$. Thus we get a real scalar particle with mass $\sqrt{2}\mu$. Using eq. (2) and the parameters of ref. [3] we obtain

$$\sqrt{2}\mu = 2\sqrt{2}B \frac{4\pi R^3}{3} \approx 780 \text{ MeV}. \quad (3)$$

The remaining states involving $(\text{TE})^2 2^{++}$, $(\text{TE})(\text{TM})$, and $(\text{TE})^3$ modes are computed with the ad hoc assumption $e_{\text{TM}} = e_{\text{TE}}$ in ref. [3]. The resulting spectrum is shown in Fig. 1 and labelled CHP. As is seen from this figure, the two Bag model approaches agree well despite the different algorithms (results from ref. [4] were not given for $(\text{TE})^3$; we have included them here using their parameters and methods). The agreement for the $(\text{TE})^2 0^{++}$ -state is perhaps a coincidence. In ref. [4] this is a conventional hadronic state whereas as shown above it is a collective excitation in ref. [3].

3. Lattice Gauge Theories

During the last two years several attempts have been made to extract glueball masses in lattice QCD using Monte Carlo methods [5-7]. The technique is to choose appropriate corrected two point correlation functions that have the relevant quantum numbers (e.g. between plaquettes for the 0^{++} state). This correlation function is then evaluated numerically and from its large imaginary time falloff the mass of the lowest state with the quantum numbers in question can be extracted. We should remark that it is only for the 0^{++} and the 2^{++} -states (the latter verified only in ref. [6]) that an appropriate scaling "window" exists, which makes MC predictions for higher lying states less convincing; it is possible that these predictions are only upper limits [6]. The concept of valence gluons do not enter in lattice QCD calculations [11], although in ref. [12] an attempt is made to measure $\langle G | \vec{B}^2 | G \rangle$ inside the 2^{++} glueballs with a suggestive dipole-structure as a result, perhaps indicating a few valence gluon structure.

The results from the Monte Carlo calculations are shown also in Fig. 1 and as can be seen the lattice predictions and the Bag model predictions agree remarkably well for both the absolute mass scale and the spin-splitting pattern. In addition, this may itself suggest that a two or three valence gluon structure for the glueballs is substantially correct.

Figure Caption

The lowest lying glueball states for each of the indicated quantum numbers. Lines without dots are Bag model calculations (our work in Ref. 3 is supplemented by the 0^{++} calculation reported here) and lines with dots are Monte Carlo calculations on a lattice. We have used fit I with $C_{TE}/C_{TM} = 1/2$ from ref. 4 and in plotting ref. 5 we used $M_{0^{++}} = 750$ MeV following the third article of that reference. Large errors, perhaps ± 150 MeV for the lower lying states and more for the higher lying states, should be assigned to each prediction.

References

1. A review is given by S. Meshkov in Proceedings of the Experimental Meson Spectroscopy Conference (Brookhaven, 1983).
2. See e.g. the talk by D. Hitlin at the International Symposium on Lepton and Photon Interactions, Cornell, August 1983.
3. C. E. Carlson, T. H. Hansson and C. Peterson, Phys. Rev. D27, 1556 (1983) and errata to appear in Phys. Rev. D.
4. M. Chanowitz and S. Sharpe, Nucl. Phys. B222, 211 (1983).
5. B. Berg and A. Billoire, Phys. Lett. 113B, 65 (1982), ibid 114B, 324 (1982), and Nucl. Phys. B221, 109 (1983).
6. K. Ishihawa, M. Teper and G. Schierholz, Phys. Lett. 116B, 429 (1982) and DESY 83-061 (July 1983).
7. H. W. Hamber and U. M. Heller, Preprint, Institute for Advanced Study (March 1983).
8. T. DeGrand, R. L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. D12, 2060 (1975).
9. For a recent discussion see H. Flensburg, C. Peterson and L. Shöld, Lund University preprint LU-TP 83-11 (1983).
10. T. H. Hansson, K. Johnson and C. Peterson, Phys. Rev. D26, 2069 (1982).
11. This is in contrast to meson and baryon MC mass calculations in the so-called quenched approximation (see e.g. H. Lipps et al., CERN TH3548 (1983)), where correlation functions between static quark sources are considered.
12. K. Ishakawa, G. Schierholz, H. Schneider and M. Teper, DESY 82-087.