THE MULTISTRING FRAGMENTATION MODEL AND
VIOLATION OF KNO SCALING

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ABSTRACT

The violation of Koba-Nielsen-Olesen scaling of the multiplicity distribution is studied in detail within the dual multistring fragmentation model. Comparison with recent data from the CERN-SPS proton-antiproton collider reveals a close agreement.

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Within the last 10 years the KNO (Koba-Nielsen-Olesen) scaling hypothesis has become the dominant framework to study experimentally and theoretically the behaviour of the multiplicity distribution of secondary hadrons produced in high energy collisions. This hypothesis, which was derived originally assuming Feynman scaling of the inclusive particle production cross-sections, states that the multiplicity distribution in the form

$$\Psi(z) = \frac{\langle n \rangle}{\sigma_{\text{inel}}}$$  \hspace{1cm} (1)

should become an energy-independent function of $z = n/\langle n \rangle$ where $n$ refers to the charge multiplicity. Experimentally up to ISR energies, KNO scaling or slightly modified versions of it were found to agree rather well with the data. However, it was also found that the KNO function $\Psi(z)$ depended on the reaction studied and that small deviations from KNO scaling could never be excluded.

Most of the theoretical models for particle production studied in the seventies did have the property of KNO scaling with the important exceptions of the multicomponent models. We study here a model of this type. Recent theoretical work related to the KNO or non-KNO behaviour of the multiplicity distributions include studies of the multistring fragmentation model, geometrical approach, the description in the framework of QCD jet calculus, QCD cluster models and soft QCD bremsstrahlung models, as well as the suggestion that the shape of the multiplicity distribution does not reflect any dynamical details but is essentially a consequence of general statistical laws.

Here we study the violation of KNO scaling in detail using a multistring fragmentation model described in detail before. The model is formulated as a Monte Carlo event generator. This formulation has the advantage that all kinematic constraints, energy and momentum conservation, as well as quantum number conservations, are taken into account, and that therefore a comparison with experiments is possible without introducing approximate kinematics. Our aim is to study the violations of KNO scaling and not to describe the data in the best possible way. Since these violations are most noticeable in the high multiplicity tail of the KNO distribution when the topological cross-sections are smallest, we need much higher statistics than were necessary to obtain inclusive quantities. Our analysis is based on 16000 "events" at 62 GeV and...
8000 "events" at 540 GeV and 2000 GeV which gives statistics comparable to experimental results. Otherwise, all conventions and parameters are as in Ref. 9).

The recent data from the CERN-SPS collider\textsuperscript{3,16,17} refer mainly to non-single diffractive events. Our model at present includes only the non-diffractive component.

What is the basic physical reason of the violation of KNO scaling within the multistring fragmentation model? There are three different mechanisms:

i) The multiplicity distribution is the superposition of contributions from i chains

\[ p_n = \frac{\sum_{i=0}^{\infty} \sigma_i(s) p_i(s_n)}{\sum_{i=0}^{\infty} \sigma_i(s)} \]  

where \( p_i(s, n) \) is the multiplicity distribution for the component involving i chains. If one assumes for \( p_i(s, n) \) a Poisson distribution in n \textsuperscript{5)}

\[ p_i(s, n) = \frac{\langle n \rangle}{n!} e^{-\langle n \rangle} \]  

then the KNO multiplicity distribution in the limit, \( s \to \infty \), will not be a smooth curve as found at low energies, but instead a series of \( \delta \)-function-like spikes, one for each i chain component.

ii) For kinematical reasons, at low energies, only a limited number of chains \( i \leq i_{\text{max}} \) can be produced. With rising primary energy, contributions with more and more chains become possible. This effect leads to the expectation that the tail at large z of the KNO curve \( \psi(z) \) will rise with rising primary energy. Our formulation of the model is of great advantage to study this effect realistically.

iii) In the parton interpretation of the dual multistring model\textsuperscript{7)-10)} partons sit at the ends of the fragmenting chains. These partons each carry a fraction of the energy of the incoming hadrons according to generalized structure functions \( \rho(x_1, x_2, \ldots, x_{21}) \) so that the string formed of partons
$x_j^L$ and $x_j^R$ has a mass squared $x_j^L x_j^R$ and decays into $n_j$ charged particles with a probability $P_{n_j}(x_j^L x_j^R)$. The normalized topological cross-section $P_n$ is then given by

$$G_{int} P_n = \frac{2}{n} \prod_{j=1}^{2i} d x_j^L d x_j^R \prod_{j=1}^{2i} f(... x_j^L,...) f(... x_j^R,...)$$

which can easily be obtained from Eq. (1) of Ref. 9 to which we send the reader for details. It is important to remember that in this model, there will be essentially two long "valence" chains and many short "sea" chains contributing to the central rapidity region. In this model, there are in each $i$-chain component considerable fluctuations of the energies associated to the chains. These fluctuations have the effect that the multiplicity distributions $P_i(s,n)$ of the $i$-chain components become much wider than Poisson distributions. We do not expect therefore that our KNO distribution at asymptotic energies will become a sequence for $\delta$-functions.

In Fig. 1, this effect is illustrated. We present three different multiplicity distributions in KNO form. The first curve is the KNO distribution obtained for six identical fragmenting chains. Up to kinematic effects in our energy-conserving Monte Carlo chain fragmentation model\(^ {\text{18}}\), this distribution is expected to have Poisson shape. The second curve is the multiplicity distribution obtained for the six-string component (three cut Pomerons) in our model, where the energy fractions of the partons at the ends of the chains are sampled from the generalized structure functions according to (4). This multiplicity distribution has become much wider than the first curve due to the fluctuations of the chain energies in the model. The third curve gives for comparison the KNO multiplicity distributions of the full multichain model with contributions from all $i$-chain ($i \geq 1$) components.

From the structure of the model described above, one can already draw some conclusions about the shape of the KNO curve and its energy variation in different rapidity regions. If one considers a rapidity interval in the
centre, each of our valence and sea quark strings will contribute roughly similar multiplicities so that the relative spread in the average multiplicity will reflect approximately the string multiplicity distribution. This spread is therefore much larger than if one considers the whole rapidity interval which receives contribution of two long strings and many short (low multiplicity) strings. As this spread in the average multiplicities of various subprocesses controls the width of the KNO curve, one expects a wider distribution in a fixed central rapidity interval than in the whole rapidity range. When the energy increases the situation does not change appreciably in the central region, whereas in the whole interval the dominance of the long strings decreases leading to large fluctuations and a widening of the KNO distribution.

In Figs. 2-4, we present our results and the comparison to the KNO violation found by the UA5 experiment at the CERN-SPS collider. We discuss separately the KNO multiplicity distributions referring to the full rapidity range and the one referring to the central rapidity region $|y| < 1.5$.

In Fig. 2, we present results for the full rapidity range. The KNO scaling violation found experimentally in Fig. 2a is compared to the KNO violation found in the model in Fig. 2b. The data in Fig. 2a are for three energies: Fermilab [$\sqrt{s} \sim 20 \text{ GeV}^{19}$], ISR [$\sqrt{s} \sim 50-60 \text{ GeV}^{19}$] and the CERN-SPS collider [$\sqrt{s} = 540 \text{ GeV}^{16}$]. The calculations in Fig. 2b are for ISR and collider energies and in addition the energy $\sqrt{s} = 2000 \text{ GeV}$.

The unscaled behaviour, the rise of the high $z$-tail of the distribution, found in the model agrees well with the behaviour of the data. According to the model, the present trend is expected to continue at higher energies. In Figs. 2c and 2d, we compare the model at ISR and collider energies directly with the data. The differences in the shape found in the low multiplicity region arise from too narrow a distribution at the one cut-Pomeron level. It can be changed by a more careful adjustment of parameters. This is not connected, however, with the non-KNO-scaling dynamics of the model. A similar adjustment of parameters could be used to improve agreement of our large multiplicity tail with the data.
In Fig. 3a-d, we present the same sets of distributions for the limited central rapidity range $|y| \leq 1.5$. In this region we observe the same type of non-KNO-scaling as in the full range, but the effect is considerably weaker. In Fig. 4, we finally compare the shapes of the KNO distributions at one of the energies for the full rapidity range and for the central region. We stress that the shapes differ considerably. No conclusions should be drawn from the comparison of KNO scaling if one does not use the same rapidity range in both cases. We conclude:

the main effect of the non KNO-scaling behaviour is the rise of the high-z-tail of the distribution. The multistring model and the data agree remarkably well in this non-scaling behaviour. A similar but much weaker non KNO-scaling behaviour is found in the central rapidity range. There are considerable differences of the KNO distributions in the full rapidity range and in the central region. This effect is important for comparison of data at different energies and of data and models. The multichain fragmentation model, together with a parton interpretation of the chains leads to a significant modification of the model. No multipeak structure of the KNO curve is expected at energies within reach.

Since the multichain fragmentation model describes the important features of non-KNO-scaling, one should not yet conclude that this non-KNO-scaling found experimentally indicates the onset of "new physics".

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Fig. 1: KNO distributions for a fixed number of strings in the case of
a) six identical strings with a Poisson multiplicity distribution
(solid line);
b) six strings in the model studied in the text, including
smearing over the parton momenta (dashed line). For comparison,
we also show the full model prediction (dash-dotted line). The
average multiplicity is fixed at 29 charged particles.

Fig. 2: Experimental KNO distributions and the multistring model
predictions in the whole rapidity range.
a) The data from FNAL [Ref. 19], ISR [Ref. 20] and SPS collider
energies [Ref. 16]. The figure is taken from K. Böckmann [Ref.
16].
b) KNO scaling violations obtained in the multistring model.
c) Comparison of the model to the data at $\sqrt{s} = 62$ GeV.
d) Comparison of the model to the data at $\sqrt{s} = 540$ GeV.

Fig. 3: Experimental KNO distributions and the multistring model
predictions in a limited pseudorapidity range ($|\eta| < 1.5$).
a) The data at $\sqrt{s} = 52$ GeV [Ref. 21] and at $\sqrt{s} = 540$ GeV [Refs.
16 and 17].
b) KNO distributions as obtained in the model.
c) Comparison of the model to the data at $\sqrt{s} = 52$ GeV.
d) Comparison of the model to the data at $\sqrt{s} = 540$ GeV. The data
are from UAL [Ref. 17] and UA5 [Ref. 16].

Fig. 4: Comparison of the shape of the KNO distributions for $|\eta| < 1.5$
and for the whole rapidity range at $\sqrt{s} = 540$ GeV.
Fig. 1
Fig. 3a

Fig. 3b
Multistring model
$\sqrt{s} = 62 \text{ GeV}$

$x \ l_{yl} < 1.5$
$\bullet \ l_{yl} < \infty$

$z = \frac{n_{ch}}{\langle n_{ch} \rangle}$

Fig. 4