UNITARITY AND APPROACH TO SCALING AT THE SPS COLLIDER

P. Valin^{*)}

CERN — Geneva

A B S T R A C T

We use variable t_1 bounds on preliminary SPS Collider data to show that these data saturate the unitarity bounds over a wide range of t . The bounds can be used to normalize different sets of data and to estimate reliably a self-consistent set of values for global quantities such as the total and the elastic cross-sections and the forward elastic slope. The approach to scaling is argued to be non-uniform in t .

*) Permanent address: High Energy Physics Group, McGill University, 3600 University St., MONTREAL, Quebec H3A 2T8, Canada.

1- INTRODUCTION

By collecting its first data in 1981, the SPS Collider opened up a new realm of energies that would seriously test our prejudices concerning asymptotic theorems and unitarity bounds. It was generally believed that the total cross-section σ_{tot} would continue to qualitatively saturate the Froissart bound, as was predicted by dispersion relation calculations¹. The forward elastic slope was also expected to follow the same $\ln^2 s$ dependence eventually², but could do so at energies as low as those accessible at the Collider³ if the ratio $(\sigma_{\text{el}}/\sigma_{\text{tot}})$ remained close to its ISR value⁴ of 0.1747 ± 0.0012 or close to the Van Hove limit of 0.1853... The simple reasoning behind this startling prediction was based on the eventual saturation of the MacDowell-Martin lower bound⁵ for the forward absorptive elastic slope and this $\ln^2 s$ trend now seems to be observed⁶ in $B(t \approx 0) = d/dt(\ln d\sigma/dt)|_{t=0}$ in contradiction with lower energy fits⁷ which varied linearly in $\ln s$. Such a linear dependence is however expected⁸ for $B(t \neq 0)$ and seen in the data⁶ at $-t \approx 0.2$ but not at $-t \approx 0.4$.

We, of course, make the usual assumption that the pp and $\bar{p}p$ channels are asymptotically equal, at least for "global" quantities like σ_{tot} , σ_{el} , $B(t \approx 0)$ and the ratio ρ of the real to imaginary parts of the amplitude. The validity of Pomeron dominance has been tested before^{2,8}, with the conclusion that any odderon contribution must be very small. The recent R211 data⁹ is particularly restrictive on this point while the R210 experiment is less so¹⁰. We feel justified, for the purposes of this paper, to consider pp and $\bar{p}p$ data on an equal footing, confident that the trend seen in the ISR data implies equality of these global quantities for the two channels in the regime of energies above the ISR.

It is well known³ that a particular combination of these global quantities, namely the ratio of the measured forward elastic slope to the MacDowell-Martin lower bound

$$R = \frac{B_A(t \approx 0)}{B_{\text{MM}}} = 18\pi \frac{\sigma_{\text{el}}^A}{\sigma_{\text{tot}}^2} B_A(t \approx 0) \geq 1 \quad (1)$$

is a particularly sensitive parameter in the determination of upper unitarity bounds for the absorptive elastic amplitude at all measured values of $-t$ (the quantities with the sub(super)script A refer to the contribution of the imaginary part of the amplitude only, which can be estimated from the measured experimental quantities as we shall see later). The new SPS Collider data now seems to prefer the scenario³ of R remaining relatively constant, since $\sigma_{\text{el}}/\sigma_{\text{tot}}$ is approximately constant and both

$B(t \approx 0)$ and σ_{tot} vary as $\ln^2 s$. The most recent UA4 data combining both the total interaction rate and optical point extrapolation methods in determining these quantities (as was done at the ISR for pp data by the CERN-Pisa-Rome-Stony Brook Collaboration¹¹) give the following results¹⁰

$$\sigma_{\text{tot}} = 65 \pm 3.5 \text{ mb} \quad (2)$$

$$\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} = 0.195 \pm 0.0 \quad (3)$$

which do not include systematic errors but are more precise than ever before and assume a $\rho(t \approx 0) = 0.15$. The world average of all forward slope measurements gives

$$B(t \approx 0) = 17.3 \pm 1.0 \quad (4)$$

which translates into a mean value for R of

$$R \approx 1.12 \quad (5)$$

where we do not quote errors because the values of these global quantities are highly correlated experimentally by the approximate relationship

$$\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} \approx \frac{(1+\rho^2)}{16\pi} \cdot \frac{\sigma_{\text{tot}}}{B} \quad (6)$$

which is exact for a purely exponential $d\sigma/dt$ and a t -independent ρ . This value of R is lower than the range of values (1.153, 1.195) spanned at the ISR³ and signals the fact that the upper unitarity bound will be at least as stringent a constraint on data as it was at the ISR. Indeed if R were exactly 1, the absorptive elastic amplitude would be of the form $f(s, t) = f(s, 0) (8/T^2) J_2(T)$ with $T = \sqrt{-\sigma_{\text{tot}}^2 t / (3\pi\sigma_{\text{el}})}$. The slight deviation of R from 1 allows the upper and lower bounds to separate with the former being more restrictive on the data.

We will attempt in this article to quantify the saturation of unitarity at the SPS Collider, as was done before at the ISR³. We therefore, in section 2, review the bounds of ref.3 and outline the results of their application to the ISR data, so as to be able to contrast the situation at the Collider. We will then, in the main section 3, use the bounds on the SPS data to check its internal consistency and estimate the degree of saturation of unitarity, paying particular attention to the extraction of the real part for better comparison with the purely absorptive differential cross-section $d\sigma^A/dt$. We will then discuss the signals for approach to scaling in T (or related variables) and find that we expect this approach to be non-uniform in t as the energy increases. Although this will not constitute a rigorous proof of scaling, we expect the conclusion to be valid as long as the data continues to saturate the upper unitarity bound over a proportionately larger range of t as the energy increases.

2- Variable- t_1 unitarity bounds for the absorptive elastic amplitude

In the impact parameter (b) representation of the absorptive elastic amplitude

$$f_A(s, t) = \int_0^\infty h(\beta, s) J_0(\sqrt{-\beta t}) d\beta \quad (7)$$

the elastic profile $h(\beta=b^2, s)$ satisfies the unitarity and positivity requirement that, for any fixed b and high s

$$0 \leq h(\beta, s) \leq 1 \quad (8)$$

where the normalization is such that

$$\frac{d\sigma}{dt} = \pi |f(s, t)|^2 = \pi |f_R(s, t) + i f_A(s, t)|^2 \quad (9)$$

and the optical theorem is in the form

$$\sigma_{tot}(s) = 4\pi f_A(s, t) \quad (10)$$

It turns out that, in the calculation of the bounds, there exists a maximum squared impact parameter β_M which truncates (7) to a finite interval. This allows rescaling of (7) in terms of the dimensionless variable $x = \beta/\beta_M \in [0, 1]$ and the renormalized profile function

$$H(x) = 4\pi \beta_M h(\beta, s) / \sigma_{tot} \quad (11)$$

such that (7) now reads

$$f_A(s, t) = f_A(s, 0) \int_0^1 H(x) J_0(\sqrt{-\beta_M t x}) dx \quad (12)$$

The existence of β_M as well as the definitions for the input experimental quantities σ_{tot} , σ_{el}^A and $B_A(s, 0)\sigma_{tot}$ have respectively the simple forms³

$$H(x \gg 1) = 0 \quad (13)$$

$$1 = \int_0^1 H(x) dx \quad (14)$$

$$4d/3 = \int_0^1 H(x)^2 dx \quad (15)$$

$$R/(3d) = \int_0^1 H(x) x dx \quad (16)$$

where the parameter d is multiplicatively related to β_M by

$$\beta_M = \frac{d}{3\pi} \cdot \frac{\sigma_{tot}^2}{\sigma_{el}^A} \quad (17)$$

Because of the high inelasticity of high energy collisions (σ_{el}/σ_{tot} is only about 0.2), the upper bound of (8) is not really needed and unitarity only requires that

$$H(0 \leq x \leq 1) \geq 0 \quad (18)$$

It was shown, with the help of ISR data, that an additional measurement of the normalized absorptive amplitude at a suitably chosen t -value (say t_1)

$$\frac{f_A(s, t_1)}{f_A(s, 0)} = \int_0^1 H(x) J_0(\sqrt{-\beta_M t_1} x) dx \quad (19)$$

considerably improved the upper unitarity bound on $f_A(s, t)$ of (12).

Extremization of (12) which is a functional of $H(x)$ subject to the 4 integral constraints due to measured experimental quantities (equations (14)-(16) and (19)) involves 4 Lagrange multipliers which can be chosen to enter in the solution for the renormalized elastic profile in the form

$$H(x) = a_1 J_0(\sqrt{-\beta_M t_1} x) - a_2 - a_3 x - a_4 J_0(\sqrt{-\beta_M t_1} x) \quad (20)$$

In addition, whenever this expression becomes negative, it is truncated by the unitarity constraint (18).

At any given t -value in (20), substitution of (20) into the 5 equations (13)-(16) and (19) determines the 5 parameters $d, a_1 \dots a_4$, which, due to the constraints (13)-(16) and (19), are therefore implicitly functions of t . Hence substitution of the resulting $H(x)$ into (12) with the 5 parameters $d, a_1 \dots a_4$ determined by the 5 constraints, generates extremal values for $f_A(s, t)$ at that value of t . By continuously varying t , we thus generate continuous extremal curves for $f_A(s, t)$. The positivity condition (18) was found to be relevant for a substantial range of t , but, when it isn't ($H(x)$ being naturally positive for the solved values of d and the a_i), the explicit integration of (12) as well as (14)-(16) and (19) is possible in simple closed form (this occurred at the ISR for $0.25 \leq |t| \leq t_1$). These explicit expressions³ provide a check on the numerical integration procedure for these values of t . From (12) one can see that they involve only the reduced dimensionless variable $\tau = -\beta_M t = -6dB_{MM}t$ and Bessel functions of $\sqrt{\tau}$ (J_0 and J_1). If d is a slowly varying function of t (as it turns out to be), we have scaling of the bound in the variable $B_{MM}t$ or $(\sigma_{tot}^2/\sigma_{el}^A)t$ or the variable of Auberson and Roy¹² (note that it is the bound normalized to 1 in the forward direction that scales; from now on, we will not mention this fact explicitly). Clearly if the data saturate the bound over a given τ interval and d is sufficiently slowly varying there, then the data must approximately scale in that variable. We will return to this concept in the last section. We now turn to some of the ISR results which will also hold at the SPS Collider.

The lower bound curve was found to be a constraint on the ISR data only in the very forward region ($|t| \leq 0.04 \text{ GeV}^2$). Since this region is not well explored at the SPS Collider at the moment, we shall disregard it from now

on. However, the upper bound can be quite restrictive depending on the choice of t_1 (hence the name "variable- t_1 bound"). With a choice of t_1 at approximately half the value of the dip position, the bounds are saturated by the data over a wide t -range ($0.1 \leq |t| \leq 0.55 \text{ GeV}^2$). In order to compare later with SPS data, it is more meaningful to say that the bound is saturated by the data for approximately 40% of the t -range to the dip position, since any such t -structure would move inwards with increasing s if scaling in τ holds for the data (as geometrically scaling models would like it to). For sufficiently small $|t|$, the bound reverts to the Roy-Singh bound¹³, corresponding to removing the constraint on the elastic slope embodied in equation (16).

In fact, there are several instances of a mild violation of the upper bound in the ISR data: for $0.25 \leq |t| \leq 0.4 \text{ GeV}^2$ at $\sqrt{s}=23.5 \text{ GeV}$, for $0.25 \leq |t| \leq 0.35 \text{ GeV}^2$ at $\sqrt{s}=52.8 \text{ GeV}$ and $0.2 \leq |t| \leq 0.3 \text{ GeV}^2$ at $\sqrt{s}=62.5 \text{ GeV}$. We call this a mild violation since it can be removed by changing the input global quantities involved in R within their quoted error bars. In this way, the saturation of the bound by the data can lead to a better determination of these global quantities consistent with unitarity and the whole of the data set. Alternatively, the violation can be removed by a small rescaling of data gathered by different experimental groups. The bounds then help in normalizing data sets in non-overlapping t -ranges by taking t_1 such that $f(t_1)$ is a representative value of the data set at larger $|t|$. We shall return to all these points again for the SPS data. In addition, the bounds can rule out any meaningful physical explanation for possible rapid oscillations in the data³. We conclude by mentioning that choosing t_1 to be the location of the zero in $f_A(s,t)$ provides very little improvement over the Auberson-Roy bound¹², since the bound actually increases (!) with $|t|$ over a wide t -range before the zero³.

3- Application of the bounds at the SPS Collider

Because of the increased importance of the real part contributions to $d\sigma/dt$ at the SPS, as is evident by the evolution of the ISR dip into a broad shoulder around $-t=0.8 \text{ GeV}^2$ at the SPS, one has to be more careful in extracting the real part in order to compare the data with the bounds for $f_A(s,t)$ or $d\sigma^A/dt=\pi f_A^2(s,t)$. This should be done for all t , even in the forward direction as $\rho(s,t \approx 0)$ is expected³ to be ≈ 0.15 at the SPS. Therefore, even in the forward direction, the real part contribution to $d\sigma/dt$ is more than twice as important as it was at the ISR. As a starting

working hypothesis, we will parametrize the full amplitude

$f(s,t)=f_R(s,t)+if_A(s,t)$ as

$$f(s,t)=\begin{cases} 13.284 (i+\rho) e^{8.65t} & , \quad 0 \leq |t| \leq .2 & (21a) \\ 9.268 (i+\rho) e^{6.85t} & , \quad .2 \leq |t| \leq .5 & (21b) \\ 0.0844 i e^{i\theta} \left(e^{5.45(t-t_0)} + e^{2.3(t-t_0)+i\phi} \right) & , \quad |t| \geq .5 & (21c) \end{cases}$$

with¹⁰ $-t_0=0.81 \pm 0.01$ and $\phi=\pi-0.39 \pm 0.16$. We extend the validity of (21c) beyond the region over which it was obtained ($0.5 \leq -t \leq 1.5$). The slopes (in GeV^{-2}) have the most recent values^{10,14} which are the result of weighing the UA1 and UA4 results in the forward direction and the latest analysis of UA4 data otherwise. The normalization factors (also in GeV^{-2}) are determined by the optical point value (equations (2) and (10)) and by continuity of $d\sigma/dt$ in the successive t ranges. The Barger-Phillips type parametrization for $d\sigma/dt$ defines f only up to an arbitrary phase factor $e^{i\theta}$ which we will take as 1 for the moment. Similarly, continuity of $d\sigma/dt$ alone, allows both signs for the real part for $0.2 \leq -t \leq 0.5$. In the absence of any model bias for the real part for $-t \leq 0.5$ we take it to have the same t -dependence as the imaginary part, as is commonly done in Coulomb interference experiments. There is no good reason to do so over such large ranges of t . Indeed³, at the ISR, derivative analyticity relations predict a zero for $f_R(s,t)$ near $-t=0.2$; this can be mimicked somewhat by taking the lower sign in (21b). However the essential point is that the real part is only really important for $-t > 0.5$ and its strength, relative to the imaginary part, can be fixed with θ , which we will do later. Note that, as a check on our normalization procedure, we can compute $d\sigma/dt$ near $t=t_0$ by (21c) and obtain approximately $1.3 \times 10^{-3} \text{ mb/GeV}^2$ in good agreement with the data¹⁴. Consequently, the largest source of error is through the overall normalization via σ_{tot} of (2).

Using the numerical procedure described in reference 3, it is a straightforward task to compute the upper bound for the values (2)-(4) leading to the small value of R in (5) (one can replace C05NAF, which has been dropped by NAGLIB, with C05NCF). We can estimate σ_{el}^A by $\sigma_{\text{el}}/(1+\rho^2)$ but the parametrization (21) forces us to equate $B_A(t=0) = B(t=0)$. The values $\sigma_{\text{el}}^A=12.4 \text{ mb}$ and $R=1.118$ are then obtained. The startling result is shown in Figure 1a for $-t_1=0.5 \text{ GeV}^2$ using an $f(t_1)$ given by (21b); in this figure, we show as a solid line the upper bound for $d\sigma^A/dt$ and we compare it to the "experimental" parametrization for $d\sigma^A/dt$ with $f_A(s,t)$ given by equation (21) (dashed line). The upper bound is actually lower than the "data" over a wide t -range and is not consistent with it within the errors

shown! There is however no cause for alarm. The reader may have gotten the impression that R is a very well determined quantity by the fact that I quote it to three decimal places (for purposes of reproducibility). In fact this is not so, as one can see from the errors of about 5% in each of the global quantities (2)-(4). However, because these errors are highly correlated by equation (6), an estimate of the error in R is difficult. It would be useful if experimenters could provide us with such an estimate, as a "band" of upper bounds could then be generated. Lacking this information on the saturation of the MacDowell-Martin bound, we can give only a crude estimate of the variability of the upper bound by changing only one of these quantities, so as to increase R and reduce the power of the bound, as mentioned in the introduction. It is convenient to pick σ_{tot} as this quantity (thus $\sigma_{\text{tot}}=61.5$) since this only rescales the "experimental" $d\sigma^A/dt$ by $(61.5/65)^2$ while it rescales the bound for $d\sigma^A/dt$, changes the scaling variable τ and modifies R , which can be thought of as a label for the possible bounds at fixed τ . The total cross-section also happens to be measured by both UA1 and UA4 and seen to agree fairly well. Indeed UA1 reports¹⁵ $\sigma_{\text{tot}} = 67.6 \pm 5.9 \pm 2.7$ mb in good agreement with (2) despite the fact that $p=0$ was used. If one uses instead $p=0.15$, one gets a best value of 66.85 mb which is even closer to (2). However UA1 also reports¹⁵ $\sigma_{\text{el}}/\sigma_{\text{tot}} = 0.209 \pm 0.018 \pm 0.008$ which is only in fair agreement with (3). This may be due to the use of an $\exp(bt+ct^2)$ fit to normalize their data taken over different t -ranges, while UA4 doesn't follow the same procedure. The new $0.03 \leq -t \leq 0.3$ preliminary¹⁰ 1983 UA4 data, while still perfectly consistent with their two exponentials (equations (21a) and (21b)) could also show a curvature term, which would be more compatible with model calculations in which an upward curvature in $\ln(d\sigma/dt)$ is interpreted as a rescattering effect both in the near forward direction as well as after the dip (or shoulder)¹⁶. Hence with the lowest value for σ_{tot} in (2) and the same values (3) and (4) as input for σ_{el}^A and $B(t=0)$, we obtain $R=1.181$ and the solid curve drawn in Figure 1b, which is again compared with the "experimental" $d\sigma/dt$. Clearly the presumed violation of the bound has disappeared, but two important lessons have been learned:

- 1) As the statistics improve, the final value of R will probably settle down between the two values mentioned above and the elastic data will probably saturate the corresponding upper bound over the range shown ($0 \leq -t \leq 0.5$) thus suggesting that the true $d\sigma/dt$ in the SPS energy range scales in the variable $B_{\text{MM}}t$. For example, with $B=17.6$ and the mean values (2)-(3), the bound passes through the data within the existing errors which can thus be thought to scale. Hence a 2-3% change in any of the quantities entering in R is sufficient to remove the "violation"

observed in Figure 1a.

- 2) If the final value of R is too low and some violation of the unitarity bound persists in one particular larger $-t$ section of the data set, the bound can then be used to normalize that larger $-t$ data with respect to smaller $-t$ data. In this context, note that the clear violation of the bound in Figure 1a starts around $-t=0.2$ where the larger $-t$ data begins (around $-t=0.2$). Presumably, the new 1983 data in the range $0.03 \leq -t \leq 0.3$ will help in this regard.

A natural question now comes to mind. Since we are so close to saturation of unitarity over two-thirds of the t range to the shoulder is it possible that unitarity is saturated all the way to the zero in $f_A(s,t)$ at the Collider? To answer this question, we need to be able to reliably extract the real part contribution to $d\sigma/dt$, i.e. to evaluate θ in (21c). We now turn to this task in as model-independent a way as possible.

By (21b) we can have $\rho(-t_1=0.5) = -0.15$ as argued earlier, while for $\theta=0$ by (21c) we have $\rho(-t_1=0.5) = -0.22$. Neither need be a good indication of the true value of ρ there. We have argued that the data approximately scale in $B_{MM}t$ up to $-t_1$ since they nearly saturate the bound which is a function of $dB_{MM}t$ with d a slowly varying function of t (for example d varies between 1.46 and 1.70 while $|t|$ more than doubles its value between $-t=0.23$ and 0.49 for the bound of Figure 1a). We would therefore hope to be able to use the well-known equation¹⁷

$$\rho(s,t) = \frac{\rho(s,0)}{\Phi(\tau)} \frac{d}{d\tau} (\tau \Phi(\tau)) \quad (22)$$

to normalize $\rho(s,t_1)$ given the scaling function $\Phi(\tau)$ for the absorptive amplitude normalized to 1 in the forward direction $f_A(s,t)/f_A(s,0)$. At least for $|t| \leq 0.5$ the parametrization of (21) for $f_A(s,t)$ should be a good ansatz to estimate the t -dependence of a more plausible small $f_R(s,t)$ than the one of (21). For a purely exponential behaviour $\Phi(\tau) = e^{-C\tau} = e^{bt}$, equation (22) simply yields

$$\rho(s,t) = \rho(s,0) (1+bt) \quad (23)$$

Hence equation (21a) would force $\rho(s,t)=0$ at $-t=0.116 \text{ GeV}^2$ while from (21b) this would happen at $-t=0.146 \text{ GeV}^2$. In view of such an uncertainty, the prediction, using (21b) and (23) that $\rho(s,t_1) \cong -0.364$ may be taken with a grain of salt. We should first test the validity of scaling in τ or $B_{MM}t$, to which we now turn.

Using $f_A(s,t)$ of (21), we can extract $h(\beta,s)$ at the Collider by taking the inverse transform of (7) and then compare it to the same quantity at

the ISR, appropriately scaled by $B_{MM}(s)$. This exercise yields an overlap function $G(\beta, s) = 4h(\beta, s)(1-h(\beta, s))$ at $\sqrt{s}=540$ GeV which is displayed in Figure 2 as a function of $\beta=b^2$, with the dashed lines indicating the uncertainty due to $\sigma_{tot} = 65 \pm 3.5$ (the upper (lower) signs corresponding naturally to the upper (lower) dashed lines). As a first approximation $G(\beta, s)$ is exponential in β , i.e. we have a nearly Gaussian overlap function in b at least up to $\beta \approx 6.5 \text{ fm}^2$. At larger β , artificial b -structures can occur due to the break at $-t=0.2$ and the small discontinuity in f_A at $-t=0.5$ (recall that we required $d\sigma/dt$ to be continuous there, yet $\rho(s, -t=0.5)$ has different values according to either (21b) or (21c)). A similar exercise can be performed¹⁸ at the ISR energy of $\sqrt{s}=52.8$ GeV with a sum of five exponentials fit whose quality can be seen in reference 19. Comparing this ISR overlap function with the values of Amaldi-Schubert⁴, we obtain good agreement, the maximal deviation being of the order of 0.0028 around 1.4 fm.

We can then test scaling of f_A in $(B_{MM}t)$ by looking at scaling of h or G as a function of β/B_{MM} , as is evident from the Fourier-Bessel transformation of equation (7). In Figure 3 is plotted the experimental difference $\Delta G(b, s) = G(b, \sqrt{s}=540) - G(b, \sqrt{s}=52.8)$ showing a slight rise at zero impact parameter which is inconsistent with the overlap function G scaling in β/B_{MM} . The strength of this violation is smaller than what was found previously¹⁸ because σ_{tot} is now only 65 ± 3.5 as opposed to the 71 mb used in reference 18. Taking into account the above errors in σ_{tot} (which generate the dashed lines), a signal for the violation of scaling is still present at $b=0$. One can actually do better if one plots the quantity

$$\Delta G(\beta) = G_{SPS}(\beta) - G_{ISR}\left(\beta' = \beta \frac{B_{MM}^{ISR}}{B_{MM}^{SPS}}\right) \quad (24)$$

where ISR and SPS refer to the same two energies as before. In doing so we remove the large signal of the peripheral increase due to approximate β/B_{MM} scaling which was already seen over the ISR energy range⁴. If there truly existed a scaling function for G , $\Delta G(\beta)$ should be identically zero; from Figure 4, it is obvious that this is not so, both at $b=0$ (as shown in Figure 3) but also for a substantial range of b (up until $b \approx 2.4$ fm at the SPS, at which point $G(b)$ starts to artificially decrease quickly in Figure 2). This remaining energy dependence in the data, additional to the one obtained by rescaling with $B_{MM}(s)$, is the signal for a blacker, edgier proton (or antiproton) found earlier¹⁸ and reported by other investigators²⁰ in the context of specific models. It appears here as an additional peripheral increase in $\Delta G(\beta)$ of Figure 4 centered around 0.8 fm, in good agreement with previous results¹⁸ using earlier data which revealed

a stronger signal for scaling violation.

Confronted with the above probable violation of scaling over a substantial b -range, one needs to confirm that the utilization of (22) and (23) yields a reasonable $\rho(s, t_1)$ at the Collider. This confirmation requires the use of a model such as the BEL ansatz¹⁸ for $G(b, s)$ either in its original form or in a form applicable over a wide energy range²¹. The net result is that, for real parts not obeying (22) but reproducing the SPS shoulder very well, $\rho(s, t_1)$ can take values between $-.32$ and $-.37$, in remarkable agreement with the prediction of $-.364$ of equations (22) and (23). For this t -value, it therefore seems that the data scale well and that ρ has a value around $-.36$. Using this value, the small angle θ in (21c) is fixed to $\theta \approx .13$. We then have $f_A(s, t)$ for all $-t \geq 0.5$. This is shown in Figure 5 along with the bound using (2)-(4) and specifying $f_A(t_1=0.6)$. There is again reasonable saturation of unitarity, given the present experimental uncertainties. Increasing t_1 further, for example to $-t_1=0.818$, the location of the zero in f_A , does not further constrain the data, as can be seen in Figure 6. It therefore seems that, although the data scale over a proportionately larger range of t at the SPS than at the ISR (60-70% of the t -range to the shoulder as opposed to only $\approx 40\%$ of the t -range to the dip at the ISR), the data immediately before the zero in $f_A(s, t)$, and after it also, are unrestricted by the bound and completely free to choose not to scale, which is precisely what they seem to do! It is not surprising therefore that geometrically scaling (GS) models are too low by a factor 10 at the shoulder²², since the bound shown in Figure 6 would allow the data to attain values as high as $d\sigma/dt \approx 1.5 \text{ mb/GeV}^{-2}$ around $-t=1.3 \text{ GeV}^2$! We are therefore led to a picture (although this is not rigorous) where the approach to scaling is non-uniform in t but is closer to being attained (over a larger t range) at the SPS than at the ISR. The good agreement of GS over the small ISR energy range may have been more of an accident than was previously thought. Only when the c.m. energy is nearly decoupled do the weaknesses of exact GS show up and these occur at large $-t$ where bounds still impose no constraint (therefore violations of GS also show up at small b in Figure 4). The global tests of GS such as the constancy of σ_{el}/σ_{tot} and $B(t=0)/\sigma_{tot}$ work relatively well because they are sensitive to smaller $-t$ data where the bounds are quite restrictive. We consider therefore these ratios as tests of GS that are not sensitive enough to its slight violation (we emphasize the word slight since, after all, $\Delta G(b)$ of Figure 3 is so much larger than $\Delta G(\beta)$ of Figure 4).

Finally, most of the other features found in the ISR analysis of

reference 3 are enhanced at the SPS.

- 1) Because the bounds are saturated over a wider range of τ , equation (20) is a slower function of t and the bounds' average profile better reflects the true profile $h(\beta, s)$.
- 2) The bound's profile $h(\beta, s)$ is nearly exponential in β despite the fact that no such function is explicitly in (20). The parameters $d, a_1 \dots a_4$ adjust themselves so as to mimic pretty well a Gaussian overlap function in b such as the one in Figure 2. As an example, the bound's profile at $-t=0.3$ for the solid curve of Figure 1 is nearly exponential until $\beta = 5.8 \text{ fm}^2$.
- 3) Given that the SPS Collider may operate at different energies soon, a qualitative treatment of the energy dependence of the true $h(\beta, s)$ may be done by specifying just one value $f(t_1)$ at a carefully chosen moderate t_1 -value, by using the fact that the data saturate the bound up to that t_1 value.
- 4) The addition of the local t_1 constraint (19) to the set of global constraints (14)-(16) (which constitute the Auberson-Roy bound¹²) significantly improves the useful range of the bound. The reason for this is quite simple: while the integral constraints (14)-(16) involve slowly varying integrands, equation (19) has an oscillatory factor $J_0(\sqrt{-\beta_M} t_1 x)$ which we can vary at our leisure to probe more finely the structures in $H(x)$. It is therefore to be expected that a suitably averaged $H(x)$ represents more closely the true $h(\beta, s)$ (thus explaining points 1) and 2)).

4- Conclusions

The constraints imposed by the variable- t_1 bounds upon the preliminary SPS Collider data have been found to be quite restrictive. We can envisage their future usefulness in obtaining a better determination of the global quantities entering in the definition of R , i.e. σ_{el} , σ_{tot} and $B(t \approx 0)$. They could also be used to normalize data gathered over different non-overlapping t -regions in a way which would preserve the internal consistency of the data set. The SPS data were seen to saturate the upper bound over a proportionately wider range of t than at the ISR, suggesting that scaling is becoming applicable over a proportionately wider range of t , given the present experimental uncertainties. At larger $-t$ ($-t \geq 0.6$) where the bounds provide no restrictions, the data is seen not to scale. This reflects itself in scaling violation at small b . The approach to scaling is thus naturally non-uniform in t . The extent of this slight

scaling violation was also quantitatively studied in impact parameter space.

ACKNOWLEDGEMENTS

The author wishes to thank A. Martin for reading the manuscript and the CERN Theoretical Physics Division for its hospitality.

REFERENCES

- 1- U.Amaldi et al., Phys. Lett. 66B (1977) 390
- 2- A.Martin, Z. Phys. C15 (1982) 185
- 3- P.Valin, Nucl. Phys. B218 (1983) 215
- 4- U.Amaldi and K.R.Schubert, Nucl. Phys. B166 (1980) 301
- 5- S.W.MacDowell and A.Martin, Phys. Rev. 135 (1964) 960
- 6- G.Matthiae, Plenary Session P10 talk on "Soft Hadronic Interactions" at the EPS High Energy Physics Conference (Brighton, England, 20-27 July 1983)
- 7- J.P.Burq et al., Phys. Lett. 109B (1982) 124
- 8- A.Martin, Journal de Physique, Supp. au #12, Tome 43 (1982) p.C3-164
- 9- N.Amos et al., Phys. Lett. 128B (1983) 343
- 10- Discussion Meeting on SPS Physics held on Nov. 11th 1983 at the CERN TH Division.
- 11- U.Amaldi et al., Nucl. Phys. B145 (1978) 367
- 12- G.Auberson and S.M.Roy, Lett. Nuovo Cim. 19 (1977) 97
- 13- S.M.Roy and V.Singh, Phys. Rev. Lett. 24 (1970) 281 and Phys. Rev. D1 (1970) 2638
- 14- M.Bozzo et al., "Elastic Scattering at the CERN SPS Collider", Contributed Paper #116 to the EPS HEP Conference (Brighton, England, 20-27 July 1983)
- 15- G.Arnison et al., Phys. Lett. 128B (1983) 336
- 16- R.Henzi and P.Valin, "What is Elastic Scattering Past the Diffraction Dip Telling Us?", Contributed paper #260 to the EPS HEP Conference

(Brighton, England, 20-27 July 1983)

- 17- A.Martin, Lett. Nuovo Cim. 7 (1973) 811
J.Dias de Deus and P.Kroll, Acta Phys. Pol. B9 (1978) 159
- 18- R.Henzi and P.Valin, Phys. Lett. 132B (1983) 443
- 19- S.Sanielevici and P.Valin, "Hadronic Matter Form Factors: πp , pp and the Valon Model", to appear in Phys. Rev. D (Jan. 1 1984)
- 20- T.T.Chou and C.N.Yang, Phys. Lett. 128B (1983) 457
- 21- P.Valin, " pp and $\bar{p}p$ Elastic Scattering from $p_{lab} = 30$ to 300,000 GeV/c" (in preparation)
- 22- P.Kroll, "Recent Theoretical Advances in Elastic Scattering", Talk at the Parallel Session D2 of the EPS HEP Conference (Brighton, England, 20-27 July 1983)

FIGURE CAPTIONS

- Fig. 1: Upper bounds for the absorptive elastic differential cross-section compared with its value $d\sigma^A/dt$ given by the parametrization for f_A in (21) (dashed straight lines). Typical experimental error bars are shown for the medium t -range only¹⁴, as they are much larger for the small $-t$ run; a) with the best values for σ_{tot} , σ_{el}^A and $B(t\approx 0)$ of (2)-(4) leading to a value of $R=1.118$; b) with the lowest value for σ_{tot} allowed by (2) but the same σ_{el}/σ_{tot} and $B(t\approx 0)$ as in a), thus leading to a value of $R=1.181$.
- Fig. 2: $G(\beta)$ as a function of $\beta=b^2$ at the SPS ($\sqrt{s} = 540$ GeV) with the solid line and dashed lines corresponding to normalizations of f_A in (21) with $\sigma_{tot} = 65\pm 3.5$ mb.
- Fig. 3: $\Delta G(b) = G(b, \sqrt{s}=540) - G(b, \sqrt{s}=52.8)$ shows the peripheral increase of the overlap function G in going from the ISR to the Collider and the small but significant increase at $b=0$ indicating scaling violation.
- Fig. 4: The residual difference between the experimental $G(\beta)$ at the SPS and the rescaled $G(\beta)$ obtained from the ISR, according to equation (24). The solid and dashed lines correspond to the different $G_{SPS}(\beta)$ of Figure 2 and correspond to different σ_{tot} values.
- Fig. 5: Upper bound for $d\sigma^A/dt$ obtained by specifying $f_A(-t_1=0.6)$ with equation (21c) and $\theta\approx 0.13$, using the mean values (2)-(4) for the global quantities compared with f_A of (21) (dashed line). Errors in the data for $-t\leq 0.5$ are shown in Fig. 1. For $-t\geq 0.5$, one cannot use the errors for $d\sigma/dt$, since we have extracted a significant and slightly model dependent real part contribution.
- Fig. 6: As in Figure 5, but specifying $f_A(-t_1=0.818)=0$. At the ISR, such a constraint on the zero in f_A is even less restrictive, since the bound actually goes up before reaching the zero! This is yet another indication of the non-uniform approach to scaling.

Fig. 1a

Fig. 1b

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6