SPONTANEOUS CP VIOLATION IN LEFT-RIGHT
SYMMETRIC GAUGE THEORIES

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A B S T R A C T

Motivated by arguments in favour of spontaneous CP violation, we investigate the general conditions imposed by P and CP invariance on the Yukawa couplings in $SU(2)_L \times SU(2)_R \times U(1)$ gauge theories. A complete discussion of these constraints is presented for two minimal Higgs sectors. In addition to the standard case of manifest CP invariance, we find two phenomenologically interesting models where the P and CP transformations are non-aligned in flavour and Higgs spaces. To a certain extent, P and CP invariance may be responsible for the existence of horizontal symmetries. An intimate connection between CP violation and the general flavour problem emerges.

1. INTRODUCTION

The origin of CP violation is still an open problem. In the standard model¹⁾ with a single scalar doublet and with three generations, CP violation is parametrized by a phase in the Kobayashi-Maskawa mixing matrix²⁾. This phase derives from complex Yukawa couplings between the quarks and the scalar Higgs field. There are at least two shortcomings associated with such an explicit hard breaking of CP:

- i) CP violation is put in by hand and there is no relation to the gauge symmetry;
- ii) if the Higgs field is composite and if the underlying theory contains only gauge bosons and fermions, phases can only be generated spontaneously³⁾.

There is a competing class of gauge models⁴⁾ based on the group $SU(2)_L \times SU(2)_R \times U(1)$ where both parity and charge conjugation may be symmetries of the Lagrangian. The spontaneous breaking of both discrete symmetries occurs together with the gauge symmetry breaking. All symmetries of the theory appear on the same footing. Although the mechanism of spontaneous symmetry breaking still needs to be clarified, such a unified treatment of gauge and discrete symmetry breaking seems to offer a better chance for an explanation of CP violation.

In $SU(2)_L \times SU(2)_R \times U(1)$ gauge models there are two different scales of spontaneous symmetry breaking, the smaller one being essentially the same as in the standard model. If the larger scale where parity invariance is restored is sufficiently high, the low energy phenomenology of such models will be almost indistinguishable from the standard model. The best way to locate possible deviations is to concentrate on processes where certain unique features of the standard model play an essential rôle. From studies of both flavour changing neutral interactions⁵⁾⁻⁸⁾ and of the neutron electric dipole moment⁹⁾, it has recently become clear that the scale of parity violation must be at least in the several TeV range. Consequently, most of the "low energy" aspects, such as the structure of neutral currents and the masses of the lowest-lying charged and neutral intermediate vector bosons, are practically identical in the standard and in the left-right symmetric models. However, this need not be the case for the manifestations of CP violation⁸⁾ as indicated by the neutron electric dipole moment⁹⁾.

In this paper we want to investigate the general structure of $SU(2)_L \times SU(2)_R \times U(1)$ gauge theories with spontaneous CP violation. More precisely, we shall be concerned with the structure of P and CP conserving Yukawa couplings between quarks and scalar fields. It is well known that a sufficient condition for P and CP invariant Yukawa couplings is the existence of a basis of weak eigenstates in which the coupling matrices are real and symmetric. It will be shown that this is actually only a special case where P and CP are in a certain sense aligned in flavour and Higgs spaces. We shall argue that a more general treatment of P and CP transformations may be useful.

In both the standard and the left-right symmetric models, the question of CP violation is one aspect of the more general problem of weak mixing angles. Therefore, it seems almost inevitable that a satisfactory explanation of CP violation should concur with an at least partial understanding of the generation problem¹⁰⁾. The standard approach assumes some remainder of the original chiral symmetry of the fermion-gauge boson interaction to manifest itself in the Yukawa interactions (and in the Higgs potential which will not concern us here). Upon spontaneous symmetry breaking, such horizontal symmetries yield constrained fermion mass matrices implying in particular restrictions for CP violation. Although sometimes considered ad hoc and therefore unattractive, horizontal symmetries have the virtue of allowing in general only very few inequivalent quark mass matrices for a given set of fundamental fields¹¹⁾⁻¹³⁾. For $SU(2)_L \times SU(2)_R \times U(1)$ models with a minimal Higgs sector, it was shown earlier^{12),13)} that for any possible horizontal symmetry all non-trivial models have CP conserving Yukawa interactions.

We may now try to turn the argument around. Assuming P and CP invariance, but no horizontal symmetries, can we obtain any models with constrained mass matrices? It will turn out that the answer is affirmative and that, moreover, most of the horizontal models can be recovered from P and CP invariance alone. In those models, P and CP determine the structure of mass matrices and there is an obvious connection between CP and the flavour problem. In a way, CP invariance explains the origin of horizontal symmetries for such models.

In Section 2, we consider the most general CP transformations for an $SU(2)_L \times SU(2)_R \times U(1)$ gauge theory with an arbitrary number of matter fields. If both the Lagrangian and the ground state are CP invariant, those

transformations assume the familiar form for the quark mass eigenstates and the left- and right-handed mixing matrices may be chosen real. In Section 3 we determine all P and CP conserving Yukawa couplings for two minimal Higgs sectors and for at most three generations. The solutions corresponding to constrained quark mass matrices are compared in Section 4 with models obtained from horizontal symmetry. For each of the two Higgs sectors, only one model survives phenomenological scrutiny. In Section 5 we present our conclusions. Two appendices contain some details of the analysis of Section 3.

2. GENERAL CP TRANSFORMATIONS IN $SU(2)_L \times SU(2)_R \times U(1)$

The Lagrangian involving only the gauge fields $\vec{W}_{L\mu}$, $\vec{W}_{R\mu}$, B_μ and the quark fields $q_L = \begin{pmatrix} p \\ n \end{pmatrix}_L$, $q_R = \begin{pmatrix} p \\ n \end{pmatrix}_R$ transforming under the gauge group as $(1/2, 0, 1/6)$ and $(0, 1/2, 1/6)$, respectively, is invariant under the general CP transformations

$$\begin{aligned} q_L(x^0, \vec{x}) &\rightarrow U_{CP} \gamma^0 C \bar{q}_L^T(x^0, -\vec{x}) = -U_{CP} C q_L^*(x^0, -\vec{x}) \\ q_R(x^0, \vec{x}) &\rightarrow V_{CP} \gamma^0 C \bar{q}_R^T(x^0, -\vec{x}) = -V_{CP} C q_R^*(x^0, -\vec{x}) \\ \vec{W}_{L(R)\mu}(x^0, \vec{x}) \vec{\tau} &\rightarrow (-1)^{\delta_{0\mu}} \vec{W}_{L(R)\mu}(x^0, -\vec{x}) \vec{\tau}^T \\ B_\mu(x^0, \vec{x}) &\rightarrow (-1)^{\delta_{0\mu}} B_\mu(x^0, -\vec{x}) \end{aligned} \quad (2.1)$$

where U_{CP} , V_{CP} are unitary n_G -dimensional matrices (n_G is the number of generations). Before spontaneous symmetry breaking, there is no guiding principle how to choose these matrices because the weak eigenfields are indistinguishable at this stage.

Of all the scalar fields in the theory we are only concerned with the fields Φ_α and their charge conjugates $\tilde{\Phi}_\alpha = \tau_2 \Phi_\alpha^* \tau_2$ ($\alpha=1, \dots, h$). Only these Higgs fields transforming as $(1/2, 1/2, 0)$ under the gauge group can have Yukawa couplings to the quarks. Their gauge couplings are given by

$$\mathcal{L}_\Phi = \sum_{\alpha=1}^h \text{tr} (\mathcal{D}_\mu \Phi_\alpha)^\dagger (\mathcal{D}^\mu \Phi_\alpha) = \frac{1}{2} \sum_{\beta=1}^{2h} \text{tr} (\mathcal{D}_\mu X_\beta)^\dagger (\mathcal{D}^\mu X_\beta) \quad (2.2)$$

where D_μ denotes the covariant derivative and $X_\alpha = \Phi_\alpha$, $X_{\alpha+h} = \tilde{\Phi}_\alpha$ ($1 \leq \alpha \leq h$). From (2.1) and (2.2) the most general CP transformation for the fields X leaving $\mathcal{L}_\Phi$ invariant is found to be

$$X(x^0, \vec{x}) \rightarrow H^{CP} X^*(x^0, -\vec{x}) \quad (2.3)$$

with a unitary, $2h$ -dimensional matrix H^{CP} . Because the components of X are not all independent H^{CP} has the structure

$$H^{CP} = \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \quad (2.4)$$

with $h \times h$ matrices A, B.

The Yukawa interaction of quarks and scalar fields is of the general form

$$\mathcal{L}_Y = - \sum_{\alpha=1}^h (\bar{q}_L \Gamma_\alpha \Phi_\alpha q_R + \bar{q}_L \Delta_\alpha \tilde{\Phi}_\alpha q_R + h.c.) \quad (2.5)$$

with n_G -dimensional coupling matrices Γ_α , Δ_α . CP invariance of (2.5) is expressed by the equations

$$\begin{aligned} \Gamma_\alpha^* &= U_{CP}^\dagger (A_{\beta\alpha} \Gamma_\beta + B_{\beta\alpha}^* \Delta_\beta) V_{CP} \\ \Delta_\alpha^* &= U_{CP}^\dagger (B_{\beta\alpha} \Gamma_\beta + A_{\beta\alpha}^* \Delta_\beta) V_{CP} \\ \alpha, \beta &= 1, \dots, h \end{aligned} \quad (2.6)$$

or, in a more compact notation,

$$G_\alpha^* = H_{\beta\alpha}^{CP} U_{CP}^\dagger G_\beta V_{CP} \quad (\alpha, \beta = 1, \dots, 2h) \quad (2.7)$$

with $G_\alpha = \Gamma_\alpha$, $G_{\alpha+h} = \Delta_\alpha$ ($\alpha = 1, \dots, h$).

The relations (2.6) or (2.7) will be the starting point for the general investigation of the subsequent section. Note that the special choice $U_{CP} = V_{CP} = 1$, $H^{CP} = 1$ leads to the familiar case of real Yukawa couplings without any further restrictions on Γ_α , Δ_α (manifest CP invariance).

In order to set our notation, we write the quark mass matrices as

$$\begin{aligned} M_p &= (v_\alpha \Gamma_\alpha + w_\alpha^* \Delta_\alpha) / \sqrt{2} \\ M_n &= (w_\alpha \Gamma_\alpha + v_\alpha^* \Delta_\alpha) / \sqrt{2} \end{aligned} \quad (2.8)$$

with vacuum expectation values

$$\langle \Phi_\alpha \rangle_0 = \begin{pmatrix} v_\alpha / \sqrt{2} & 0 \\ 0 & w_\alpha / \sqrt{2} \end{pmatrix}.$$

Unitary matrices $U_{L,R}^{p,n}$ can be chosen to diagonalize M_p, M_n :

$$\begin{aligned} U_L^{p\dagger} M_p U_R^p &= \hat{M}_u \\ U_L^{n\dagger} M_n U_R^n &= \hat{M}_d \end{aligned} \quad (2.9)$$

with positive, diagonal $\hat{M}_{u,d}$. The CP invariance conditions (2.6) restrict the mass matrices and, consequently, the left- and right-handed mixing matrices

$$K_{L(R)} = U_{L(R)}^{p\dagger} U_{L(R)}^n. \quad (2.10)$$

Let us consider a theory with both a CP invariant Lagrangian and a CP invariant ground state, i.e.,

$$H_{\alpha\beta}^{CP} \langle X_\beta \rangle_0^* = \langle X_\alpha \rangle_0. \quad (2.11)$$

We want to prove that in this case the CP transformations for the mass eigenfields u, d assume the familiar form

$$\psi(x^0, \vec{x}) \rightarrow -C \psi^*(x^0, -\vec{x}) \quad (\psi = u_{L(R)}, d_{L(R)}) \quad (2.12)$$

and the mixing matrices K_L, K_R may be chosen real. This fact provides an additional justification for calling the transformations (2.1) CP transformations.

From (2.7) and (2.11) we obtain

$$U_{CP}^\dagger M_p V_{CP} = M_p^*$$

$$U_{CP}^\dagger M_n V_{CP} = M_n^* \quad (2.13)$$

(2.9) and (2.13) then imply

$$[U_L^{p(n)\dagger} U_{CP} U_L^{p(n)*}, \hat{M}_{u(d)}^2] = [U_R^{p(n)\dagger} V_{CP} U_R^{p(n)*}, \hat{M}_{u(d)}^2] = 0. \quad (2.14)$$

Assuming non-degenerate quark masses and using (2.13) once more, we find

$$\begin{aligned} U_L^{p\dagger} U_{CP} U_L^{p*} &= U_R^{p\dagger} V_{CP} U_R^{p*} = P_u \\ U_L^{n\dagger} U_{CP} U_L^{n*} &= U_R^{n\dagger} V_{CP} U_R^{n*} = P_d \end{aligned} \quad (2.15)$$

where the diagonal matrices P_u, P_d represent the original U_{CP}, V_{CP} on the mass eigenfields u, d . The diagonalizing unitary matrices in (2.9) are only defined up to common phase transformations on U_L^p, U_R^p and U_L^n, U_R^n , respectively. Those phases may be chosen such that $P_u = P_d = 1$ which is nothing but (2.12).

In this basis, the relations (2.15) imply

$$\begin{aligned} U_{CP} &= U_L^p U_L^{pT} = U_L^n U_L^{nT} \\ V_{CP} &= U_R^p U_R^{pT} = U_R^n U_R^{nT} \end{aligned} \quad (2.16)$$

and therefore

$$K_L = K_L^*, K_R = K_R^* \quad (2.17)$$

completing the proof.

3. P AND CP CONSERVING YUKAWA COUPLINGS

The constraints on Yukawa couplings implied by general parity invariance were analyzed in Refs. 12),13) for the minimal Higgs sectors H_1 ($h=1$) and H_2 ($h=2$, but $\tilde{\Phi}_1, \tilde{\Phi}_2$ do not couple to quarks). It was shown that for all phenomenologically acceptable models with $n_G \leq 3$ there exists a basis of weak eigenstates such that the coupling matrices $\Gamma_\alpha, \Delta_\alpha$ are Hermitean. This basis corresponds to choosing $U_P = V_P = 1, H^P = 1$ for the parity transformations in flavour and Higgs space, respectively.

Demanding now both P and CP invariance, therefore amounts to finding all solutions of (2.6) in terms of Hermitean $\Gamma_\alpha, \Delta_\alpha$ if we restrict ourselves again to the minimal Higgs sectors H_1, H_2 . Obviously, there is always a solution for $U_{CP} = V_{CP} = 1, H^{CP} = 1$ in terms of real, symmetric $\Gamma_\alpha, \Delta_\alpha$ (manifest CP invariance). The purpose of this section is to determine all other solutions corresponding to non-aligned P and CP transformations in flavour and Higgs spaces. As in Refs. 12),13), we assume that all quarks are massive and that no generation decouples.

Let us first consider the Higgs sector H_1 . For $h = 1$ H^{CP} in (2.4) is determined by complex numbers A, B. Since unitarity implies $AB = 0$, $|A|^2 + |B|^2 = 1$ we have either $A = 0, B = e^{i\phi}$ or $A = e^{i\phi}, B = 0$.

For $A = 0, B = e^{i\phi}$ equations (2.6) reduce to

$$\begin{aligned} \Gamma^* &= e^{-i\phi} U_{CP}^\dagger \Delta V_{CP} = \Gamma^T \\ \Delta^* &= e^{i\phi} U_{CP}^\dagger \Gamma V_{CP} = \Delta^T \end{aligned} \quad (3.1)$$

implying

$$\text{tr } M_p M_p^\dagger = \text{tr } M_n M_n^\dagger \quad (3.2)$$

which is certainly excluded for $n_G \leq 3$.

We are thus left with $A = e^{i\phi}, B = 0$ and

$$\begin{aligned}\Gamma^* &= e^{i\psi} U_{CP}^\dagger \Gamma V_{CP} \\ \Delta^* &= e^{-i\psi} U_{CP}^\dagger \Delta V_{CP} .\end{aligned}\tag{3.3}$$

A basis transformation on the quark fields

$$q_{L(R)} \rightarrow W_{L(R)} q_{L(R)}\tag{3.4}$$

leads to

$$\begin{aligned}\Gamma &\rightarrow W_L^\dagger \Gamma W_R , \quad \Delta \rightarrow W_L^\dagger \Delta W_R \\ U_{CP} &\rightarrow W_L^\dagger U_{CP} W_L^* , \quad V_{CP} \rightarrow W_R^\dagger V_{CP} W_R^* .\end{aligned}\tag{3.5}$$

We may therefore choose $W_L = W_R$ such that Γ is real and diagonal and Δ is still Hermitean. In this basis, which we adopt henceforth, we get from (3.3)

$$[\Gamma^2, U_{CP}] = [\Gamma^2, V_{CP}] = 0 .\tag{3.6}$$

In discussing (3.3) we now distinguish different cases depending on the degree of degeneracy of Γ^2 . Without loss of generality, Δ^2 is assumed to be at least as degenerate as Γ^2 .

I) Γ^2 non-degenerate

In this case U_{CP} and V_{CP} are diagonal because of (3.6). By an appropriate phase transformation with diagonal $W_L = W_R$ in (3.5) $U_{CP} = 1$ can be achieved without changing Γ . Ordering the diagonal elements of Γ according to increasing absolute magnitude, the equations (3.3) reduce to

$$\begin{aligned}e^{i\psi} V_{CP} &= \text{diag} (e^{i\psi}, 1, \dots, 1) \\ \Delta^* &= e^{-2i\psi} \Delta \text{diag} (e^{i\psi}, 1, \dots, 1)\end{aligned}\tag{3.7}$$

where $(e^{i\psi}-1)\Gamma_{11} = 0$.

For $e^{i\psi} = 1$, (3.7) and Hermiticity imply

$$\Delta^* = e^{-2i\psi} \Delta = e^{2i\psi} \Delta. \quad (3.8)$$

For non-trivial mixing $\Delta \neq 0$ and thus $e^{2i\psi} = \pm 1$. In addition to the familiar solution of manifest CP invariance ($e^{2i\psi}=1$) we obtain a new solution for $e^{2i\psi} = -1$ with

$$\begin{aligned} \Gamma & \text{ real, diagonal} \\ \Delta & = -\Delta^* = -\Delta^T \end{aligned} \quad (3.9)$$

and the corresponding CP transformations $U_{CP} = 1$, $V_{CP} = \mp i$, $e^{i\psi} = \pm i$.

For $e^{i\psi} \neq 1$ we get $\Gamma_{11} = 0$ and

$$\Delta^* = e^{-2i\psi} \Delta \text{diag}(e^{i\psi}, 1, \dots, 1) = e^{2i\psi} \text{diag}(e^{-i\psi}, 1, \dots, 1) \Delta \quad (3.10)$$

from (3.7). To prevent decoupling of the first generation, we must have $e^{4i\psi} \neq 1$ so that Δ is of the form

$$\Delta = \begin{pmatrix} \Delta_{11} & \Delta_{1i} \\ \Delta_{1i}^* & 0 \end{pmatrix} \quad i=2, \dots, n_G \quad (3.11)$$

Non-trivial mixing requires $\Delta_{1i} \neq 0$ for at least one index i and therefore $e^{4i\psi} = e^{i\psi}$ which in turn implies $\Delta_{11} = 0$. We have thus obtained a further solution

$$\begin{aligned} \Gamma & = \text{diag}(0, g_2, \dots, g_{n_G}) \\ \Delta & = \begin{pmatrix} 0 & \Delta_{1i} \\ \Delta_{1i}^* & 0 \end{pmatrix} \quad i=2, \dots, n_G \end{aligned} \quad (3.12)$$

corresponding to CP transformations

$$V_{CP}=1, V_{CP}=\text{diag}(e^{3i\varphi}, e^{-i\varphi}, \dots, e^{-i\varphi}), e^{4i\varphi} \neq 1. \quad (3.13)$$

This completes the discussion for non-degenerate Γ^2 and arbitrary n_G . Since there is no mixing for n_G -fold degenerate Γ^2 and Δ^2 the case $n_G = 2$ is already settled. For $n_G = 3$ one further possibility remains to be discussed.

II) Γ^2 twice degenerate ($n_G=3$)

The analysis requires a little more work than in the non-degenerate case and is postponed to Appendix A. Up to basis transformations, a single model

$$\Gamma = \begin{pmatrix} 0 & a & 0 \\ a & 0 & 0 \\ 0 & 0 & b \end{pmatrix}, \quad \Delta = \begin{pmatrix} 0 & 0 & d \\ 0 & c & 0 \\ d & 0 & 0 \end{pmatrix} \quad (3.14)$$

emerges with real a, b, c, d . P and CP invariance may require $bc = 0$ but $b^2 + c^2 \neq 0$ is necessary for $\det M_{p,n} \neq 0$.

Let us recapitulate our results for the Higgs sector H_1 . P and CP invariance admit four inequivalent solutions for the coupling matrices :

- A) Γ real diagonal, $\Delta = \Delta^* = \Delta^T$ (manifest CP invariance)
 - B) Γ real diagonal, $\Delta = -\Delta^* = -\Delta^T$
 - C) Model (3.12)
 - D) Model (3.14)
- (3.15)

The models A,B,C apply for arbitrary n_G while model D is restricted to three generations.

The Higgs sector H_2 is characterized by coupling matrices Γ_1 , Γ_2 and $\Delta_1 = \Delta_2 \equiv 0$. For non-trivial mixing the CP invariance conditions (2.6) imply $B = 0$ so that we are left with

$$A_{\alpha\beta}^* \Gamma_\beta^* = U_{CP}^\dagger \Gamma_\alpha V_{CP} \quad \alpha, \beta = 1, 2 \quad (3.16)$$

where A is now unitary. It is immediately evident by comparison with (3.3) that all four solutions in (3.15) will also be solutions of (3.16) with the replacements $\Gamma, \Delta \rightarrow \Gamma_1, \Gamma_2$.

To find the additional solutions, we perform an orthogonal transformation on the scalar fields to bring A^* into the form

$$O^T A^* O = e^{i\varphi} \begin{pmatrix} \cos \theta & \sin \theta e^{-i\rho} \\ -\sin \theta e^{i\rho} & \cos \theta \end{pmatrix} \quad (3.17)$$

without destroying the Hermiticity of Γ_1, Γ_2 . It can easily be shown that for $\sin \theta \cos \rho = 0$ only the solutions (3.15) are possible and we therefore restrict ourselves to

$$\sin \theta \cos \rho \neq 0 \quad (3.18)$$

from now on.

In the basis (3.17) it is convenient to diagonalize (3.16) using

$$U_\Phi^\dagger A^* U_\Phi = e^{i\varphi} \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \quad (3.19)$$

$$U_\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ ie^{i\rho} & -ie^{i\rho} \end{pmatrix}$$

With $\Gamma_\alpha = U_{\Phi, \alpha\beta} E_\beta$ and absorbing the phase factor $e^{-i(\phi+\theta)}$ into V_{CP} , (3.16) is equivalent to

$$\begin{aligned} E_1^T &= U_{CP}^+ E_1 V_{CP} \\ E_2^T &= e^{2i\theta} U_{CP}^+ E_2 V_{CP} \end{aligned} \quad (3.20)$$

The price we have to pay consists of the more complicated Hermiticity conditions

$$\begin{aligned} E_1 &= a_1 E_1^+ + a_2 E_2^+ \\ E_2 &= a_2 E_1^+ + a_1 E_2^+ \end{aligned} \quad (3.21)$$

with $a_1 = i \sin \rho e^{-i\rho}$, $a_2 = \cos \rho e^{-i\rho}$ ($a_1 + a_2 = 1$). Because of (3.18) we get from (3.21)

$$E_2 = (E_1^+ - a_1^* E_1) / a_2^* \quad (3.22)$$

Inserting (3.22) into (3.20) we obtain the final form of the CP invariance conditions

$$E_1^T = Z_{CP}^T V_{CP}^+ E_1 V_{CP} \quad (3.23a)$$

$$e^{-2i\theta} E_1^+ - Z_{CP} E_1^+ Z_{CP} = a_1^* (e^{-2i\theta} - 1) E_1 \quad (3.23b)$$

with $Z_{CP} = V_{CP}^T U_{CP}^*$.

On the other hand, the original equations (3.16) imply

$$U_{CP}^+ \sum_{\alpha=1}^2 \Gamma_{\alpha}^2 U_{CP} = V_{CP}^+ \sum_{\alpha=1}^2 \Gamma_{\alpha}^2 V_{CP} = \sum_{\alpha=1}^2 \Gamma_{\alpha}^{*2} \quad (3.24)$$

Considering first the case of non-degenerate $\sum_{\alpha} \Gamma_{\alpha}^2$ (n_G arbitrary), we find from (3.24) that U_{CP} , V_{CP} and therefore also Z_{CP} are diagonal in the basis where $\sum_{\alpha} \Gamma_{\alpha}^2$ is diagonal. But in this case

$$E_1^+ = Z_{CP} E_1^+ Z_{CP} \quad (3.25)$$

from (3.23a) so that (3.23b) can be written

$$(e^{-2i\theta} - 1) E_1^\dagger = a_1^* (e^{-2i\theta} - 1) E_1. \quad (3.26)$$

Because of (3.18) only the trivial solution $E_1 \equiv 0$ satisfies (3.26). Thus, there are no solutions in addition to (3.15) for a non-degenerate $\sum_{\alpha} \Gamma_{\alpha}^2$. In particular, there are no further solutions at all for $n_G = 2$ because a completely degenerate $\sum_{\alpha} \Gamma_{\alpha}^2$ leads to trivial mixing.

For $n_G = 3$, the case of a twice degenerate $\sum_{\alpha} \Gamma_{\alpha}^2$ requires a tedious, but straightforward analysis which is sketched in Appendix B. The final outcome is one (always up to basis transformations) further model

$$E: \quad \Gamma_1 = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & b \\ 0 & a & 0 \\ b & 0 & 0 \end{pmatrix} \quad (3.27)$$

with real a, b .

To summarize the results of this section, we have shown that for both minimal Higgs sectors H_1 and H_2 , P and CP invariance admit four inequivalent solutions A, B, C, D for the coupling matrices Γ , Δ or Γ_1, Γ_2 and one additional solution E for H_2 only. The models A, B, C are valid for arbitrary n_G whereas D, E are special solutions for $n_G = 3$.

4. PHENOMENOLOGY OF MASS MATRICES

Solution A of Section 3 is the standard case of manifest CP invariance with real, symmetric Yukawa coupling matrices. The ensuing mass matrices are symmetric and can therefore be diagonalized with $U_R^{p(n)} = U_L^{p(n)*}$ in (2.9). Consequently, the left- and right-handed mixing matrices can be chosen to obey $K_R = K_L^*$ but there are in general no further constraints on mixing angles or quark masses.

Although of pedagogical interest only, let us look for a moment at $n_G = 2$. It turns out that the two remaining solutions B, C are identical to the models predicted by horizontal symmetries^{12), 13)}. For the Higgs sector H_1 only B is compatible with the known quark masses and it implies the bounds

$$\left(m_d/m_s - m_u/m_c\right)^{1/2} \lesssim \theta_c \lesssim \left(m_d/m_s + m_u/m_c\right)^{1/2} \quad (4.1)$$

for the Cabibbo angle θ_c in well-known agreement with experiment. On the other hand, for the Higgs sector H_2 only model C leads to a constraint on θ_c which is, however, weaker than (4.1). Actually, the first attempt to predict the Cabibbo angle within a gauge theory framework¹⁴⁾ was based on model C.

The remarkable feature of $n_G = 2$ is that, for minimal Higgs sectors H_1 or H_2 , all models with horizontal symmetry can be derived from P and CP invariance alone. In other words, P and CP invariance explain the existence of a horizontal symmetry.

Before discussing the different models for $n_G = 3$, we briefly recall the status of the Kobayashi-Maskawa mixing angles. An important step forward has been made recently by the measurements of the B-meson lifetime¹⁵⁾. Quoting from a recent combined analysis¹⁶⁾ of all available data, the following discussion will be based on the one standard deviation domains

$$\begin{aligned} s_1 &= 0.231 \pm 0.003 \\ 0.015 &< s_2 < 0.09 \\ s_3 &< 0.04 \end{aligned} \quad (4.2)$$

for the KM parameters $s_i = \sin\theta_i$. Quark masses are to be interpreted as current quark masses and we take $m_t/m_b \gtrsim 5$ as a lower bound for m_t .

Starting with the last model first, we recognize E in (3.27) as a model obtained in Ref. 13) from a horizontal symmetry. However, it leads to the unacceptable mass relation $m_t/m_b \approx (m_u m_c / m_d m_s)^{1/2}$. Likewise, model D in (3.14) was derived earlier from horizontal symmetries^{12), 13)}. It is incompatible with the lower bound on m_t for the Higgs sector H_1 and barely compatible with the upper bound on s_3 for H_2 . Moreover, it does not really contain a prediction for s_1 .

Continuing backwards in the list (3.15), also C turns out to correspond to a model with a horizontal symmetry. It is ruled out experimentally for H_1 ¹²⁾, but it exhibits an unusual feature for the Higgs sector H_2 ¹³⁾: all mixing angles depend very strongly on m_t . The precisely known s_1 and the bounds for s_2, s_3 in (4.2) [the model predicts $s_2 \approx s_3$] then require the approximate mass relation $m_t \approx m_b m_c / m_s$. With quark

masses from Ref. 17) and employing first order renormalization group equations [see also Ref. 18)], $m_b m_c / m_s$ corresponds to a constituent mass of approximately 27 GeV (for $\Lambda_{\text{QCD}} = 100$ MeV) with rather large uncertainties mainly due to the poorly determined value of m_s . Although the model does not comply with the usual prejudice that the Cabibbo angle as a measure of mixing among the lighter quarks should not depend drastically on m_t , it is in fact the only completely constrained model for the Higgs sector H_2 which is fully consistent with (4.2).

Model B in (3.15) deserves closer investigation. The coupling matrices for the Higgs sector H_1 ^{*)} are

$$\begin{aligned} \Gamma &= \text{diag}(g_1, g_2, g_3) \\ \Delta &= i \begin{pmatrix} 0 & h_1 & h_2 \\ -h_1 & 0 & h_3 \\ -h_2 & -h_3 & 0 \end{pmatrix} \end{aligned} \quad (4.3)$$

with real g_i, h_i and $|g_1| \leq |g_2| \leq |g_3|$ without loss of generality. It is convenient to redefine the phases of weak eigenstates in such a way that in the new basis

$$\begin{aligned} \Gamma &= \text{diag}(g_1, g_2, g_3) \\ \Delta &= \begin{pmatrix} 0 & h_1 & -ih_2 \\ h_1 & 0 & h_3 \\ ih_2 & h_3 & 0 \end{pmatrix} \end{aligned} \quad (4.4)$$

with $g_3 > 0$, $h_1 \geq 0$, $h_3 \geq 0$ and real g_1, g_2, h_2 . Although this model has not been treated before, it reduces to a known model with horizontal symmetry ¹²⁾ in the limit ^{**) h₂ → 0.}

Let us recall the relevant properties of the mass matrices for $h_2 = 0$ from Ref. 12). First of all, the model predicts s_1 to be practically independent of m_t and to good accuracy $s_1 \approx \sqrt{m_d/m_s}$. In addition, one finds the approximate bounds

$$|m_s/m_b - m_c/m_t| \lesssim s_2^2 \lesssim m_s/m_b + m_c/m_t \quad (4.5)$$

for s_2 . Since $\sqrt{m_s/m_b} \approx 0.2$ these bounds are consistent with (4.2) only if m_t is of the same order as $m_c m_b / m_s$. Finally, the model yields an approximate relation

*) We do not consider model B for the Higgs sector H_2 because in that case the mass matrices contain too many parameters to be of phenomenological interest.

**) As a matter of fact, the model exhibits a horizontal symmetry for any $h_1 = 0$, but $h_2 = 0$ is the most interesting case phenomenologically ¹²⁾.

$$S_3 \simeq S_2 m_s/m_e \quad (4.6)$$

and therefore a very small s_3 in agreement with (4.2). This relation also ensures that CP violating effects from the left-handed KM-matrix are naturally small.

Contrary to the horizontal symmetry invoked in Ref. 12), P and CP invariance alone do not force $h_2 = 0$ in (4.4). It is clear that with one additional parameter some predictive power is lost. For arbitrary values of h_2 the model will actually look completely different from what we have described so far. However, it turns out that some features are relatively insensitive to h_2 as long as $|h_2| \lesssim h_1$. We have performed an approximate analytic diagonalization and checked the outcome numerically. For $|h_2| \lesssim h_1$ the following results were obtained :

- i) $s_1 \simeq \sqrt{m_d/m_s}$ is rather insensitive to h_2 ;
- ii) the bounds (4.5) for s_2 are practically unchanged and thus again $m_t = 0$ ($m_b m_c/m_s$) for consistency;
- iii) for $|h_2|/h_1 > s_2$ one gets approximately

$$S_3 \simeq \frac{|h_2| m_s}{h_1 m_e} \quad (4.7)$$

instead of (4.6) and therefore $|h_2| \lesssim h_1$ to satisfy (4.2);

- iv) for $h_2 = 0$, Γ, Δ in (4.4) are symmetric and thus $K_R = K_L^*$ with a certain choice of phases. For $h_2 \neq 0$ one finds $K_R \neq K_L^*$ but the differences between the right- and left-handed mixing angles are relatively small.

Although the model does not predict the magnitude of h_2/h_1 , the mass matrices are consistent with all experimental constraints on quark masses and weak mixing angles for a finite range of values of h_2/h_1 . We are not aware of many other constrained models which allow similar statements to be made.

In addition to the manifestly CP invariant case, we have thus found only two phenomenologically acceptable P and CP invariant models for $n_G = 3$: model B for Higgs sector H_1 and model C for H_2 , in complete analogy to $n_G = 2$.

5. CONCLUSIONS

In the introduction we have pointed out two theoretical arguments in favour of spontaneous versus explicit hard CP violation. There may be another even more compelling reason to reject the CP violation mechanism of the standard model. The rather small mixing parameters s_2, s_3 due to the long B-meson lifetime could make it difficult or even impossible to account for the magnitude of CP violation in the K^0 system unless m_t is very heavy¹⁹⁾.

A minimal extension of the standard model which both exhibits spontaneous CP violation and circumvents the phenomenological problem in the K^0 system is provided by the $SU(2)_L \times SU(2)_R \times U(1)$ gauge theory with a CP conserving Lagrangian and with a single scalar multiplet Φ (Higgs sector H_1). In this case there is again a single phase $\arg(vw)$ which is responsible for all CP violating effects^{12), 20)}. However, in contrast to the standard model CP is now violated also in the right-handed gauge interactions. If M_{W_R} is not too heavy, these contributions cannot be neglected in general. In a recent paper⁸⁾ an approximate upper bound of 20 TeV was estimated for M_{W_R} along these lines.

Concentrating therefore on the minimal Higgs sector H_1 we have shown that there are only two possibilities for a P and CP conserving Lagrangian which are still phenomenologically acceptable. The first possibility is the standard case of manifest CP invariance with real, symmetric Yukawa coupling matrices. The left- and right-handed mixing matrices are related by $K_R = K_L^*$ but there are no further constraints on quark masses and mixing angles.

The alternative scenario for spontaneous CP violation is given by model B in (3.15) with one of the coupling matrices real and symmetric, but the other one purely imaginary and antisymmetric. For $n_G = 2$ this model turns out to be equivalent to the unique horizontal symmetry model¹²⁾ which determines θ_c in terms of quark masses. For $n_G = 3$ there is one additional

parameter compared to the unique model found earlier¹²⁾ from horizontal symmetries. For a certain range of this extra parameter the model reproduces the successful prediction $s_1 \approx (m_d/m_s)^{\frac{1}{2}}$ and satisfies the experimental bounds for s_2, s_3 as long as m_t is of the order of $m_b m_c / m_s$. Although $K_R \neq K_L^*$ the differences between the right- and left-handed mixing angles are rather small.

In this paper we have exhibited a possible connection between the discrete symmetries P and CP and horizontal symmetries. The mechanism is based on non-aligned P and CP transformations in flavour space and in the space of Higgs fields. It requires a theory where both P and CP are violated only spontaneously and therefore enjoy the same status as the gauge symmetry. We have investigated this approach to the flavour problem in the simplest $SU(2)_L \times SU(2)_R \times U(1)$ gauge theory but we expect the underlying idea to be of more general relevance.

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APPENDIX A

In this Appendix the main features of the analysis for a twice degenerate Γ^2 (Higgs sector H_1 , $n_G=3$) will be sketched.

We choose to work in a basis where $\Gamma = \text{diag}(g_1, \varepsilon g_1, g_3)$ with $\varepsilon = \pm 1$. Without loss of generality, $g_1 \neq 0$ to avoid massless quarks. From (3.3) and (3.6) one obtains

$$U_{CP} = \begin{pmatrix} U_1 & 0 \\ 0 & 1 \end{pmatrix}, \quad e^{i\varphi} V_{CP} = \begin{pmatrix} V_1 & 0 \\ 0 & e^{i\beta} \end{pmatrix}$$

$$U_1 = \begin{pmatrix} \cos\theta & \sin\theta e^{-i\rho} \\ -\sin\theta e^{i\rho} & \cos\theta \end{pmatrix}, \quad V_1 = \begin{pmatrix} \cos\theta & \varepsilon \sin\theta e^{-i\rho} \\ -\varepsilon \sin\theta e^{i\rho} & \cos\theta \end{pmatrix} \quad (\text{A.1})$$

with a proper choice of phases. It is easily seen that CP invariance cannot enforce degenerate Γ^2 , Δ^2 unless $\sin\theta \neq 0$. Writing Δ as

$$\Delta = \begin{pmatrix} \Delta_1 & d \\ d^\dagger & d_{33} \end{pmatrix}$$

with a two-dimensional matrix Δ_1 and $d^T = (d_1, d_2)$, the equations (3.3) take the form

$$g_3 (1 - e^{i\beta}) = 0 \quad (\text{A.2})$$

and

$$\begin{aligned} \Delta_1^* &= e^{-2i\varphi} U_1^\dagger \Delta_1 V_1 \\ U_1 d^* &= e^{i(\beta-2\varphi)} d \\ V_1^T d^* &= e^{2i\varphi} d \\ d_{33} (1 - e^{i(\beta-2\varphi)}) &= 0. \end{aligned} \quad (\text{A.3})$$

From (A.3) we deduce

$$(U_1 - U_1^T) d^* = 0 \quad (\text{A.4})$$

which implies $e^{i\rho} = -e^{-i\rho} = \pm i$ and $U_1 = U_1^T$, $V_1 = V_1^T$ because of $\sin\theta \neq 0$ and $d \neq 0$ to prevent flavour decoupling. For $\varepsilon = 1$ ($V_1 = U_1$) we can perform a further basis transformation to attain $U_1 = V_1 = 1$ contrary to the assumption $\sin\theta \neq 0$.

For $\varepsilon = -1$, one gets $V_1 = U_1^*$ and

$$U_1 = \begin{pmatrix} \cos \theta & -i \sin \theta \\ -i \sin \theta & \cos \theta \end{pmatrix}$$

leading to

$$\begin{aligned} e^{i(\beta - 4\varphi)} &= e^{2i\eta\theta} \\ \cos \theta (\eta d_1 + d_2) &= 0 \end{aligned} \quad \eta = \pm 1 \quad (\text{A.5})$$

as a consequence of (A.3).

Considering $\cos \theta = 0$ first, some straightforward algebra shows that only $e^{i\beta} = -e^{4i\phi} = 1$ can produce a solution with twice degenerate Γ^2 , Δ^2 and $\det M_{p,n} \neq 0$. The model is given by

$$\begin{aligned} \Gamma &= \text{diag}(g_1, -g_1, g_3) \\ \Delta &= \begin{pmatrix} 0 & 0 & d_1 \\ 0 & 0 & \pm d_1^* \\ d_1^* & \pm d_1 & 0 \end{pmatrix} \end{aligned} \quad (\text{A.6})$$

and it is equivalent to (3.14) with $c = 0$ after a basis transformation.

For the same reasons as before, it turns out that for $\sin \theta \cos \theta \neq 0$, acceptable solutions of (A.3) are only possible for

$$\begin{aligned} e^{2i\varphi} &= e^{2i\eta\theta} \neq \pm 1 \\ e^{i\beta} &= e^{6i\varphi} \end{aligned} \quad (\text{A.7})$$

leading to

$$\begin{aligned} \Gamma &= \text{diag}(g_1, -g_1, g_3) \\ \Delta &= \begin{pmatrix} \delta_1 & \eta \delta_1 & d_1 \\ \eta \delta_1 & \delta_1 & -\eta d_1 \\ d_1^* & -\eta d_1^* & 0 \end{pmatrix} \end{aligned} \quad (\text{A.8})$$

with $g_3(1 - e^{6i\phi}) = 0$. Once again, (A.8) is identical to (3.14) after a basis transformation.

APPENDIX B

The main steps of the analysis of Higgs sector H_2 with a twice degenerate $\sum_{\alpha}^2 (n_G=3)$ will be outlined here.

Recalling the definition $Z_{CP} = V_{CP}^T U_{CP}^*$ we infer from (3.24) that we may choose a basis with

$$\begin{aligned} Z_{CP} &= \text{diag} (e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}) \\ V_{CP} &= \begin{pmatrix} V_1 & 0 \\ 0 & 1 \end{pmatrix}. \end{aligned} \quad (B.1)$$

If V_1 is diagonal the analysis proceeds as in the non-degenerate case and we therefore assume a non-diagonal V_1 from now on.

Parametrizing E_1 as

$$\begin{aligned} E_1 &= \begin{pmatrix} E & e \\ f^T & e_{33} \end{pmatrix}, \quad E = (E_{ij}) \quad i,j=1,2 \\ e^T &= (e_1, e_2), \quad f^T = (f_1, f_2) \end{aligned} \quad (B.2)$$

one finds from (3.23)

$$\begin{aligned} e_{33} &= e^{i\alpha_3} e_{33} \\ (e^{-2i\theta} - e^{2i\alpha_3}) e_{33}^* &= a_1^* (e^{-2i\theta} - 1) e_{33}. \end{aligned} \quad (B.3)$$

Keeping in mind $|a_1| < 1$ and $e^{2i\theta} \neq 1$ because of (3.18), the relations (B.3) imply $e_{33} = 0$. The remaining equations to be solved are then

$$(e^{-2i\theta} - e^{i(\alpha_i + \alpha_j)}) E_{1,ji}^* = a_1^* (e^{-2i\theta} - 1) E_{1,ij} \quad i,j=1,2,3 \quad (B.4a)$$

$$\text{diag} (e^{-i\alpha_1}, e^{-i\alpha_2}) E^T = V_1^+ E V_1 \quad (B.4b)$$

$$\text{diag} (e^{-i\alpha_1}, e^{-i\alpha_2}) f = V_1^+ e \quad (B.4c)$$

$$e^{-i\alpha_3} e = V^T f. \quad (B.4d)$$

Considering $a_1 = 0$ first, we get

$$e^{i\alpha_1} = e^{i\alpha_2} = e^{i\alpha_3} = \varepsilon e^{-i\theta} \quad (\varepsilon = \pm 1) \quad (B.5)$$

from (B.4a) because we must have $e \neq 0$, $f \neq 0$, $E \neq 0$ for all quarks to be massive. The relations (B.5) permit a further basis transformation which we use to set $e^T = (e_1, 0)$. From (B.4c) and (B.4d) one then obtains

$$V_1 = \begin{pmatrix} 0 & e^{i\alpha_1} \\ e^{-i\alpha_1} & 0 \end{pmatrix}, \quad f^T = (0, e_1) \quad (B.6)$$

with a convenient choice of phases. Finally, (B.4b) and $E \neq 0$ lead to

$$E_1 = \begin{pmatrix} 0 & 0 & e_1 \\ E_{21} & 0 & 0 \\ 0 & e_1 & 0 \end{pmatrix}, \quad E_2 = E_1^+ \quad (B.7)$$

with $e^{3i\alpha_1} = 1$ (but $e^{i\alpha_1} \neq 1$). Through a basis transformation, (B.7) is equivalent to (3.27).

For $0 < |a_1| < 1$ (B.4a) implies

$$|E_{1,ij}| = |E_{1,ji}|. \quad (B.8)$$

Remembering that V_1 is non-diagonal, some algebra shows that the equations (B.4) can only have a solution for

$$\begin{aligned} e^{i\alpha_2} &= e^{-i\alpha_1} \neq \pm 1 \\ e^{i\alpha_3} &= \varepsilon e^{-2i\theta} \quad (\varepsilon = \pm 1) \end{aligned} \quad (B.9)$$

From (B.4a), (B.4b) and (B.9), one derives

$$\begin{aligned} E_{12} &= E_{21} = 0 \\ E_{22} &= e^{-i\alpha_1} E_{11} \end{aligned} \quad (B.10)$$

On the other hand, (B.4c), (B.4d) and (B.8) imply $e^{2i\theta} = -1$ and

$$|e_1| = |e_2| = |f_1| = |f_2|. \quad (B.11)$$

With a proper choice of phases we can achieve $e_1 = e_2 > 0$.

The remaining equations (B.4a) now assume the simpler form

$$(1 + e^{i(\alpha_i + \alpha_j)}) E_{1,ji}^* = 2a_i^* E_{1,ij} \quad (B.12)$$

which have non-trivial solutions only for

$$e^{i\alpha_1} = \varepsilon e^{-i\pi t/3} \quad (t = \pm 1), \quad a_1 = \frac{is}{2} e^{-i\pi s/6} \quad (s = \pm 1) \quad (\text{B.13})$$

taking $\text{cosp } 0$ without loss of generality. The solutions are

$$\begin{aligned} E_{11}^* &= -is e^{i\pi(s+2t)/6} E_{11} \\ f_1^* &= -ste_1 e^{i\pi(s+t)/6}, \quad f_2^* = ste_1 e^{i\pi(s-t)/6} \end{aligned} \quad (\text{B.14})$$

and they yield the coupling matrices (3.27) after appropriate basis transformations.

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